Stabilization of Synchronous Machine through Integral Action

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Katsumi YAMASHITA* and Tsuneo TANIGUCHI**

(Received June 16, 1980)

Integral control proposed by C.D. Johnson[1] takes a feedback form of all state variables. In power systems, however, it is difficult to measure all state variables, and then his controller is not practically acceptable. It is the purpose of the present paper to design an integral controller based on quantities easily measurable at the generator location, using the algorithm of C.D. Johnson for optimization. This controller will be physically realizable and practical to implement.

List of Principal Symbols

M =machine (generator) inertia constant, second.
M_u =machine (induction motor) inertia constant, second.
x_d =p.u. direct axis synchronous reactance.
x'_d =p.u. direct axis transient reactance.
x' $=x_{11}-x_{12}^2/x_{22}$.
$\dot{E}_1$ =p.u. internal voltage behind transient reactance x'_d.
$\dot{E}_3$ =p.u. internal voltage behind transient reactance x'.
V_t =p.u. machine (generator) terminal voltage.
T'_{d0} =direct axis field time constant, second.
T_0 $=x_{22}/(\omega_0 r_{22})$.
ω_0 $=2\pi f_0$ ($f_0=60$ Hz).
Δ =incremental operator.

1. Introduction

In recent years, considerable works[2),3)] have been done on the application of optimal control theory to the design of regulators for linear machine models. A great part of the previous works, however, are based on proportional feedback of the state variables, and the equilibrium condition cannot be maintained by such a method of feedback, if a constant external disturbance is present.

C.D. Johnson[1)] proposed the integral controller to elminate this defect, but his controller was not usually accepted in power systems, because it assumed that all state variables of the systems were available for control.

This paper presents the method for designing on integral controller based on only

* Speciality Student, Department of Electrical Engineering, College of Engineering.
** Department of Electrical Engineering, College of Engineering.

those variables that are easily measurable at the generator location, using the algorithm of C.D. Johnson for optimization. The proposed controller is applied to stabilization of a synchronous generator with dynamic behavior of load. The results are then compared with those given by the optimal control in the conventional linear regulator problem[4], and the proposed controller is found to be useful practically in power systems.

2. Suboptimal Control

Consider a linearized power system model described by the state variable form

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{Ax}+\boldsymbol{Bu}+\boldsymbol{Dw}\,, \qquad \boldsymbol{y} = \boldsymbol{Cx}\,, \tag{1}$$

where $\boldsymbol{x}$ is the n-dimensional state vector, $\boldsymbol{y}$ is the m-dimensional measurable output vector, $\boldsymbol{u}$ is the r-dimensional control vector, $\boldsymbol{w}$ is a p-dimensional constant disturbance vector, and $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ are constant matrices of appropriate dimensions. It is assumed that $\boldsymbol{B}$ has the maximal rank r and the range of $\boldsymbol{D}$ is contained in the range of $\boldsymbol{B}$. Further the system description is assumed to be completely controllable.

Optimal control can be achived by minimizing a quadratic cost function of the form

$$PI = \mathrm{E}\left[\int_0^\infty \left\{\boldsymbol{x}^T\boldsymbol{Qx}+\frac{d\boldsymbol{u}^T}{dt}\boldsymbol{R}\frac{d\boldsymbol{u}}{dt}\right\}dt\right], \tag{2}$$

subject to the system control constraint Eq. (1), where E is the expectation operator over the random initial state, $\boldsymbol{Q}$ is an $n\times n$ symmetric positive semi-definite constant matrix and $\boldsymbol{R}$ is an $r\times r$ symmetric positive definite constant matrix.

Now, to introduce integral action on the input variables[1], we rewrite Eq. (1) as

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{Ax}+\boldsymbol{Bu}+\boldsymbol{B\Gamma w}\,, \qquad \boldsymbol{y} = \boldsymbol{Cx}\,, \tag{3}$$

where

$$\boldsymbol{\Gamma} = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{D}\,,$$

and define a new state variable vector $\boldsymbol{V}$ as

$$\boldsymbol{V} = \frac{d\boldsymbol{u}}{dt}\,. \tag{4}$$

The following augmented system may then be obtained:

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{u}+\boldsymbol{\Gamma w}\end{bmatrix} = \begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{O} & \boldsymbol{O}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{u}+\boldsymbol{\Gamma w}\end{bmatrix}+\begin{bmatrix}\boldsymbol{O}\\ \boldsymbol{I}\end{bmatrix}\boldsymbol{V}, \tag{5}$$

where $\boldsymbol{I}$ is the $r\times r$ identity matrix.

Next, assume the new variable in Eq. (5) as follows:

$$\boldsymbol{V} = -\boldsymbol{K}\boldsymbol{y}\,, \tag{6}$$

where $\boldsymbol{K}$ is an $r \times m$ feedback gain matrix.

On the basis of this assumption, and by defining the augmented state vector

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{u}+\boldsymbol{\Gamma}\boldsymbol{w} \end{bmatrix} \tag{7}$$

the plant equation becomes

$$\frac{d\boldsymbol{Z}}{dt} = (\hat{\boldsymbol{A}} - \hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}})\boldsymbol{Z}\,, \tag{8}$$

and the performance index is also expressed as

$$PI = \mathrm{E}\left[\int_0^\infty \boldsymbol{Z}^T(\hat{\boldsymbol{Q}} + \hat{\boldsymbol{C}}^T\boldsymbol{K}^T\boldsymbol{R}\boldsymbol{K}\hat{\boldsymbol{C}})\boldsymbol{Z}dt\right], \tag{9}$$

where

$$\hat{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{O} & \boldsymbol{O} \end{bmatrix}, \quad \hat{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{I} \end{bmatrix},$$

$$\hat{\boldsymbol{C}} = [\boldsymbol{C} \quad \boldsymbol{O}], \quad \hat{\boldsymbol{Q}} = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} \end{bmatrix}.$$

Further, by substituting Eq. (8) into Eq. (9), the performance index becomes

$$PI = \mathrm{E}[\boldsymbol{Z}(0)^T\boldsymbol{P}\boldsymbol{Z}(0)]\,, \tag{10}$$

where $\boldsymbol{P}$ is the solution of the Lyapunov Matrix equation

$$(\hat{\boldsymbol{A}} - \hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}})^T\boldsymbol{P} + \boldsymbol{P}(\hat{\boldsymbol{A}} - \hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}}) + \hat{\boldsymbol{C}}^T\boldsymbol{K}^T\boldsymbol{R}\boldsymbol{K}\hat{\boldsymbol{C}} + \hat{\boldsymbol{Q}} = \boldsymbol{O}\,. \tag{11}$$

On assuming now that $\boldsymbol{Z}(0)$ is uniformly distributed over the surface of a hypersphere[5], the performance index becomes

$$PI = \mathrm{tr}\,\boldsymbol{P}\,, \tag{12}$$

where tr $\boldsymbol{P}$ is the sum of the diagonal terms of $\boldsymbol{P}$.

Therefore $\boldsymbol{K}$ for the suboptimal controller must be determined in such a manner that $\boldsymbol{P}$ can be minimized subject to the constraint given by Eq. (11). For minimizing PI, the Hamiltonian

$$\begin{aligned} L = \mathrm{tr}\,\boldsymbol{P} + \mathrm{tr}[\boldsymbol{S}^T\{&(\hat{\boldsymbol{A}} - \hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}})^T\boldsymbol{P} + \boldsymbol{P}(\hat{\boldsymbol{A}} - \hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}}) \\ &+ \hat{\boldsymbol{C}}^T\boldsymbol{K}^T\boldsymbol{R}\boldsymbol{K}\hat{\boldsymbol{C}} + \boldsymbol{Q}\}] \end{aligned} \tag{13}$$

is chosen and the necessary conditions for minimization of L

$$\frac{\partial L}{\partial \boldsymbol{K}} = \boldsymbol{O}\,, \quad \frac{\partial L}{\partial \boldsymbol{P}} = \boldsymbol{O}\,, \quad \frac{\partial L}{\partial \boldsymbol{S}} = \boldsymbol{O}\,, \tag{14}$$

yield the following solutions:

$$\boldsymbol{K} = \boldsymbol{R}^{-1}\hat{\boldsymbol{B}}^T\boldsymbol{P}\boldsymbol{S}\hat{\boldsymbol{C}}^T(\hat{\boldsymbol{C}}\boldsymbol{S}\hat{\boldsymbol{C}}^T)^{-1},$$
$$(\hat{\boldsymbol{A}}-\hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}})\boldsymbol{S}+\boldsymbol{S}(\hat{\boldsymbol{A}}-\hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}})^T+\boldsymbol{I} = \boldsymbol{O},$$
$$(\hat{\boldsymbol{A}}-\hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}})\boldsymbol{P}+\boldsymbol{P}(\hat{\boldsymbol{A}}-\hat{\boldsymbol{B}}\boldsymbol{K}\hat{\boldsymbol{C}})+\hat{\boldsymbol{C}}^T\boldsymbol{K}^T\boldsymbol{R}\boldsymbol{K}\hat{\boldsymbol{C}}+\hat{\boldsymbol{Q}} = \boldsymbol{O}. \tag{15}$$

By solving Eq. (15) simultaneously with respect to $\boldsymbol{K}$, $\boldsymbol{S}$ and $\boldsymbol{P}$, and then making use of Eqs. (4) and (6), the suboptimal control is found to be

$$\boldsymbol{u} = -\boldsymbol{K}\int_0^t \boldsymbol{y}\,dt+\boldsymbol{u}_0, \tag{16}$$

where $\boldsymbol{u}_0$ is the initial value of $\boldsymbol{u}$.

This controller with integral action will put the state $\boldsymbol{x}$ back in its normal value even under the presence of a constant disturbance such as $\boldsymbol{w}$.

3. Example

A synchronous machine connected to an infinite bus as shown in Fig. 1 is used to evaluate the effectiveness of the foregoing controller. In this system, an induction machine and a shunt impedance are connected in parallel at local terminal.

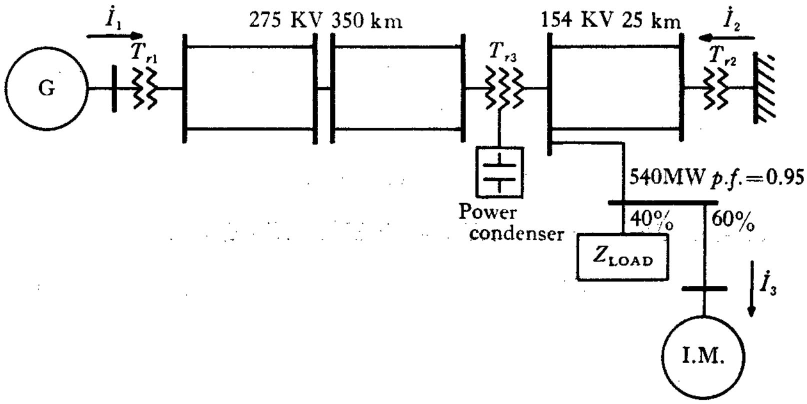

Fig. 1. Model system.

The nodal equation which gives the current-voltage relation of this model is as follows;

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \end{bmatrix} = \begin{bmatrix} \dot{Y}_{11} & \dot{Y}_{12} & \dot{Y}_{13} \\ \dot{Y}_{21} & \dot{Y}_{22} & \dot{Y}_{23} \\ \dot{Y}_{31} & \dot{Y}_{32} & \dot{Y}_{33} \end{bmatrix} \begin{bmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \dot{E}_3 \end{bmatrix}. \tag{17}$$

Then, the electro-mechanical oscillation of the synchronous generator can be written as

$$\frac{d\delta}{dt} = \omega,$$
$$\frac{d\omega}{dt} = \frac{\omega_0}{M}(P_m-P_e), \tag{18}$$

where

$$P_e = \mathrm{Real}[\bar{\dot{E}}_1 \dot{I}_1]\,.$$

The electromagnetic oscillation of the power system can be written, by neglecting the time lag in the exciter, as

$$\frac{dE_1}{dt} = u - \frac{1}{T'_{d0}}\{(x_d - x'_d)I_d + E_1 - E_{ex}\}\,, \tag{19}$$

where

$$I_d = \mathrm{Im}[\bar{\dot{E}}_1 \dot{I}_1 / |\dot{E}_1|]\,.$$

Further, for the induction machine[6], the equations are;

$$\begin{aligned} \frac{dS}{dt} &= \frac{1}{M_u}(P_u - P_s) \\ \frac{d\dot{E}_3}{dt} &= -jS\omega_0\dot{E}_3 - \frac{1}{T_0}\{\dot{E}_3 - j(x_{11} - x')\dot{I}_3\}\,, \end{aligned} \tag{20}$$

where

$$\dot{E}_3 = E_{md} + jE_{mq}\,,$$
$$P_s = \mathrm{Real}[\bar{\dot{E}}_3 \dot{I}_3]\,.$$

Table 1. System conditions.

Generator	x_d			0.6051
	x'_d			0.2017
	T'_{d0}			6.0
	M			14.4
Transformer	Reactance	T_{r1}		0.0756
		T_{r2}		0.0722
		T_{r3}	1_{ry}	0.0290
			2_{ry}	0.0087
			3_{ry}	0.0058
Trans. line (1 route)	Impedance	350 km		0.0622+j0.5581
		25 km		0.0166+j0.1070
Induction motor	r_{11}	1_{ry}		0.0135
	r_{22}	2_{ry}		0.0135
	x_{11}	1_{ry}		1.2001
	x_{22}	2_{ry}		1.1923
	x_{12}			1.1597
	M_u			7.26

(250 KV, 200 MVA base)

Table 2. Initial conditions.

δ	ω	E_1	E_2	P_m	S	E_{md}	E_{mq}
0.5643	0.0	1.3452	1.15	1.3458	0.0241	0.9032	−0.2064

Therefore, linearizing Eqs. (18), (19) and (20) about the steady state operating point and taking the presence of a constant disturbance into account, we can obtain the first order linear differential equation given by Eq. (1).

The output variables considered for the design of a suboptimal controller is formed in terms of the angular velocity, terminal voltage and electrical power of the synchronous generator, which are quantities easily measurable at the generator location.

The system conditions and initial conditions are shown in Tables 1 and 2. Then, by solving Eq. (15), the integral control u is expressed as

$$u = -0.225\int_0^t \Delta\omega\, dt - 1.379\int_0^t \Delta V_t\, dt - 0.769\int_0^t \Delta P_e\, dt. \tag{21}$$

On the other hand, the optimal control[4] in the conventional undisturbed linear regulator problem with

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{Ax} + \boldsymbol{Bu} \tag{22}$$

and

$$PI = \int_0^\infty (\boldsymbol{x}^T\boldsymbol{Qx} + ru^2)dt\,, \tag{23}$$

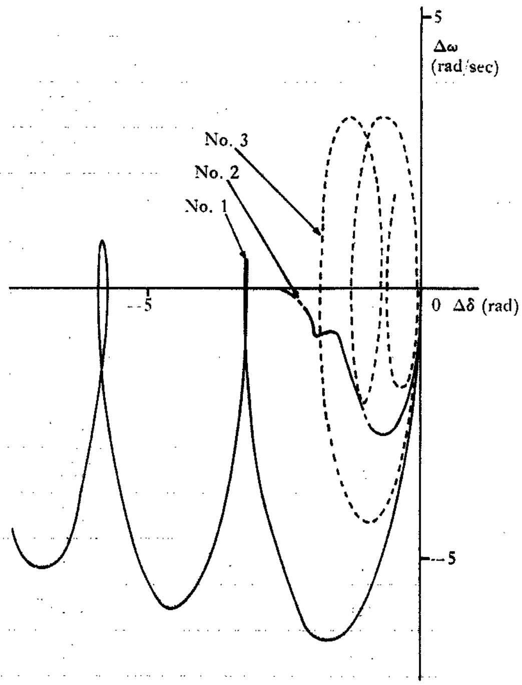

Fig. 2. Relation between $\Delta\delta$ and $\Delta\omega$.

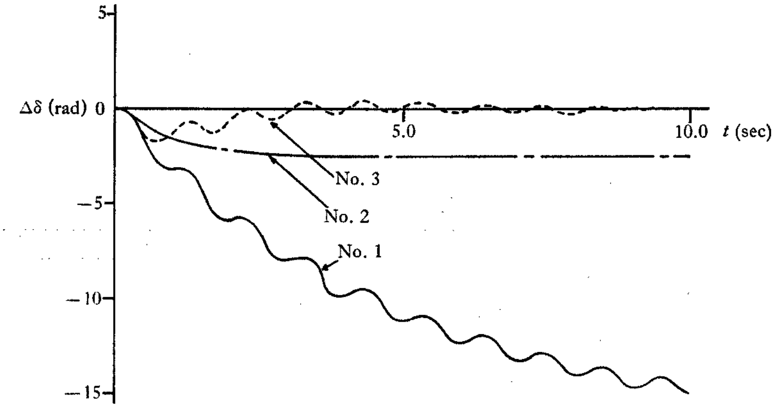

Fig. 3. Relation between $\Delta\delta$ and t.

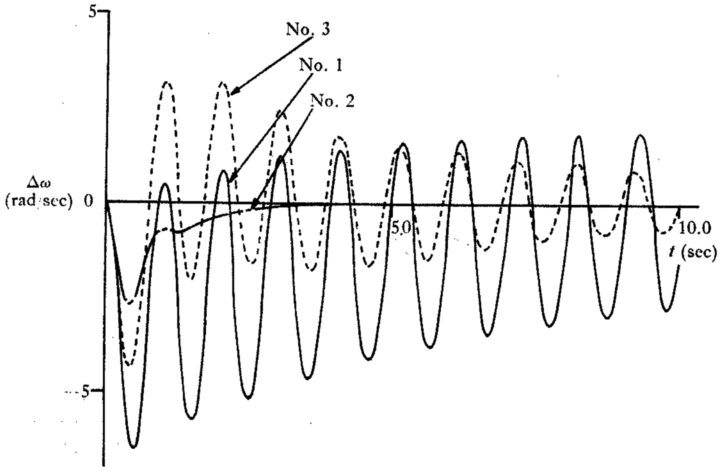

Fig. 4. Relation between $\Delta\omega$ and t.

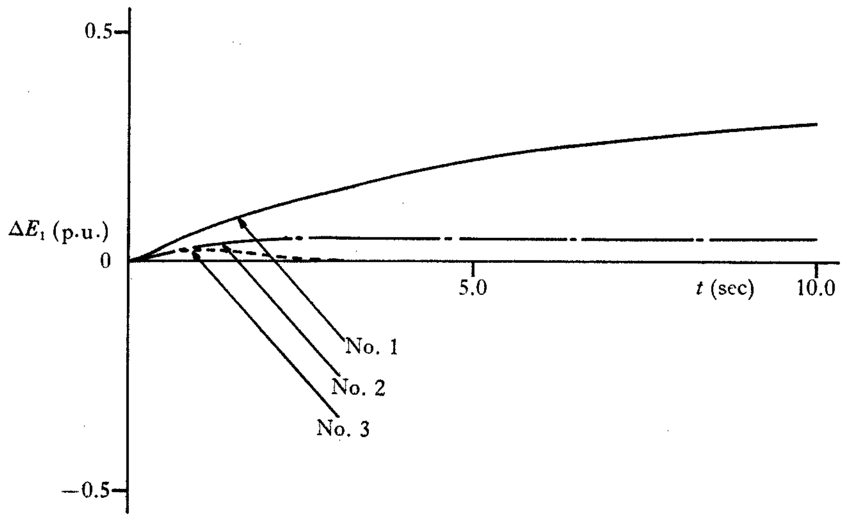

Fig. 5. Relation between ΔE_1 and t.

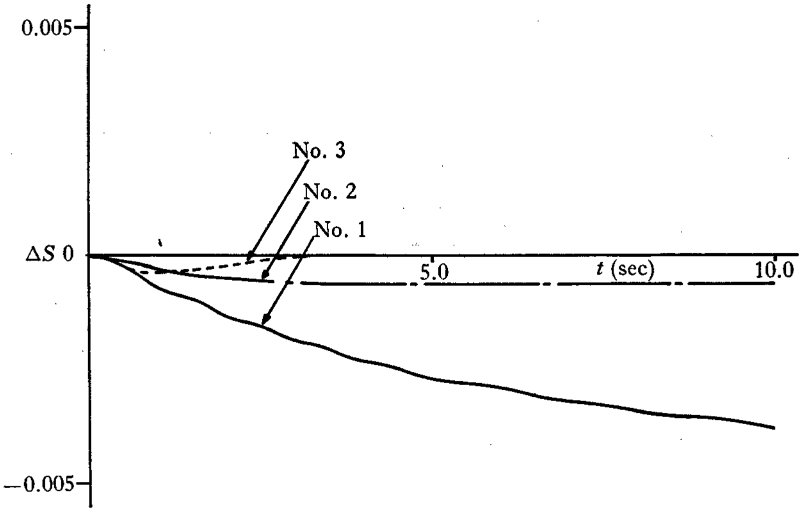

Fig. 6. Relation between ΔS and t.

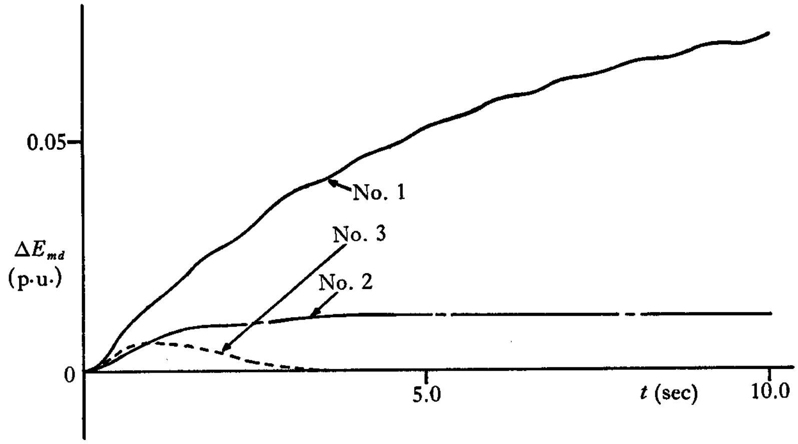

Fig. 7. Relation between ΔE_{md} and t.

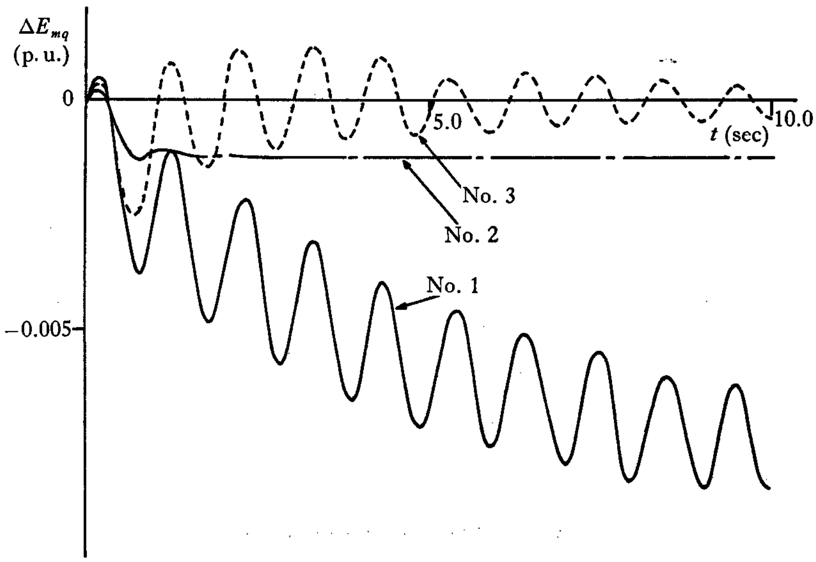

Fig. 8. Relation between ΔE_{mq} and t.

is found to be

$$u = -5.157\Delta\delta + 0.519\Delta\omega - 5.633\Delta E_1 \\ -12.68\Delta S - 0.446\Delta E_{md} + 0.050\Delta E_{mq}\,. \tag{24}$$

The responses of uncontrolled system (No. 1), optimal system given by Eq. (24) (No. 2) and proposed suboptimal system given by Eq. (21) (No. 3) for a value of $w=0.07$ are shown in Fig. 2～Fig. 8.

The results, shown in Fig. 2～Fig. 8, indicate that the proposed controller with integral action puts the state $\boldsymbol{x}$ back in its normal value even under the presence of a constant disturbance.

4. Conclusion

The present paper has considered the development of an integral controller based on measurement of only the avilable output quantities, using the algorithm of C.D. Johnson for optimization. The proposed controller has been applied to stabilization of a synchronous machine and found to result in good control.

Acknowledgment

The authors acknowledge Mr. K. Okano, Student of University of Osaka Prefecture, for his assistance of numerical calculation. Numerical calculation were performed by using ACOS-600 at the computer center, Univ. of Osaka Prefecture.

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