A lattice Boltzmann method for turbulent flow simulation

A LATTICE BOLTZMANN METHOD FOR TURBULENT FLOW SIMULATION

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INTRODUCTION

The lattice Boltzmann method (LBM) is free from difficulties of grid generation and of convergence of a pressure solver since it usually applies regular cubic grids and does not require the Poisson equation for pressure fields. The LBM is based on a discretized Boltzmann equation expressed by an explicit time-marching formula and hence it is very much suitable for parallelized computation with multi GPUs.

However, because the LBM is young compared with the other CFD methods, it still has many important issues to overcome, particularly for turbulence simulation. Therefore, in the present report, we point out some of those issues and show how to overcome them referring to our recent studies.

D3Q27 MRT LBM

For three-dimensional flows, the LBM usually applies the D3Q15, D3Q19 or D3Q27 model. (D3 corresponds to three-dimensional and Qn corresponds to the number of discrete velocity vectors.) If the simulated results are virtually the same, the smaller number of the discrete velocities is desirable due to the computational costs. Indeed, for laminar pipe flows, as seen in Fig.1, all those discrete velocity models produce ideally symmetrical velocity profiles. However, when the flow becomes turbulent, the D3Q15 and D3Q19 models produce unphysically patterned profiles. Only the D3Q27 model predicts a reasonable velocity profile. The reason why such anomaly occurs was analysed by the authors [1] as follows.

Applying the error analysis method of Holdych et al. (2004) to each discrete velocity model, the error term $\mathscr{E}_i$ arising in the momentum equation:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u_i}{\partial x_j^2} + \mathscr{E}_i, \tag{1}$$

was estimated. After projecting the error terms to the cylindrical coordinate, the circumferential components were written with the relaxation time τ of the LBM:

$$\mathscr{E}_\theta^{D3Q15} = \Delta^2 \frac{(-6\tau^2+6\tau-1)}{6}\left[\sin 4\theta \left\{ \frac14 \left(\frac{\partial^3}{\partial r^3} + \frac{3}{r}\frac{\partial^2}{\partial r^2} - \frac{3}{r^2}\frac{\partial}{\partial r}\right)\left(\overline{u_z}^2 + \overline{u'_z u'_z}\right) - \frac14 \left(-\frac{\partial^3}{\partial r^3} + \frac{4}{r}\frac{\partial^2}{\partial r^2} - \frac{8}{r^2}\frac{\partial}{\partial r} + \frac{4}{r^3}\right) \times \overline{u'_r u'_r} - \frac14 \left(-\frac{1}{r}\frac{\partial^2}{\partial r^2} + \frac{5}{r^2}\frac{\partial}{\partial r} - \frac{4}{r^3}\right)\overline{u'_\theta u'_\theta}\right\}\right] + O(\Delta^4), \tag{2}$$

$$\mathscr{E}_\theta^{D3Q19} = \Delta^2 \frac{(-6\tau^2+6\tau-1)}{6}\left[\sin 4\theta \left\{ -\frac14 \left(\frac{\partial^3}{\partial r^3} + \frac{3}{r}\frac{\partial^2}{\partial r^2} - \frac{3}{r^2}\frac{\partial}{\partial r}\right)\left(\overline{u_z}^2 + \overline{u'_z u'_z}\right) - \frac14 \left(-\frac{\partial^3}{\partial r^3} + \frac{4}{r}\frac{\partial^2}{\partial r^2} - \frac{8}{r^2}\frac{\partial}{\partial r} + \frac{4}{r^3}\right) \times \overline{u'_r u'_r} - \frac14 \left(-\frac{1}{r}\frac{\partial^2}{\partial r^2} + \frac{5}{r^2}\frac{\partial}{\partial r} - \frac{4}{r^3}\right)\overline{u'_\theta u'_\theta}\right\}\right] + O(\Delta^4), \tag{3}$$

$$\mathscr{E}_\theta^{D3Q27} = \Delta^2 \frac{(-6\tau^2+6\tau-1)}{6}\left[\sin 4\theta \left\{ \frac14 \left(\frac{\partial^3}{\partial r^3} - \frac{4}{r}\frac{\partial^2}{\partial r^2} + \frac{8}{r^2}\frac{\partial}{\partial r} - \frac{4}{r^3}\right)\overline{u'_r u'_r} + \frac14 \left(\frac{1}{r}\frac{\partial^2}{\partial r^2} - \frac{5}{r^2}\frac{\partial}{\partial r} + \frac{4}{r^3}\right)\overline{u'_\theta u'_\theta}\right\}\right] + O(\Delta^4), \tag{4}$$

respectively for the D3Q15, D3Q19 and D3Q27 models. When the pipe flow is laminar, since the Reynolds stresses are negligible, the error term of the D3Q27 model (Eq.4) vanishes and the total contributions by the parabolic profile of the streamwize mean velocity $\bar{u}_z$ in Eqs.2,3 also vanish. Accordingly, the space accuracy of all those models become $O(\Delta^4)$ or higher. However, in turbulent pipe flows all those contributions by the Reynolds stress and the mean velocity are not negligible and the errors act as asymmetrical force terms. In this sense, even the D3Q27 model is not satisfactory. However, it was found that the magnitude of the error term of the D3Q27 model is two order smaller than those of the other models. Thus, the D3Q27 model is desirable for turbulence simulation.

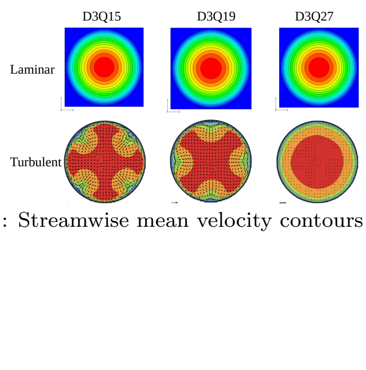

Figure 1: Streamwise mean velocity contours of pipe flows.

For high Reynolds number flows, it is known that the multiple-relaxation-time (MRT) form of the LBM is desirable:

$$|\mathbf{f}(\mathbf{x}+\xi_\alpha \delta t, t+\delta t)\rangle - |\mathbf{f}(\mathbf{x},t)\rangle = -\mathbf{M}^{-1}\widehat{\mathbf{S}}\left[|\mathbf{m}(\mathbf{x},t)\rangle - |\mathbf{m}^{eq}(\mathbf{x},t)\rangle\right] + \mathbf{M}^{-1}\left(\mathbf{I} - \frac{\widehat{\mathbf{S}}}{2}\right)\mathbf{M}\,|\mathbf{F}\rangle\,\delta t, \tag{5}$$

where ξ_α is the discrete velocity and $\mathbf{M}$ is a $Q \times Q$ matrix that transforms the distribution function $\mathbf{f}$ to moments $\mathbf{m}$ as $|\mathbf{m}\rangle = \mathbf{M} \cdot |\mathbf{f}\rangle$. For the D3Q27 model, the present authors' group derived a 27×27 transformation matrix [2]. The diagonal collision matrix $\hat{\boldsymbol{S}}$ is:

$$\hat{\boldsymbol{S}} \equiv diag(0,0,0,0,s_4,s_5,s_5,s_7,s_7,s_7,s_{10},s_{10},s_{10},s_{13}, s_{13},s_{13},s_{16},s_{17},s_{18},s_{20},s_{20},s_{20},s_{23},s_{23},s_{23},s_{26}), \tag{6}$$

and the proposed relaxation parameters [2] are

$s_4 = 1.54, \quad s_7 = s_5, \quad s_{10} = 1.5, \quad s_{13} = 1.83, \quad s_{16} = 1.4,$

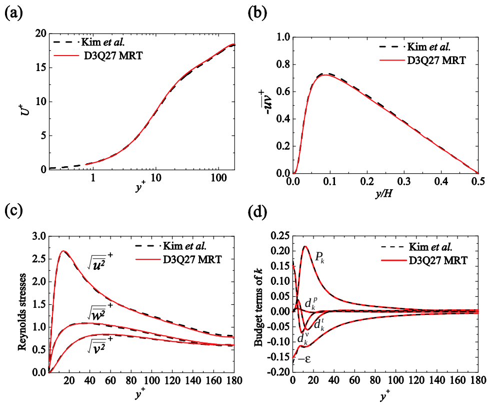

Figure 2: Turbulent channel flow quantities at Re_τ=180.

$$s_{17} = 1.61, \quad s_{18} = s_{20} = 1.98, \quad s_{23} = s_{26} = 1.74. \tag{7}$$

The relaxation parameter s_5 is related to the fluid viscosity, which is sum of the molecular viscosity ν and the sub-grid-scale viscosity ν_{SGS} for the LES, $\nu + \nu_{SGS} = c_s^2 \left(\frac{1}{s_5} - \frac{1}{2}\right) \delta t$, where c_s is the sound speed of the LBM.

The accuracy of the D3Q27 MRT LBM for turbulent channel flow DNS is confirmed in Fig.2. DNS was performed by regular grids of $1539 \times 240 \times 754$ for the region of $4\pi\delta \times 2\delta \times 2\pi\delta$ with the resolution of $\Delta^+ = 1.5$. Indeed, the identical data to those by Kim et al. (1987) were obtained.

IBC LOCAL MESH REFINEMENT

To resolve boundary layers, local mesh refinement is essential for the method employing regular grids. In the context of the LBM, the widely applied local mesh refinement applies the communication rule between the distribution functions of course and fine grids: $f_\alpha^{eq,c} \Leftrightarrow f_\alpha^{eq,f}, f_\alpha^{neq,c} \Leftrightarrow n\frac{\tau^c}{\tau^f} f_\alpha^{neq,f}$, with $n = \Delta^c/\Delta^f = \delta t^c/\delta t^f$ where superscripts c and f correspond to "course" and "fine" grids. The non-equilibrium distribution function is $f_\alpha^{neq} = f_\alpha - f_\alpha^{eq}$.

The well known difficulty of the conventional method by Dupuis and Chopard (2003) is that kinked profiles of turbulent flow quantities are inevitable at the interface of the grids. The present authors pointed out the reason of the difficulty and proposed the following correction method [3].

The conditions of mass and momentum conservation: $\Sigma_\alpha f_\alpha^{neq} = 0, \Sigma_\alpha \xi_\alpha f_\alpha^{neq} = \mathbf{0}$ are not always satisfied at the interface of the conventional method. To correct such imbalances, the residuals were calculated at the interface shown in Fig.3 as

$$\varepsilon_\rho = \sum_{\beta c} f_{\beta c}^{neq,c} + \sum_{\hat{\beta} c} f_{\hat{\beta} c}^{neq,f\to c}, \tag{8}$$

$$\varepsilon_{\mathbf{u}} = \sum_{\beta c} \xi_{\beta c} f_{\beta c}^{neq,c} + \sum_{\hat{\beta} c} \xi_{\hat{\beta} c} f_{\hat{\beta} c}^{neq,f\to c}. \tag{9}$$

When $\varepsilon_\rho, \varepsilon_{\mathbf{u}}$ vanish, the conservation of mass and momentum is achieved. Hence, by setting the re-calculated residuals with the corrected density and velocity: $\rho* = \rho + \lambda_\rho \varepsilon_\rho, \mathbf{u}* = \mathbf{u} + \lambda_{\mathbf{u}} \varepsilon_{\mathbf{u}}$, to zero, the correction coefficients were obtained as

$$\lambda_\rho = \lambda_{\mathbf{u}x} = \left\{1 - \frac{1}{6}\left(1 - n\tau^c/\tau^f\right)\left(1 + \frac{c^2}{c_s^2}\right)\right\}^{-1}, \tag{10}$$

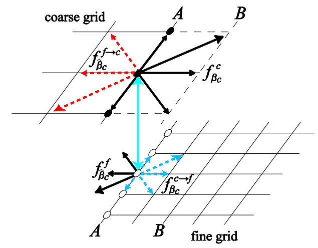

Figure 3: Stencil of the IBC local mesh refinement.

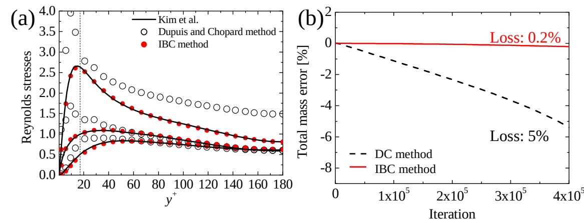

Figure 4: Performance of the IBC method in LES; grid interface is set at $y^+ = 17.7$.

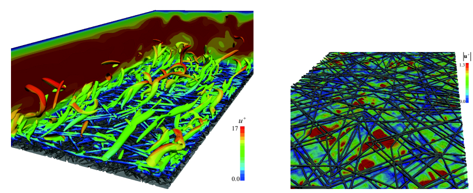

Figure 5: LES of turbulence inside and over a carbon paper at Re_b=3000: left: iso-surfaces of the second invariant of the velocity gradient tensor, right: close-up view of carbon paper.

$$\lambda_{\mathbf{u}y} = \lambda_{\mathbf{u}z} = \left\{1 - \frac{1}{18}\left(1 - n\tau^c/\tau^f\right)\left(\frac{c^2}{c_s^2}\right)\right\}^{-1}, \tag{11}$$

where $c = \Delta/\delta t$. Those corrected variables were used for the simulation.

The performance of this imbalance correction (IBC) method is shown in Fig.4. It does not produce kinks in the Reynolds stress distributions and improves mass conservation drastically.

CONCLUSIONS

For turbulent flow simulation by the LBM, the D3Q27 MRT LBM with the IBC mesh refinement method is recommended. It is certainly very powerful for turbulence simulation in a very complex flow region such as that shown in Fig.5.

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