Delay-Independent Design for Synchronization in Delayed-Coupled One-Dimensional Map Networks

著者	Sugitani Yoshiki, Konishi Keiji
journal or publication title	IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences
volume	E101-A
number	10
page range	1708-1712
year	2018-10-01
権利	(C) 2018 The Institute of Electronics, Information and Communication Engineers
URL	http://hdl.handle.net/10466/00017400

doi: https://doi.org/10.1587/transfun.E101.A.1708

LETTER

Delay-Independent Design for Synchronization in Delayed-Coupled One-Dimensional Map Networks

Yoshiki SUGITANI[†a)] ***and*** **Keiji KONISHI**[††], ***Members***

SUMMARY The present Letter proposes a design procedure for inducing synchronization in delayed-coupled one-dimensional map networks. We assume the practical situation where the connection delay, the detailed information about the network topology, and the number of the maps are unknown in advance. In such a situation, it is difficult to guarantee the stability of synchronization, since the local stability of a synchronized manifold is equivalent to that of a linear time-variant system. A sufficient condition in robust control theory helps us to derive a simple design procedure. The validity of our design procedure is numerically confirmed.

key words: *synchronization, delayed-coupled maps*

1. Introduction

Synchronization in coupled oscillators has been widely observed in the world [1], and has been extensively investigated [2]. It is well known that the local stability of synchronization is equivalent to that of a linear time-variant system [3]. Since it is generally difficult to analyze time-variant systems theoretically, numerical calculations are required to analyze the stability of synchronization.

Connection delays between oscillators inevitably exist due to the finite propagation speed of signals. Thus, delayed-coupled oscillators have attracted attention in recent years [4]. Considering connection delays, the local stability of synchronization is equivalent to that of a linear time-variant system with a time delay. Such systems are quite difficult to analyze because of the following two facts: (i) the systems are infinite dimensional; and (ii) the systems include time-variant parameters depending on the synchronized state.

Facts (i) and (ii) can be avoided. First, fact (i) can be avoided by employing maps as oscillators. This is because delayed-coupled maps are discrete dynamical systems; that is, they are finite dimensional. Second, by employing Bernoulli maps, fact (ii) also can be avoided, since the slope of the maps is constant with respect to the state of the map. As a result, the stability of the synchronized manifold for delayed-coupled Bernoulli maps is equivalent to that of a linear time-*invariant* system. Thus, we can easily analyze the stability of synchronization theoretically [5]–[7].

For engineering applications of dynamical systems, the design procedures for inducing desired dynamics are currently under intense investigation [8], [9]. In a previous study, we proposed a design procedure for inducing chaotic synchronization in delayed-coupled Bernoulli maps [10]. Our design procedure can be used in the situation where the connection delay, the detailed information about the network topology, and the number of the maps are unknown in advance. Unfortunately, the procedure can be applied only to Bernoulli maps; applications of this procedure are thus quite limited.

The present Letter proposes a design procedure for inducing synchronization in delayed-coupled *general* one-dimensional-map networks. For general one-dimensional maps, stability analysis of the synchronization is difficult due to fact (ii). However, a sufficient condition in robust control theory helps us to analyze the stability. In contrast to that in our previous study [10], the present design procedure does not designate the dynamics of the synchronized state (i.e., chaotic, periodic, or stable equilibrium point). The design procedure is confirmed by numerical simulations.

The following notation is used in this Letter. $\{\boldsymbol{A}\}_{ij}$ is the (i, j) element of matrix $\boldsymbol{A}$. $\boldsymbol{I}_m$ is an $m \times m$ unitary matrix and $\boldsymbol{0}_m$ is an $m \times 1$ column vector whose all elements are zero. Moreover, $||\boldsymbol{A}||_\infty := \max_{i \in \{1,\ldots,m\}} \sum_{j=1}^{m} |\{\boldsymbol{A}\}_{ij}|$ denotes the matrix infinity norm of a matrix $\boldsymbol{A} \in \mathbb{R}^{m\times m}$.

2. Delay-Coupled Map Network

Let us consider delayed-coupled one-dimensional maps [11],

$$x_i(n+1) = f[x_i(n)] + \varepsilon u_i(n), \quad (i = 1, \ldots, N), \qquad (1)$$

where $x_i(n) \in [0, 1]$ and $u_i(n) \in \mathbb{R}$ denote the state variable and input signal of the i-th map at time $n \in \mathbb{Z}$, respectively. $f : [0, 1] \to [0, 1]$ is the nonlinear function of the map. $\varepsilon \in [0, 1]$ is the coupling strength. The input signal $u_i(n)$ is given by

$$u_i(n) = \sum_{j=1}^{N} \frac{c_{ij}}{d_i} f[x_j(n-\tau)] - f[x_i(n)], \qquad (2)$$

where $x_j(n-\tau)$ is the delayed state variable of the j-th map, and $\tau \in \mathbb{Z}^+$ is the connection delay. c_{ij} represents the (i, j) element of the adjacency matrix $\boldsymbol{C}$: if the i-th and j-th maps are coupled, then $c_{ij} = c_{ji} = 1$, otherwise $c_{ij} = c_{ji} = 0$. Here, $d_i := \sum_{j=1}^{N} c_{ij}$ is the degree of the i-th map.

It is known that the eigenvalues ρ_q $(q = 1, \ldots, N)$ of

Manuscript received November 21, 2017.
Manuscript revised May 14, 2018.

[†]The author is with the Department of Electrical and Electronic Engineering, Ibaraki University, Hitachi-shi, 316-8511 Japan.
[††]The author is with the Department of Electrical and Information Systems, Osaka Prefecture University, Sakai-shi, 599-8531 Japan.
a) E-mail: yoshiki.sugitani.0301@vc.ibaraki.ac.jp

DOI: 10.1587/transfun.E101.A.1708

Table 1 Comparison of the previous study [10] with the present letter.

	Controlled object	Assumption	Goal
Previous study [10]	Delayed coupled Bernoulli maps	Given: $\rho_{\min}$ and $\rho_{\max}$. Unknown: $\boldsymbol{C}$, N and τ	Chaotic synchronization
Present Letter	Delayed coupled one-dimensional maps		Synchronization

the normalized Laplacian matrix $\boldsymbol{M} := \boldsymbol{I}_N - \boldsymbol{D}^{-1}\boldsymbol{C}$ with $\boldsymbol{D} := \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ satisfy

$$0 = \rho_1 \le \rho_2 \le \cdots \le \rho_N \le 2, \tag{3}$$

for any network topologies [12]. The synchronization manifold is given by

$$s(n) := x_1(n) = x_2(n) = \cdots = x_N(n). \tag{4}$$

Substituting Eq. (4) into Eqs. (1) and (2) yields the dynamics of the synchronization manifold,

$$s(n+1) = (1-\varepsilon) f[s(n)] + \varepsilon f[s(n-\tau)]. \tag{5}$$

The present Letter tackles the following problem:

Problem statement

Controlled object: Delayed-coupled maps (1), (2)
Assumption I: The number of maps N and the network topology c_{ij} are unknown. The lower and upper eigenvalues of the normalized Laplacian matrix $\boldsymbol{M}$ (i.e., $\rho_{\min} \le \rho_2$ and $\rho_{\max} \ge \rho_N$) are given.
Assumption II: The connection delay τ is unknown.
Design specification: Synchronized manifold (4) is stable for any number of maps, any connection delay, and any network topologies satisfying $\rho_{\min} \le \rho_2$ and $\rho_{\max} \ge \rho_N$.
Design parameters: The coupling strength ε and the map parameter.

Note that our previous study [10] considers a problem similar to the one described above. The differences between the previous study and the present one are summarized in Table 1: the previous study deals only with the Bernoulli map and aims to achieve chaotic synchronization; the present Letter deals with general one-dimensional maps and aims to achieve synchronization.

3. Stability Analysis

We focus on the local stability of the synchronized manifold (5). Substituting the perturbation $\delta x_i(n) := x_i(n) - s(n)$ into Eqs. (1) and (2), we obtain the local dynamics around manifold (4):

$$\delta\boldsymbol{x}(n+1) = (1-\varepsilon)\mathrm{D}F(n)\delta\boldsymbol{x}(n) + \varepsilon \mathrm{D}F_\tau(n)\boldsymbol{D}^{-1}\boldsymbol{C}\delta\boldsymbol{x}(n-\tau), \tag{6}$$

where

$$\delta\boldsymbol{x}(n) := \begin{bmatrix}\delta x_1(n) & \delta x_2(n) & \cdots & \delta x_N(n)\end{bmatrix}^T,$$

$$\mathrm{D}F(n) := \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=s(n)}, \ \mathrm{D}F_\tau(n) := \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x=s(n-\tau)}.$$

Note that the local dynamics (6) is a time-variant system with $\mathrm{D}F(n)$ and $\mathrm{D}F_\tau(n)$ depending on the synchronized state $s(n)$. On the other hand, the local dynamics (6) of the previous study [10] is a time-invariant system, since $\mathrm{D}F(n)$ and $\mathrm{D}F_\tau(n)$ for the Bernoulli map are constant. Thus, we cannot apply the analytical approach of the previous study to our problem.

The Laplacian matrix $\boldsymbol{M}$ can be diagonalized as $\boldsymbol{T}^{-1}\boldsymbol{M}\boldsymbol{T} = \mathrm{diag}(\rho_1, \ldots, \rho_N)$ by a transformation matrix $\boldsymbol{T}$ [12]. By introducing the change of variable $\delta\boldsymbol{x}(n) = \boldsymbol{T}\boldsymbol{z}(n)$, the stability of linear system (6) can be separated into N modes, where mode q ($q = 1, \ldots, N$) is given by

$$z_q(n+1) = (1-\varepsilon)\mathrm{D}F(n) z_q(n) + \varepsilon(1-\rho_q)\mathrm{D}F_\tau(n) z_q(n-\tau), \tag{7}$$

and mode $q = 1$ represents the time development of the synchronized manifold (4). Therefore, the synchronized manifold (4) is locally stable if and only if Eq. (7) with mode q is stable for any mode $q \in \{2, \ldots, N\}$ (i.e., excluding mode $q = 1$). Equation (7) can be rewritten in matrix form for a $(\tau+1)$-dimensional system with $\boldsymbol{x}_q(n) := [z_q(n)\ z_q(n-1)\ \cdots\ z_q(n-\tau)]^T$,

$$\boldsymbol{x}_q(n+1) = \boldsymbol{A}^*\left\{\mathrm{D}F(n), \mathrm{D}F_\tau(n), \rho_q\right\}\boldsymbol{x}_q(n), \tag{8}$$

$$\boldsymbol{A}^*\{\alpha, \beta, \gamma\} := \left[\begin{array}{cc|c} (1-\varepsilon)\alpha & \boldsymbol{0}_{\tau-1}^T & \varepsilon(1-\gamma)\beta \\ \hline \multicolumn{2}{c|}{\boldsymbol{I}_\tau} & \boldsymbol{0}_\tau \end{array}\right].$$

It is difficult to guarantee the stability of linear system (8), since system (8) includes time-variant terms $\mathrm{D}F(n)$ and $\mathrm{D}F_\tau(n)$ depending on the synchronized state $s(n)$. Here, we assume that the lower and upper limits of $\mathrm{D}F(n)$ and $\mathrm{D}F_\tau(n)$ satisfy

$$\mathrm{D}F(n),\ \mathrm{D}F_\tau(n) \in \left[\underline{\lambda}, \overline{\lambda}\right]. \tag{9}$$

Then, it is obvious that matrix $\boldsymbol{A}^*\left\{\mathrm{D}F(n), \mathrm{D}F_\tau(n), \rho_q\right\}$ in Eq. (8) belongs to the set of matrices $\boldsymbol{A}^I(q)$ for each time step, where $\boldsymbol{A}^I(q) := \left\{\sum_{i=1}^4 \eta_i \boldsymbol{A}_i(q) \mid \eta_i \ge 0, \sum_{i=1}^4 \eta_i = 1\right\}$, $\boldsymbol{A}_1(q) := \boldsymbol{A}^*\left\{\overline{\lambda}, \overline{\lambda}, \rho_q\right\}$, $\boldsymbol{A}_2(q) := \boldsymbol{A}^*\left\{\underline{\lambda}, \overline{\lambda}, \rho_q\right\}$, $\boldsymbol{A}_3(q) := \boldsymbol{A}^*\left\{\underline{\lambda}, \underline{\lambda}, \rho_q\right\}$, and $\boldsymbol{A}_4(q) := \boldsymbol{A}^*\left\{\overline{\lambda}, \underline{\lambda}, \rho_q\right\}$. The sufficient condition for system (8) to be stable is given by the following corollary.

Corollary 1: The time-variant system (8) is stable if the $(\tau+1)$-dimensional system

$$\boldsymbol{x}_q(n+1) = \boldsymbol{A}_q^{'}(n)\boldsymbol{x}_q(n), \tag{10}$$

is stable for any $\boldsymbol{A}_q^{'}(n) \in \boldsymbol{A}^I(q)$.

We now consider the stability of Eq. (10) instead of that of system (8). Equation (10) is a time-variant system with parameter uncertainty. For such system, a sufficient condition proposed by Bauer helps us to guarantee the stability [13].

Lemma 1 ([13])**:** The time-variant system (10) is stable for any $\boldsymbol{A}_q^{'}(n) \in \boldsymbol{A}^I(q)$ if and only if there exists a positive integer $k \in \mathbb{Z}^+$ such that

$$||\boldsymbol{A}_{t_1}(q)\boldsymbol{A}_{t_2}(q)\cdots\boldsymbol{A}_{t_k}(q)||_\infty < 1, \tag{11}$$
$$\forall \boldsymbol{A}_{t_j}(q) \in \{\boldsymbol{A}_1(q), \boldsymbol{A}_2(q), \boldsymbol{A}_3(q), \boldsymbol{A}_4(q)\},$$

for $j = 1, \ldots, k$.

The sufficient condition for existence of the positive integer k in Eq. (11) is derived below.

Lemma 2: There exists a positive integer $k = \tau + 1$ satisfying inequality (11) if the following inequality holds:

$$(1-\varepsilon)\lambda_{\max} + \varepsilon|1-\rho_q|\lambda_{\max} < 1, \tag{12}$$
$$\lambda_{\max} := \max\left\{|\underline{\lambda}|, |\overline{\lambda}|\right\}.$$

Proof: The proof is divided into propositions (a) and (b).

(a) For any positive integer $l \in \mathbb{Z}^+$, the following inequality always holds:

$$||\boldsymbol{A}_{t_1}(q)\boldsymbol{A}_{t_2}(q)\cdots\boldsymbol{A}_{t_l}(q)||_\infty \le \left\|\hat{\boldsymbol{A}}^l(q)\right\|_\infty, \tag{13}$$
$$\forall \boldsymbol{A}_{t_j}(q) \in \{\boldsymbol{A}_1(q), \boldsymbol{A}_2(q), \boldsymbol{A}_3(q), \boldsymbol{A}_4(q)\},$$

where $\hat{\boldsymbol{A}}(q) \in \mathbb{R}^{(\tau+1)\times(\tau+1)}$ is given by

$$\hat{\boldsymbol{A}}(q) := \left[\begin{array}{cc|c} (1-\varepsilon)\lambda_{\max} & \boldsymbol{0}_{\tau-1}^T & \varepsilon|1-\rho_q|\lambda_{\max} \\ \hline \multicolumn{2}{c|}{\boldsymbol{I}_\tau} & \boldsymbol{0}_\tau \end{array}\right].$$

(b) For $l = \tau + 1$, the right-hand side of Eq. (13) satisfies

$$\left\|\hat{\boldsymbol{A}}^{\tau+1}(q)\right\|_\infty < 1. \tag{14}$$

From Eqs. (13) and (14), propositions (a) and (b) imply that there exists an integer $\tau + 1$ satisfying the inequality

$$||\boldsymbol{A}_{t_1}(q)\boldsymbol{A}_{t_2}(q)\cdots\boldsymbol{A}_{t_{\tau+1}}(q)||_\infty \le \left\|\hat{\boldsymbol{A}}^{\tau+1}(q)\right\|_\infty < 1;$$

that is, Eq. (11) is satisfied. Propositions (a) and (b) will be proved below.

(a) All the elements of $\hat{\boldsymbol{A}}(q)$ and $\boldsymbol{A}_{1,2,3,4}(q)$ are the same except for the $(1,1)$ and $(1,\tau+1)$ elements. Moreover, the $(1,1)$ and $(1,\tau+1)$ elements in $\hat{\boldsymbol{A}}(q)$ are greater than or equal to those in $\boldsymbol{A}_{1,2,3,4}(q)$. Thus, the following relation is derived:

$$\{\boldsymbol{A}_{t_j}(q)\}_{m,n} \le \left\{\hat{\boldsymbol{A}}(q)\right\}_{m,n}, \quad \forall m, n \in \{1, \ldots, \tau+1\}.$$

Furthermore, all the elements of $\hat{\boldsymbol{A}}(q)$ being non-negative, for a positive integer l and any m, n, we get

$$\left|\{\boldsymbol{A}_{t_1}(q)\boldsymbol{A}_{t_2}(q)\cdots\boldsymbol{A}_{t_l}(q)\}_{m,n}\right| \le \left|\left\{\hat{\boldsymbol{A}}^l(q)\right\}_{m,n}\right|. \tag{15}$$

Summing both sides of Eq. (15) within each row (i.e., summing up every element in the same row), we can obtain Eq. (13).

(b) The $(\tau+1)$-th power of $\hat{\boldsymbol{A}}(q)$ is given by

$$\hat{\boldsymbol{A}}^{\tau+1}(q) := \begin{bmatrix} W + V^{\tau+1} & VW & V^2W & \cdots & V^{\tau-1}W & V^\tau W \\ V^\tau & W & VW & \cdots & V^{\tau-2}W & V^{\tau-1}W \\ V^{\tau-1} & 0 & W & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ V^2 & \vdots & & \ddots & \ddots & VW \\ V & 0 & \cdots & \cdots & 0 & W \end{bmatrix},$$

where $V := (1-\varepsilon)\lambda_{\max}$ and $W := \varepsilon|1-\rho_q|\lambda_{\max}$. Comparing the sum of elements of row $m \in \{1, \ldots, \tau\}$ in $\hat{\boldsymbol{A}}^{\tau+1}(q)$ with that of row $m+1$, we get

$$\sum_{n=1}^{\tau+1}\left\{\hat{\boldsymbol{A}}^{\tau+1}(q)\right\}_{m,n} - \sum_{n=1}^{\tau+1}\left\{\hat{\boldsymbol{A}}^{\tau+1}(q)\right\}_{m+1,n} = V^{\tau-m-1}(V+W-1). \tag{16}$$

If inequality (12) is satisfied, then the right-hand side of Eq. (16) is negative; that is, the sum of elements of row m is less than that of row $m+1$. Therefore, the infinity norm of $\hat{\boldsymbol{A}}^{\tau+1}(q)$ is given by

$$\left\|\hat{\boldsymbol{A}}^{\tau+1}(q)\right\|_\infty = \sum_{n=1}^{\tau+1}\left\{\hat{\boldsymbol{A}}^{\tau+1}(q)\right\}_{\tau+1,n} = V + W < 1.$$

From propositions (a) and (b), the proof is complete. □

Based on the above results, the following theorem is derived.

Theorem 1: Consider the delayed-coupled maps (1) with (2). The synchronized manifold (4) is stable for any connection delay τ if the following inequalities are satisfied:

$$-\frac{1-\lambda_{\max}}{\varepsilon\lambda_{\max}} < \rho_{\min}, \ \rho_{\max} < \frac{1-\lambda_{\max}}{\varepsilon\lambda_{\max}} + 2. \tag{17}$$

Proof: From Corollary 1 and Lemmas 1 and 2, time-variant system (8) is stable if inequality (12) is satisfied. Thus, synchronized manifold (4) is stable if inequality (12) is satisfied for all $q = 2, \ldots, N$. Here, inequality (12) can be rewritten as

$$-\frac{1-\lambda_{\max}}{\varepsilon\lambda_{\max}} < \rho_q < \frac{1-\lambda_{\max}}{\varepsilon\lambda_{\max}} + 2. \tag{18}$$

Inequality (18) is satisfied for all $q = 2, \ldots, N$ if the lower and upper limits of the eigenvalues satisfy the inequalities in Eq. (17). □

Based on Eq. (17) in Theorem 1, the systematic design procedure is straightforwardly derived.

Corollary 2 (Design procedure)**:** The synchronized manifold (4) is stable for any connection delay τ if the map parameter satisfies

$$\lambda_{\max} < \min\left\{\frac{1}{|1-\rho_{\min}|}, \frac{1}{\rho_{\max}-1}\right\}, \tag{19}$$

and the coupling strength ε satisfies

$$\varepsilon \in \left(\underline{\varepsilon}, 1\right], \underline{\varepsilon} := \max\left\{-\frac{1-\lambda_{\max}}{\lambda_{\max}\rho_{\min}}, \frac{1-\lambda_{\max}}{\lambda_{\max}(\rho_{\max}-2)}\right\}. \tag{20}$$

The design procedure in Corollary 2 is similar to that of the previous study (see Corollary 2 in [10]).

4. Numerical Examples

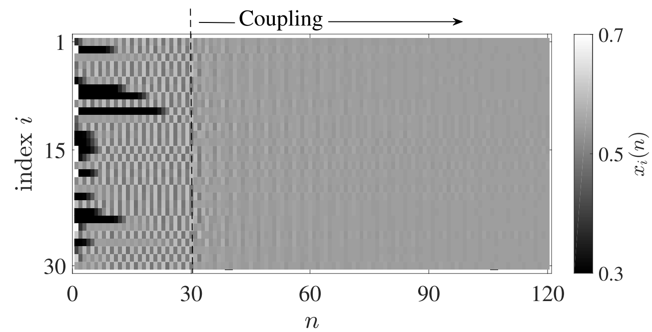
This section numerically confirms the validity of our design procedure (i.e., Corollary 2). From Assumption I, we consider the situation where $\rho_{\min} = 0.28$ and $\rho_{\max} = 1.78$ are given in advance. According to Eqs. (19) and (20), the map parameter and the coupling strength are set to $\lambda_{\max} = 1.2 < 1.2821$ and $\varepsilon = 0.8 \in (0.7576, 1]$, respectively.

Let us employ the tent map,

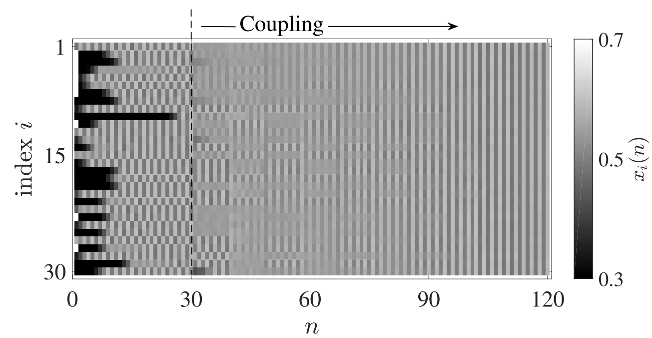
$$f(x) = \frac{a(1-|1-2x|)}{2}, \quad (0 \le a \le 2). \tag{21}$$

The map parameter $\lambda_{\max}$ is given by $\lambda_{\max} = a$; that is, we set $a = 1.2$. For this parameter, the tent map oscillates chaotically with a positive Lyapunov exponent (i.e., $\ln(1.2) > 0$). We confirm the behavior of the coupled maps in a scale-free network with $N = 30$ with lower and upper limits of the eigenvalues satisfying $\rho_2 = 0.2963 \ge \rho_{\min} = 0.28$ and $\rho_{30} = 1.7712 \le \rho_{\max} = 1.78$.

Figure 1 shows the time-series data of $x_i(n)$ for the connection delays $\tau = 1$ and 8. The initial conditions of the all the maps are uniformly randomly chosen in [0, 1] and all the maps oscillate independently until $n = 30$. After coupling at $n = 30$, synchronization occurs for both of the connection delays $\tau = 1$ and 8; that is, our procedure can induce synchronization independently of the connection delay. However, the dynamics of a synchronized manifold in Eq. (5) is totally different: in Fig. 1(a), all the maps stop oscillating (i.e., amplitude death [14]); in Fig. 1(b), all the maps continue to oscillate. This is because the designed procedure in Corollary 2 does not specify the dynamics of the synchronized manifold.



(a) $\tau = 1$



(b) $\tau = 8$

Fig. 1 Time series data of delayed-coupled tent maps in the scale-free network ($N = 30$). The eigenvalues of the scale-free network are $\rho_2 = 0.2963$ and $\rho_{30} = 1.7712$. Pixel color indicates the value of the state variable of the map.

5. Conclusion

The present Letter extended the previous study [10] to delayed-coupled general one-dimensional map networks, and proposed a simple design procedure for inducing synchronization. Future work will include confirming our design procedure in a circuit experiment [15] and extending the procedure to delayed-coupled high-dimensional map networks.

Acknowledgments

The present study is partially supported by JSPS KAKENHI (JP26289131) and (17K12748).

References

[1] S. Strogatz, Sync: The Emerging Science of Spontaneous Order, Hachette Books, 2003.
[2] T. Ikeguchi and I. Tokuda, "Special section on synchronization in

nonlinear science and engineering," Nonlinear Theory and Its Applications, IEICE, vol.3, no.2, pp.112–112, 2012.

[3] L. Pecora and T. Carroll, "Master stability functions for synchronized coupled systems," Phys. Rev. Lett., vol.80, no.10, pp.2109–2112, 1998.

[4] V. Flunkert, I. Fischer, and E. Schöll, "Dynamics, control and information in delay-coupled systems: Overview," Phil. Trans. R. Soc. A, vol.371, no.1999, 2013.

[5] W. Kinzel, A. Englert, G. Reents, M. Zigzag, and I. Kanter, "Synchronization of networks of chaotic units with time-delayed couplings," Phys. Rev. E, vol.79, no.5, 056207, 2009.

[6] A. Englert, S. Heiligenthal, W. Kinzel, and I. Kanter, "Synchronization of chaotic networks with time-delayed couplings: An analytic study," Phys. Rev. E, vol.83, no.4, 046222, 2011.

[7] I. Kanter, M. Zigzag, A. Englert, F. Geissler, and W. Kinzel, "Synchronization of unidirectional time delay chaotic networks and the greatest common divisor," Europhys. Lett., vol.93, no.6, p.60003, 2011.

[8] I.Z. Kiss, C.G. Rusin, H. Kori, and J.L. Hudson, "Engineering complex dynamical structures: Sequential patterns and desynchronization," Science, vol.316, no.5833, pp.1886–1889, 2007.

[9] A. Zlotnik, R. Nagao, István, Z. Kiss, and J.S. Li, "Phase-selective entrainment of nonlinear oscillator ensembles," Nature Commun., vol.7, no.10788, 2016.

[10] Y. Sugitani, T. Watanabe, K. Konishi, and N. Hara, "Delay-independent design for chaotic synchronization in delay-coupled Bernoulli map networks," Nonlinear Theory and Its Applications, IEICE, vol.8, no.2, pp.162–172, 2017.

[11] F.M. Atay, J. Jost, and A. Wende, "Delays, connection topology, and synchronization of coupled chaotic maps," Phys. Rev. Lett., vol.92, no.14, 144101, 2004.

[12] F. Atay, "Oscillator death in coupled functional differential equations near hopf bifurcation," J. Differ. Equations, vol.221, no.1, pp.190–209, 2006.

[13] P. Bauer, K. Premaratne, and J. Dur̍an, "A necessary and sufficient condition for robust asymptotic stability of time-variant discrete systems," IEEE Trans. Autom. Control, vol.38, no.9, pp.1427–1430, 1993.

[14] T. Watanabe, Y. Sugitani, K. Konishi, and N. Hara, "Stability analysis of amplitude death in delay-coupled high-dimensional map networks and their design procedure," Physica D, vol.338, pp.26–33, 2017.

[15] H. Nguyen and T. Tsubone, "Demonstration of chaos synchronization of sampled-data coupled piecewise-linear systems," Proc. International Symposium on Nonlinear Theory and its Applications, pp.570–573, Yugawara, Japan, 2016.