Determination of the Economically Optimum Machining Conditions under Effect of Experimental Error

Determination of the Economically Optimum Machining Conditions under Effect of Experimental Error

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Consideration is given to the application of the equations for machining cost per piece and production rate in turning. The restrictions on machining operation and the effects of experimental error of the parameters in tool life equation are also investigated. The computer can aid one in determining the minimum cost and the maximum production rate under the restrictions and the effects of experimental error.

1. Introduction

The basic mathematical model which has been used in selection of the economically optimum machining conditions was established by Gilbert.[1] While the machining economics including two variables—cutting speed and feed—are analyzed by Brewer,[2] in Gilbert's paper the cutting speed is treated as the only variable. It is possible numerically to obtain an optimum solution based on two variables at the same time but not analytically. In case of two variables with restrictions, the investigation by a principle of optimum seeking method is presented.[3] Recently, in addition to these studies, the computerized method of determining the optimum cutting speed from the sets of data on machining conditions is published.[4] The conception of maximum profit rate for selecting optimum cutting conditions is presented.[5][6] However, the variance of parameters in tool life equation, $VT^n=C$ (or $VT^nf^{n_1}=C$), is neglected in all the above papers.

This paper treats the case of machining of a kind of product by one machine tool, suggesting the equations for cost and production rate which decompose Gilbert's basic model into several elements of shop work. The computer program for determination of optimum cutting conditions is also proposed by the results of tool life experiment on the effects of the variance of the parameters on machining cost and production rate.

2. Cost and Production Rate Equations

Machining time per piece in cutting operations (in turning) consists of eight elements:

(1) actual cutting time $t_f(=DL/1000fV)$

(2) approach time $t_a(=Da/1000fV=a/Nf)$

(3) rapid traverse time $t_{rp}\{=(L+a)/r\}$

(4) cross-slide handling time t_{cs}

(5) examination time t_e

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(6) loading and unloading time t_l
(7) setup time t_0/N_L
(8) tool changing time t_{tc}

The determination of machining conditions necessitates the tool cost besides the above factors.

(9) tool cost $C_{tl}(=C_t t_f/T)$

Table 1. Symbols for cost and production rate equations.

Symbol	Definition
a	approach of tool to work; mm.
C_t	tool depreciation cost per blade; yen/blade
C_{tl}	tool cost per work piece; yen/piece
D	diameter of work piece in turning; mm.
f	feed per revolution; mm./rev.
L	length of work piece in turning; mm.
M	labor and overhead cost on lathe; yen/min.
N	revolution of spindle speed; rpm.
N_L	number of work pieces in lot
r	rapid traverse rate; mm./min.
T	tool life; min.
V	cutting speed; m./min.

The production rate which stands for the number of products per hour is

$$P=\frac{60}{t_f+t_i+t_{tc}t_f/T} \tag{1}$$

where

$$t_i=a/Nf+(a+L)/r+t_{cs}+t_l+t_m+t_0/N_L\,.$$

The machining cost per piece is

$$C=M(t_f+t_i+t_{tc}t_f/T)+C_t t_f/T. \tag{2}$$

In the following analysis it is assumed that machine tool, cutting tool and work material have been selected, and that the various nonproduction times and tool changing time are minimum. Various criteria are also postulated in the selection of the cutting conditions for turning and single-pass operations.

The tool life equation which is required for determination of optimum conditions as well as the equations for cost and production rate is given by

$$VT^n f^{n_1} d^{n_2}=K \tag{3}$$

where, V is the cutting speed in m./min., T the tool life in min., f the feed in mm./rev., d the depth of cut in mm., K a constant, and, n, n_1 and n_2 are exponents of the tool life, the feed and depth of cut, respectively.

Since the depth of cut is fixed from the drawing, the remaining cutting variables—speed and feed—must be chosen to optimize the cutting conditions. For the maximum production rate speed and feed are obtained by substituting Eq. (3) into Eq. (1). Thus

$$\frac{\partial P}{\partial V}=0 \quad \text{i.e.,} \quad V_{\text{max.prod.}}=\frac{K}{\left[t_{tc}\left(\frac{1}{n}-1\right)\left(\frac{L}{L+a}\right)\right]^{n} f^{n_1} d^{n_2}}, \tag{4}$$

$$\frac{\partial P}{\partial f}=0 \quad \text{i.e.,} \quad f_{\text{max.prod.}}=\frac{K}{\left[t_{tc}\left(\frac{1}{n_1}-1\right)\left(\frac{L}{L+a}\right)\right]^{n_1} V^{n} d^{n_2}}. \tag{5}$$

Conditions for minimum cost per piece are also obtained as

$$\frac{\partial C}{\partial V}=0 \quad \text{i.e.,} \quad V_{\text{min.}c.}=\frac{K}{\left[\left(t_{tc}+\frac{C_t}{M}\right)\left(\frac{1}{n}-1\right)\left(\frac{L}{L+a}\right)\right]^{n} f^{n_1} d^{n_2}}, \tag{6}$$

$$\frac{\partial C}{\partial f}=0 \quad \text{i.e.,} \quad f_{\text{min.}c.}=\frac{K}{\left[\left(t_{tc}+\frac{C_t}{M}\right)\left(\frac{1}{n_1}-1\right)\left(\frac{L}{L+a}\right)\right]^{n_1} V^{n} d^{n_2}}. \tag{7}$$

Eqs. (4) and (5) in the maximum production rate, and Eqs. (6) and (7) in the minimum cost cannot be simultaneously satisfied and a unique solution dose not exist in either case. Since $n_1/n<1/n$ in tool life equation, generally, the cost per piece decreases and the production rate increases as the feed increases.[7] Therefore the selection of the highest feed and the determination of cutting speed give the optimum cutting conditions from Eq. (4) or Eq. (6).

The final choice of cutting conditions must satisfy several restrictions. The method of selecting the cutting conditions for each restriction is as follows. (a) Maximum depth of cut restrictions: Maximum depth of cut, $d_{\max}$, is determined from the shape of the tool according to

$$d_{\max} \leqq W \cos C_s$$

where W is the width of tip and C_s the angle of side cutting edge. (b) Maximum feed restrictions: The feed must be chosen as much as the machine tool permits, as mentioned above. The maximum is determined from the maximum machine tool feed and surface finish restrictions. It is well-known that the surface finish varies with the feed, geometrical shape of the tool and degree of wear of the tool. In fact an empirical equation is shown[8] which represents the relationship among them. Here taken an approximation that the feed marks have connection with the surface finish $H_{\max}$, then the distance between the peak to valley for a round-nosed tool is

$$H_{\max} \fallingdotseq f^2/8R$$

where R is the radius of tool edge. (c) Cutting speed restrictions: The optimum cutting speed can be calculated provided that the depth of cut and the feed are chosen. However, one must compare the calculated speed with the maximum and minimum speed, and the cutting speed at the maximum power, V_{hp}. The former can be determined from spindle speed and the latter is given by the relation[9] between the feed and the specific cutting resistance, k_s. The equation of V_{hp} is given by

$$V_{hp}=75\times 60\times 0.8/k_s d f.$$

Thus the optimum feed and cutting speed are obtained from the flow chart shown in Fig. 1. In this figure the cutting conditions for the minimum cost can be determined, while the maximum production rate is determined by using Eq. (4) in stead of Eq. (6).

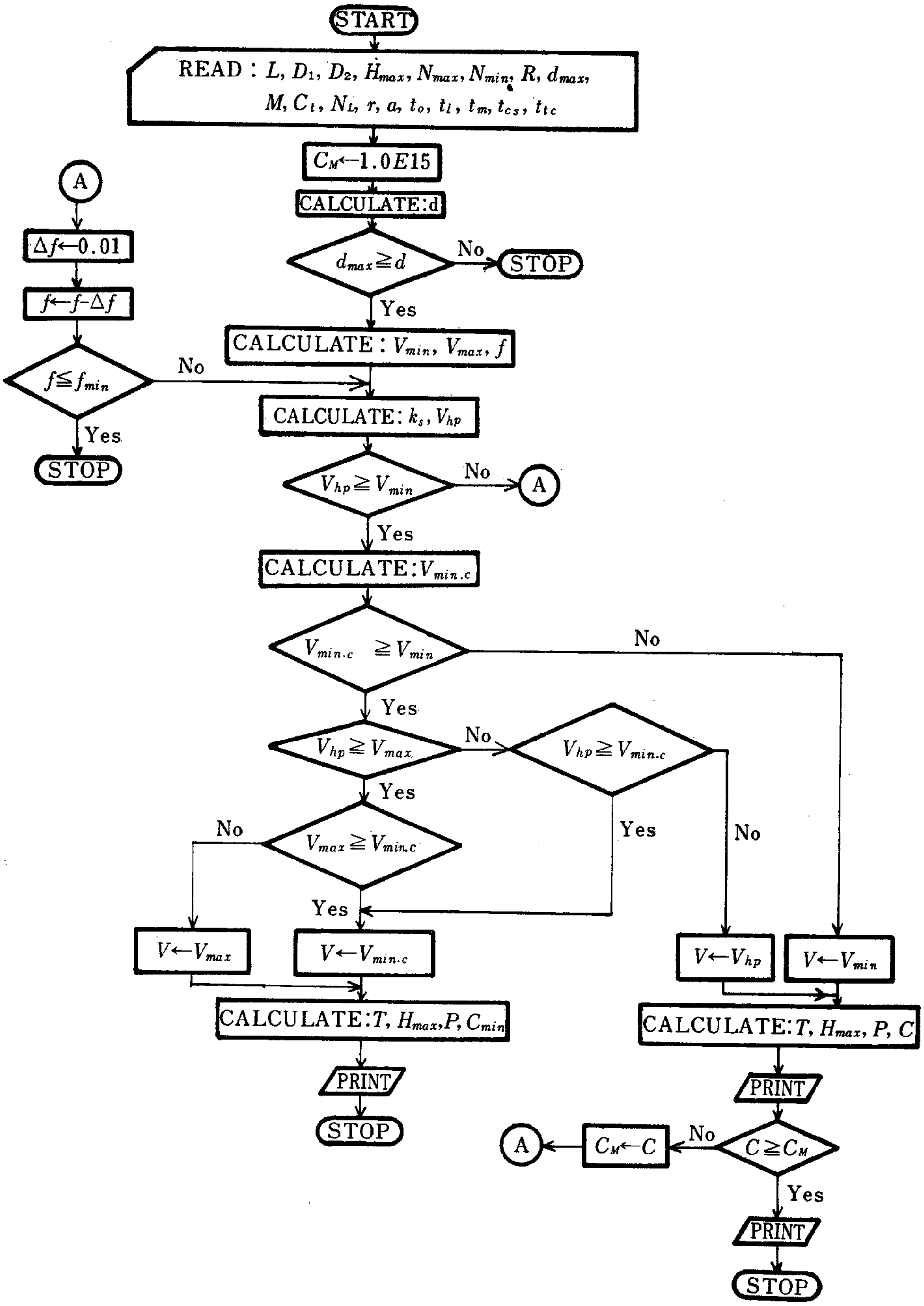

Fig. 1. Flow chart for determining the optimum machining conditions.

3. Effect of Experimental Error

The preceding section describes how to obtain the optimum cutting speed on the basis of the assumption that the parameters in tool life equation are constant. In tool life experiment, however, these are subject to the uncertainty of experimental error. When their variance is considered, it should be noticed that the optimum cutting speed cannot be uniquely defined but lies within some probable interval of cutting speeds. Under such circumstances, the minimax principle is applied in order to make a decision about the value within this interval. This method has been used by Ermer and Wu[10] in analyzing the similar problem to our study. If we decide the optimum cutting conditions and show the effects of experimenal error on determination of the optimum cutting speed, the parameters in Eq. (3) should be estimated. For this purpose, Eq. (3) can be written in a more conventional form by taking logarithms of both sides as

$$\eta=\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3$$

where η is the true response of tool life on a logarithmic scale, x_1, x_2 and x_3 are the logarithmic transformations of V, f and d, respectively, and β_0, β_1, β_2 and β_3 the parameters.

This formula can also be rewritten, namely

$$y=b_0+b_1 x_1+b_2 x_2+b_3 x_3+\varepsilon \tag{8}$$

where y is the observed tool life on a logarithmic scale, b_0, b_1, b_2 and b_3 are estimates of the parameters β_0, β_1, β_2 and β_3, respectively, and ε stands for the experimental error. The parameters of this linear equation can be easily estimated by the method of least squares.

The experimental design used in this study is a composite design proposed by Box and Wilson, and used by Wu.[11] The choice of cutting conditions and their coding are shown in Table 2. The parameters in Eq. (8) are given by

$$\boldsymbol{b}=(X'X)^{-1}X'\boldsymbol{y}$$

where X is the matrix of independent variables for twelve tests shown below:

$$\begin{array}{cc} & \begin{array}{cccc} x_0 & x_1 & x_2 & x_3 \end{array} \\ X= & \begin{pmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \end{array}$$

These estimated parameters are mutually uncorrelated on account of orthogonality of the experimental design.

The confidence interval (CI) for a given parameter is

$$CI(\beta_i) = b_i \pm t_{\nu:\alpha/2}\sqrt{V(b_i)}$$

where $V(b_i)$ can be obtained from

$$V(\boldsymbol{b}) = (X'X)^{-1}S^2$$

and

$$S^2 = \frac{\sum_{i=1}^{N}(y_i-\hat{y})^2}{N-q} .$$

In the last equation N is the number of observations, y_i the i-th observation and q the number of parameters.

4. Experimental Results

The tool life test and work measurement were performed in the following way. The engine lathe equipped with a ten *hp* was used. The cutting tool was of throw-away type, and its material was P10 and its geometrical shape was (−5, −5, 5, 5, 15, 15, 0.8). The work material used was S45C carbon steel, 80 mm. in diameter and 400 mm. in length, and its hardness was HB180 after annealing. The tool life criterion was 0.7 mm. of frank wear.

The results of experiment are shown in Table 2, and adequacy of the postulated model can be checked by analysis of variance such as in Table 3. These analysis yields

$$y = 2.79667 - 0.62125\,x_1 - 0.39125\,x_2 - 0.00625\,x_3$$

which can be transformed into a generalized tool life equation

$$VT^{0.356}f^{0.201}d^{0.006} = 431 \tag{9}$$

Table 2. Experimental conditions and observations.

No.	Spped, V, m./min.	Feed, f, mm./rev.	Depth of cut, d, mm.	Coding			Tool life, T, min.
				x_1	x_2	x_3	
1	180	0.09	1.00	−1	−1	−1	34.4
2	280	0.09	1.00	1	−1	−1	13.4
3	180	0.36	1.00	−1	1	−1	20.7
4	280	0.36	1.00	1	1	−1	6.1
5	180	0.09	2.00	−1	−1	1	38.8
6	280	0.09	2.00	1	−1	1	15.2
7	180	0.36	2.00	−1	1	1	24.9
8	280	0.36	2.00	1	1	1	3.8
9	223	0.18	1.42	0	0	0	17.7
10	223	0.18	1.42	0	0	0	20.9
11	223	0.18	1.42	0	0	0	16.8
12	223	0.18	1.42	0	0	0	18.6

Table 3. Analysis of variance.

Source	$S.S.$	$d.f.$	V	F
Total	98.5996	12		
Zero-order	93.8564	1		
First-order	4.3126	3		
Residual	0.4306	8		
Lack of fit	0.4040	5	0.0808	$9.0787 < F_3^5\ (0.025)$
Pure error	0.0267	3	0.0089	

with 95 per cent confidence interval of

$$CI(\beta_0) = 2.79667 \pm 2.3060\{(0.08333)(0.05384)\}^{1/2} = \begin{cases} 2.95112 \\ 2.64222, \end{cases}$$

$$CI(\beta_1) = -0.62125 \pm 2.3060\{(0.125)(0.05384)\}^{1/2} = \begin{cases} -0.43209 \\ -0.81041, \end{cases}$$

$$CI(\beta_2) = -0.39125 \pm 2.3060\{(0.125)(0.05384)\}^{1/2} = \begin{cases} -0.20209 \\ -0.58041, \end{cases}$$

and

$$CI(\beta_3) = -0.00625 \pm 2.3060\{(0.125)(0.05384)\}^{1/2} = \begin{cases} +0.18291 \\ -0.19541. \end{cases}$$

For example, when $f=0.35$ mm./rev. and $d=1$ mm., Eq. (9) becomes

$$VT^{0.356} = 535.$$

Using the lower limit of 95% $CI(\beta_1)$ for b_1, and the upper limit of 95% $CI(\beta_0)$ for b_0, we find the tool life equation as

$$VT^{0.511} = 761,$$

and the interchange of the limits brings

$$VT^{0.273} = 430,$$

whose results are illustrated in Fig. 2.

The operation time for turning was observed by a stop watch, the values of whose elements were

$$t_0 = 2.0, \quad t_l = 2.0, \quad t_m = 0.5, \quad t_{cs} = 0.6, \quad t_{tc} = 0.3 \text{ (min.)}.$$

By the input data shown in Table 4, the cutting conditions for the minimum cost and maximum production rate can be determined as shown in Table 5. From the results it

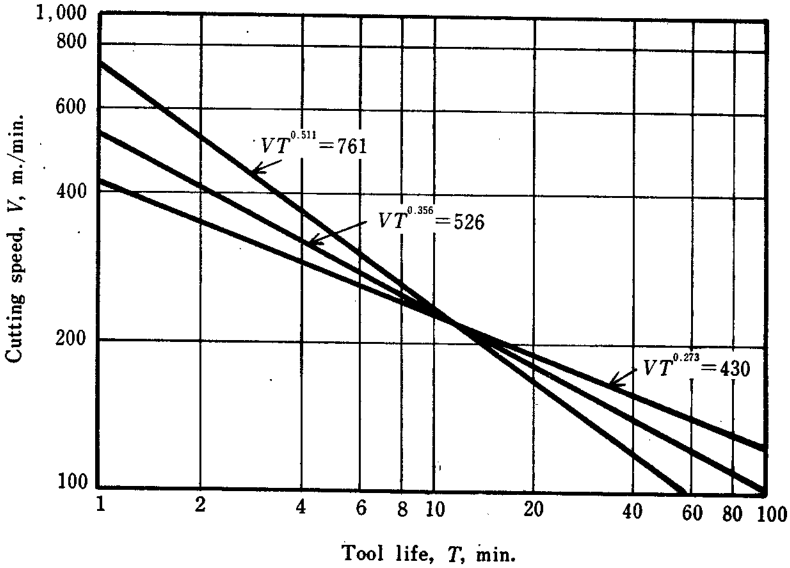

Fig. 2. Effect of experimental error on tool life equation.

Table 4. Input data in search of the optimum cutting conditions.

Machining conditions			
work piece (S45C)	length, mm.		$L=$ 350
	diameter, mm.		$D_1=$ 75
	finished dia., mm.		$D_2=$ 73
	surface finish, μ		$H_{max.}=$ 20
lathe	horse power, hp.		$hp=$ 10
	max. spindle speed, rpm.		$N_{max.}=$2000
	min. spindle speed, rpm.		$N_{min.}=$ 20
	max. feed, mm./rev.		$f_{max.}=$ 1.2
	min. feed, mm./rev.		$f_{min.}=$ 0.05
cutting tool (P10)	nose radius, mm.		$R=$ 0.8
	max. depth of cut, mm.		$d_{max.}=$ 6.0
	tool life criterion, mm.		$V_B=$ 0.7
labor and overhead cost on lathe, yen/min.			$M=$ 30
tool depreciation cost per blade, yen/blade			$C_t=$ 77.257
number of pieces in lot, pieces			$N_L=$ 80
rapid traverse rate, mm./min.			$r=$2300
approach of tool to work, mm.			$a=$ 30
setup time, min.			$t_0=$ 20
loading and unloading time, min.			$t_l=$ 2.0
examination time, min.			$t_e=$ 0.5
cross-slide handling time, min.			$t_{cs}=$ 0.6
tool changing time, min.			$t_{t2}=$ 0.3
equation of specific cutting resistance	$k_s f^{0.43}=e^{5.06}$		
tool life equation	$VT^{0.356}f^{0.201}d^{0.006}=431$		

Table 5. Output for computational results of optimum machining conditions.

	Min. cost	Max. production rate
depth of cut, mm.	$d=$ 1.00	1.00
feed, mm./rev.	$f=$ 0.35	0.35
surface finish, μ	$H_{max.}=$ 19.1	19.1
cutting speed, m./min.	$V=$304.7 (354.4)	416.0
tool life, min.	$T=$ 4.8 (4.8 ~ 2.0)	2.0
production rate, pieces/hr.	$P=$ 14.0 (14.0 ~ 13.8)	14.6
machining cost, yen/piece	$C=$141.56 (139.96~155.34)	145.23

Note: Figures in parentheses give the values when the effects of the variance of the parameters in tool life equation are taken into consideration.

may be deduced that the cutting speed used here is higher than general case, and that tool changing must be carried out once every five minutes. The practical possibility of such a high speed is not necessarily clear insofar as the rigidity of machine tool, the chatter during the work, a tool fail, and so on are not examined.

5. Conclusion

The points of our study may be now summerized as follows:

(1) The equations for cost per piece and production rate based on shop work are proposed.

(2) The criteria for selection of the optimum cutting conditions are investigated.

(3) The computer program for determination of optimum cutting conditions is constructed under the effects of the restrictions and the variance of the parameters in tool life equation on cost and production rate.

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References

1) W. W. Gilbert, Manual on Cutting of Metals, p. 313, ASME (1952).
2) R. C. Brewer, Trans. ASME, **80**, 1479 (1952).
3) E. Usui and T. Kishinami, J. Japan Soc. Precision Eng., **34**, 248 (1968).
4) M. Field, *et al.*, Trans. ASME, B**90**, 455 (1968).
5) K. Okushima and K. Hitomi, J. Japan Soc. Precision Eng., **30**, 16 (1964).
6) S. M. Wu and D. S. Ermer, Trans. ASME, B**88**, 435 (1966).
7) E. J. A. Armarego and R. H. Brown, The Machining of Metals, p. 260, Prentice-Hall (1969).
8) V. Šolaja, Wear, **2**, 40 (1958/59).
9) F. Koenighsberger, Design Principles of Metal Cutting Machine Tools, p. 4, Pergamon Press (1964).
10) D. S. Ermer and S. M. Wu, Trans. ASME, B**89**, 315 (1967).
11) S. M. Wu, Trans. ASME, B**86**, 105 (1964).