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Time-delay-induced amplitude death in chaotic map lattices and its avoiding control

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Abstract

The present paper deals with amplitude death in chaotic map lattices coupled with a diffusive delay connection. It is shown that if a fixed point of the individual map satisfies an odd-number property, then amplitude death never occurs at the fixed point for any number of the maps, coupling strength, and delay time. From the viewpoint of engineering applications that utilize oscillatory behavior in coupled oscillators, death would be undesirable. This paper proposes a feedback controller, which is added to each chaotic map, such that the fixed point of the individual map satisfies the odd-number property. Accordingly, it is guaranteed that death never occurs in the controlled chaotic-map-lattice. It is verified that the proposed controller works well in numerical simulations.

Key words: amplitude death, coupled systems, time delay, feedback control
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1 Introduction

Diffusive coupled oscillators that do not have the same parameters can cease their own oscillations [1–5]. This phenomenon, often called *Amplitude Death* or *Oscillation Death*, has been investigated theoretically [6–8] and experimentally [9–11] for nearly twenty years. It was proven that the death phenomenon never occurs in diffusive coupled *identical* oscillators [2,4,12,13]. Reddy *et al.* found that a *time delay* connection can induce the death phenomenon even in coupled identical oscillators [14]. This time-delay-induced amplitude death

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has attracted a growing interest in the field of nonlinear science [15]. It was experimentally observed that delay-induced death can occur in electrical circuits [16] and thermo-optical oscillators [17]. Furthermore, the theoretical analysis of delay-induced death has been an important subject of many research papers: stability of death in coupled simple oscillators near Hopf bifurcations [18], death induced by dynamical connections [19], sufficient condition, called the *odd number property*, under which death never occurs [12,13], oscillators coupled by a one-way time-delay connection [20], distributed delay effect [21], total and partial death in networks [22], coupled chaotic oscillators [23], and a ring of coupled limit cycle oscillators [24,25].

The previous papers [12,13] showed that if a fixed point in two coupled identical oscillators satisfies the odd number property, then death never occurs at the fixed point for any coupling strength and delay time. On the other hand, from a viewpoint of engineering applications that utilize oscillatory behavior in coupled oscillators ², the death phenomenon must be avoided. The present paper extends the previous result [12] to a large scale chaotic-map-lattices; in addition, on the basis of the extended result, a feedback controller (see Fig. 1) that never induces death is proposed. The proposed controller has the following four features: it is valid for high-dimensional local maps; it has a one-dimensional simple structure; it can be designed by a simple systematic procedure independent of coupling strength, delay time, and number of maps; it works for periodic, quasi-periodic, chaotic maps. In order to verify the theoretical results, it is demonstrated that the proposed controller works well in numerical simulations.

2 Chaotic maps with delayed coupling

Consider m -dimensional chaotic maps

$$\begin{cases} \mathbf{x}_i(n+1) = \mathbf{f}(\mathbf{x}_i(n)) + \mathbf{b}_w w_i(n) \\ z_i(n) = \mathbf{c}_z \mathbf{x}_i(n) \end{cases} \quad (i = 1, 2, \dots, N), \quad (1)$$

where $\mathbf{x}_i(n) = [x_i^{(1)}(n) \ x_i^{(2)}(n) \ \dots \ x_i^{(m)}(n)]^T \in \mathbb{R}^m$ is i -th map state at time $n \in \mathbb{N}$ and $N \in \mathbb{N}$ denotes the number of maps. $w_i(n) \in \mathbb{R}$ and $z_i(n) \in \mathbb{R}$ are the input and output signals, and $\mathbf{b}_w \in \mathbb{R}^m$ and $\mathbf{c}_z \in \mathbb{R}^{1 \times m}$ are the input and output vectors. It is assumed that the local nonlinear function $\mathbf{f} : \mathbb{R}^m \rightarrow \mathbb{R}^m$

² Oscillatory behavior in coupled oscillators is used for practical applications; for example, coupled magnetrons for high-power microwave sources [26] and a central pattern generator for controlling multi-legged robots [27].

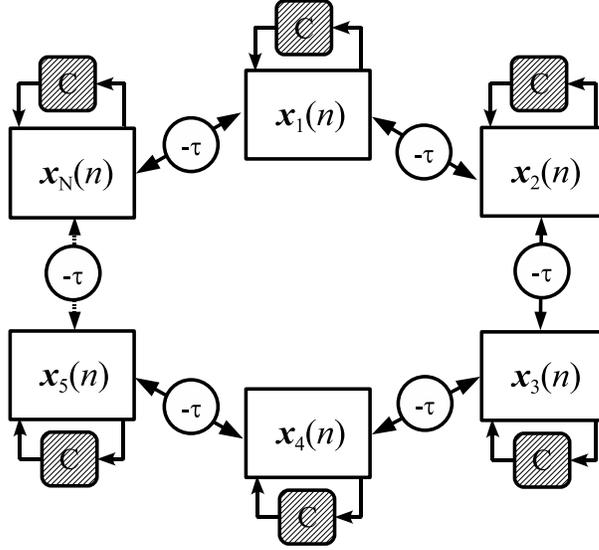


Fig. 1. Sketches of controlled chaotic maps with delayed coupling. The boxed $\mathbf{x}_i(n)$ and the circled $-\tau$ indicate the i -th chaotic map and the time delay connection, respectively. C in the shaded box represents the feedback controller.

has hyperbolic fixed points $\mathbf{x}_f : \mathbf{f}(\mathbf{x}_f) = \mathbf{x}_f$. These maps are assumed to be coupled by the following delayed diffusive connection [12]:

$$w_i(n) = \frac{1}{2}\varepsilon \{z_{i-1}(n - \tau) + z_{i+1}(n - \tau) - 2z_i(n)\}. \quad (2)$$

$\varepsilon \in \mathbb{R}$ is the coupling strength and $\tau \in \mathbb{N}$ is the delay time. As sketched in Fig 1, the boundary condition is periodic (i.e., $z_0(n) := z_N(n)$, $z_{N+1}(n) = z_1(n)$). The chaotic maps (1) with delayed coupling (2) have the steady state

$$\left[\mathbf{x}_1(n)^T \ \mathbf{x}_2(n)^T \ \cdots \ \mathbf{x}_N(n)^T \right]^T = \left[\mathbf{x}_f^T \ \mathbf{x}_f^T \ \cdots \ \mathbf{x}_f^T \right]^T, \quad (3)$$

where T represents the transpose of a vector or a matrix. This paper regards amplitude death as the phenomenon where every maps stop their oscillations and all of the coupling signals $w_i(n)$ become zero³. Consequently, the stabilization of steady state (3) is a necessary condition for death⁴. The remainder of this section will consider the odd-number property [29–31] for $N = 2$ and $N \geq 2$.

³ In other words, all of the maps converge on steady state (3). If oscillation ceases at the other steady states except for state (3), then signals $w_i(n)$ do not become zero. The present paper does not deal with the latter case.

⁴ Even if steady state (3) of the coupled maps is stable, death may not occur. This is because coupled maps happen to settle on one of the coexistent spatiotemporal patterns [28]. Thus the stability condition cannot provide sufficiency.

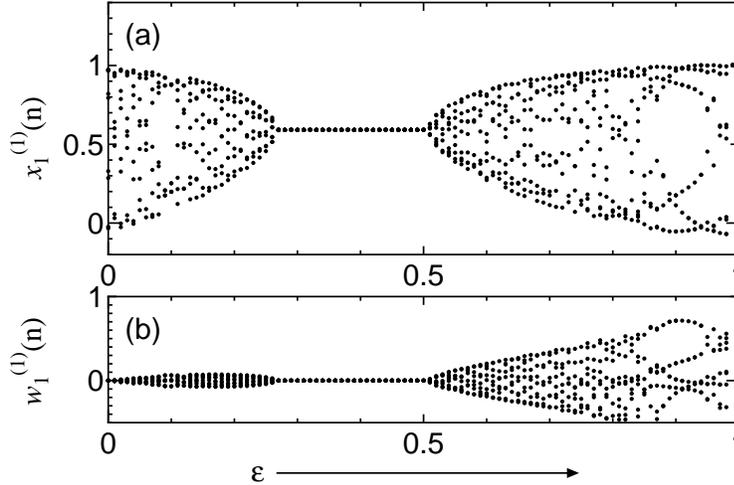


Fig. 2. Bifurcation diagrams of coupled generalized Henon maps (4) for coupling strength ε ($N = 2$). (a) System state $x_1^{(1)}(n)$. (b) Coupling signal $w_1(n)$.

2.1 In case of $N = 2$

A previous paper [12] derived the following condition in the case of $N = 2$.

Lemma 1 ([12]) *Consider the Jacobi matrix of map \mathbf{f} evaluated at hyperbolic fixed point \mathbf{x}_f ,*

$$\mathbf{A} = \left. \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_f}.$$

If \mathbf{A} has an odd number of real eigenvalues greater than 1 (i.e., \mathbf{A} satisfies the odd-number property), then steady state (3) is not stabilized for any \mathbf{b}_w , \mathbf{c}_z , τ , ε . Accordingly, amplitude death never occurs.

This Lemma was then extended to continuous-time oscillators [13]. Now Lemma 1 is checked by the following example.

[Example 1] The generalized Henon map [33]

$$\mathbf{f}(\mathbf{x}) = \left[\gamma_1 - x_2^2 - \gamma_2 x_3 \quad x_1 \quad x_2 \right]^T, \quad (4)$$

is used as a local map, where the parameters $\gamma_1 = 1.0$ and $\gamma_2 = 0.1$ are fixed. The function \mathbf{f} has two fixed points: $\mathbf{x}_{f(1)} = 0.5913 \left[1 \ 1 \ 1 \right]^T$, $\mathbf{x}_{f(2)} =$

$-1.6913 \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$. The Jacobi matrices at these points are

$$\mathbf{A}_{(1)} = \begin{bmatrix} 0 & -1.1825 & -0.1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{A}_{(2)} = \begin{bmatrix} 0 & 3.3825 & -0.1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

respectively. The eigenvalues of the matrices are estimated as $\lambda_{1,2,3}(\mathbf{A}_{(1)}) = 0.0420 \pm i1.0899, -0.0841$, $\lambda_{1,2,3}(\mathbf{A}_{(2)}) = -1.8538, 0.0296, 1.8242$. Since $\mathbf{A}_{(2)}$ satisfies Lemma 1, death never occurs at $\mathbf{x}_{f(2)}$ for any $\mathbf{b}_w, \mathbf{c}_z, \varepsilon$, and τ . On the other hand, $\mathbf{A}_{(1)}$ does not satisfy Lemma 1, so it is not known whether death occurs or not. The coupling vectors and the delay time are set to $\mathbf{b}_w = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$, $\mathbf{c}_z = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $\tau = 1$. The bifurcation diagrams of system state $x_1^{(1)}(n)$ and coupling signal $w_1(n)$ at the first site for the coupling strength ε are shown in Figs. 2(a) and (b), respectively. It can be seen that death occurs for $\varepsilon \in [0.27, 0.50]$ in which the oscillation ceases and the coupling signal $w_1(n)$ becomes zero. This is the death this paper will address. These numerical results exemplify Lemma 1.

2.2 In case of $N \geq 2$

The present paper extends the previous result [12] to large scale systems: coupled map lattices (i.e., $N \geq 2$) shown in Fig. 1. Linearizing map (1) at steady state (3) yields

$$\begin{cases} \boldsymbol{\xi}_i(n+1) = \mathbf{A}\boldsymbol{\xi}_i(n) + \mathbf{b}_w w_i(n) \\ \Delta z_i(n) = \mathbf{c}_z \boldsymbol{\xi}_i(n) \end{cases} \quad (i = 1, 2, \dots, N), \quad (5)$$

where $\boldsymbol{\xi}_i(n) := \mathbf{x}_i(n) - \mathbf{x}_f$, $\Delta z_i(n) := z_i(n) - \mathbf{c}_z \mathbf{x}_f$. The extended result of Lemma 1 is given as follows.

Theorem 2 *If \mathbf{A} has an odd number of real eigenvalues greater than 1, then steady state (3) for $N \geq 2$ is not stabilized for any $\mathbf{b}_w, \mathbf{c}_z, \tau, \varepsilon, N$. Accordingly, amplitude death never occurs.*

PROOF. The characteristic equation of linearized system (5) with time-delayed diffusive coupling (2) is written as $\det \mathbf{H}(\lambda) = 0$, where $\mathbf{H}(\lambda) \in \mathbb{R}^{N \times N}$

is described by

$$\mathbf{H}(\lambda) = \begin{bmatrix} H_0 & H_1 & 0 & \cdots & H_1 \\ H_1 & H_0 & H_1 & \cdots & 0 \\ 0 & H_1 & H_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_1 & 0 & 0 & \cdots & H_0 \end{bmatrix}. \quad (6)$$

The matrix elements are given by $H_0 := \lambda^\tau (d(\lambda) + \varepsilon n(\lambda))$ and $H_1 := -\frac{1}{2}\varepsilon n(\lambda)$. Let $n(\lambda)/d(\lambda)$ denote the transfer function of linear system (5) from $w_i(n)$ to $\Delta z_i(n)$, where

$$n(\lambda) := \mathbf{c}_z \text{adj}(\lambda \mathbf{I}_m - \mathbf{A}) \mathbf{b}_w, \quad d(\lambda) := \det(\lambda \mathbf{I}_m - \mathbf{A}). \quad (7)$$

A feature of the circulant matrix (see Appendix A) simplifies $\det \mathbf{H}(\lambda)$:

$$\det \mathbf{H}(\lambda) = \prod_{q=0}^{N-1} h_N(\lambda, q), \quad h_N(\lambda, q) := \lambda^\tau d(\lambda) + \varepsilon n(\lambda) \left\{ \lambda^\tau - \cos\left(\frac{2\pi q}{N}\right) \right\}. \quad (8)$$

It is obvious that the roots of the characteristic equation $\det \mathbf{H}(\lambda) = 0$ include those of $h_N(\lambda, 0) = 0$. At least one real root of $h_N(\lambda, 0) = 0$ is greater than 1 if $h_N(\lambda, 0)$ satisfies the following two conditions: (a) $\lim_{\lambda \rightarrow +\infty} h_N(\lambda, 0) = +\infty$; (b) $h_N(1, 0) < 0$. Apparently condition (a) is satisfied, and condition (b) can be determined by

$$h_N(1, 0) = d(1) = \det(\mathbf{I}_m - \mathbf{A}) = \prod_{j=1}^m (1 - \lambda_j). \quad (9)$$

λ_j denotes the eigenvalue of Jacobi matrix \mathbf{A} . Equation (9) implies that if \mathbf{A} has an odd number of real eigenvalues greater than 1, then $h_N(1, 0) < 0$ holds. In other words, the characteristic equation $\det \mathbf{H}(\lambda) = 0$ has at least one real root $\lambda > 1$. \square

[Example 2] Fifty chaotic maps, which are the same as in Example 1, are coupled as illustrated in Fig. 1. Theorem 2 guarantees that death never occurs at steady state (3) composed of $\mathbf{x}_{f(2)}$; however, it cannot provide any information about $\mathbf{x}_{f(1)}$. Now an average distance between system state $\mathbf{x}_i(n)$ and steady state $\mathbf{x}_{f(l)}$ ($l = 1, 2$) are proposed in order to check the death in coupled map lattices:

$$\delta_{(l)} = \frac{1}{N} \lim_{n \rightarrow +\infty} \sum_{i=1}^N \|\mathbf{x}_i(n) - \mathbf{x}_{f(l)}\|.$$

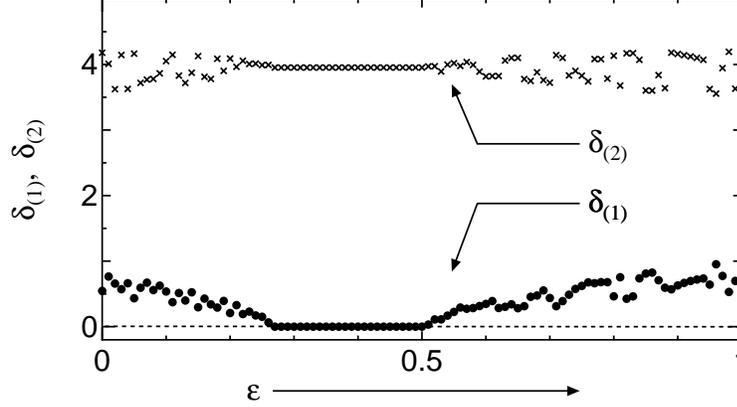


Fig. 3. Death in the coupled map lattice whose local map is generalized Henon map (4) ($N = 50$)

Apparently $\delta_{(l)}$ becomes zero when death occurs at steady state (3) composed of $\mathbf{x}_{f(l)}$. $\delta_{(1)}$ and $\delta_{(2)}$ estimated at $n = 100000$ are shown in Fig. 3. It can be seen that $\delta_{(2)}$ does not become zero for any ε . Then it is noticed that death does not occur at steady state (3) composed of $\mathbf{x}_{f(2)}$. On the other hand, $\delta_{(1)}$ remains at zero for $\varepsilon \in [0.27, 0.50]$, then the death occurs.

In the numerical simulations, we observe that this ε region of death is independent of number N . For instance, ε region of death in Fig. 3 and that in Figs. 2(a)(b) are completely identical. This fact can be explained by the following result.

Corollary 3 *If the inequality*

$$\bar{\lambda}(\varepsilon) < 1, \quad (10)$$

is held, then steady state (3) is stable for any number N of maps. $\bar{\lambda}(\varepsilon)$ is defined as

$$\bar{\lambda}(\varepsilon) := \max_{\alpha \in [-1, +1]} \hat{\lambda}(\alpha, \varepsilon) \quad (11)$$

$$\hat{\lambda}(\alpha, \varepsilon) := \max \{ |\lambda| : \lambda^\tau d(\lambda) + \varepsilon n(\lambda) (\lambda^\tau - \alpha) = 0 \}. \quad (12)$$

PROOF. In characteristic polynomial (8), $\cos\left(\frac{2\pi q}{N}\right) \in [-1, +1]$ holds for any $N \geq 2$ and $q \in \{0, 1, \dots, N-1\}$. Then, if condition (10) is satisfied, all of the roots of the characteristic equation $\det \mathbf{H}(\lambda) = 0$ are within the unit circle on the complex plane. Hence, stability of steady state (3) is independent of N . \square

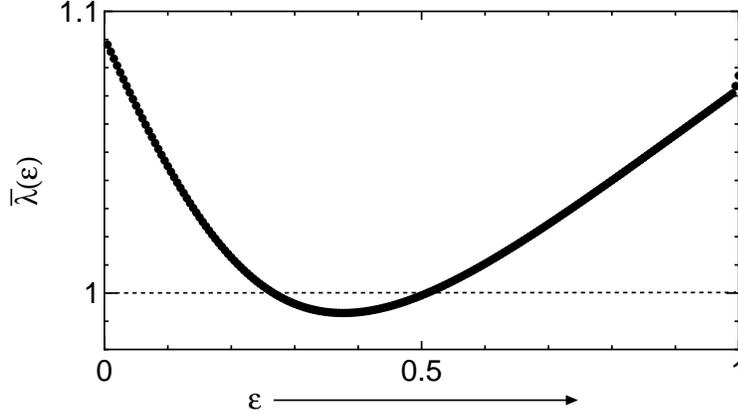


Fig. 4. $\bar{\lambda}(\varepsilon)$ for generalized Henon map (4)

Figure 4 shows $\bar{\lambda}(\varepsilon)$ for steady state (3) composed of $\mathbf{x}_{f(1)}$. As can be seen from Fig. 4, condition (10) is satisfied in the range $\varepsilon \in [0.27, 0.50]$. Therefore, it is guaranteed that steady state (3) composed of $\mathbf{x}_{f(1)}$ is stable for any $N \geq 2$ in the coupling strength range.

Corollary 4 *If the delayed feedback control (DFC) system,*

$$\mathbf{x}(n+1) = \mathbf{f}(\mathbf{x}(n)) + \mathbf{b}_w \varepsilon \{ \mathbf{c}_z \mathbf{x}(n-\tau) - \mathbf{c}_z \mathbf{x}(n) \}, \quad (13)$$

has an unstable fixed point \mathbf{x}_f , then linearized system (5) with time-delayed diffusive coupling (2) becomes unstable for any $N \geq 2$. Accordingly, death never occurs.

PROOF. The characteristic polynomial (8) includes a common polynomial $h_N(\lambda, 0)$ for any $N \geq 1$. DFC system (13) is a special case of the coupled map with $N = 1$; hence, its characteristic polynomial becomes $h_1(\lambda, 0)$, which is the same as the polynomials $h_N(\lambda, 0)$ for $N \geq 2$. Therefore, if system (13) has an unstable fixed point \mathbf{x}_f (i.e., $h_1(\lambda, 0)$ is an unstable polynomial), then steady state (3) is also unstable for any $N \geq 2$. \square

Corollary 4 can not guarantee that steady state (3) is stable even if fixed point \mathbf{x}_f of the DFC system (13) is stable. Furthermore, if one can change \mathbf{b}_w and \mathbf{c}_z , then Corollary 4 allows for avoidance of death in chaotic map lattices by the following procedure: \mathbf{b}_w and \mathbf{c}_z are adjusted such that the fixed point \mathbf{x}_f of DFC system (13) becomes unstable.

Dodla, Sen, and Johnston [24] investigated amplitude death in a ring of delayed coupled limit cycle oscillators [25]. Its network structure (i.e. ring type structure) is the same as our connection (2). These papers [24,25] employed two-dimensional limit cycle oscillators near the super critical Hopf bifurcation, and derived the death island boundaries that depend on the coupling

strength, delay time, and number of oscillators. On the contrary, Theorem 2 in the present paper uses m -dimensional chaotic maps and derived a simple sufficient condition under which death never occurs independent of the coupling strength, the delay time, the input and output vectors for coupling, and the number of maps. In the next section, it is shown that this sufficient condition is convenient for designing a feedback controller.

3 Feedback control

3.1 Control system

The input $u_i(n) \in \mathbb{R}$ and output $y_i(n) \in \mathbb{R}$ control signals are added to map (1):

$$\begin{cases} \mathbf{x}_i(n+1) = \mathbf{f}(\mathbf{x}_i(n)) + \mathbf{b}_w w_i(n) + \mathbf{b}_u u_i(n) \\ z_i(n) = \mathbf{c}_z \mathbf{x}_i(n) \\ y_i(n) = \mathbf{c}_y \mathbf{x}_i(n) \end{cases} \quad (i = 1, 2, \dots, N). \quad (14)$$

Here, $\mathbf{b}_u \in \mathbb{R}^m$, $\mathbf{c}_y \in \mathbb{R}^{1 \times m}$ are the input and output vectors for the control signals, respectively. The proposed controllers,

$$u_i(n) = \begin{cases} k(y_i(n) - y_f) & |y_i(n) - y_f| < \mu, \\ 0 & \text{otherwise.} \end{cases}, (i = 1, 2, \dots, N), \quad (15)$$

are connected to the maps as shown in Fig. 1, where y_f is given by $y_f := \mathbf{c}_y \mathbf{x}_f$ and $k \in \mathbb{R}$ is the feedback gain. If the output signal $y_i(n)$ comes close to y_f , then the i -th controller provides the input signal $u_i(n)$ for the i -th map, otherwise the controller provides no signal. Threshold μ is set to a positive small value. It should be noted that the controller may provide the signal even if the map state $\mathbf{x}_i(n)$ does not come close to \mathbf{x}_f . This is because $y_i(n) = \mathbf{c}_y \mathbf{x}_i(n)$ does not include all information of the map state $\mathbf{x}_i(n)$ ⁵.

⁵ If the all system states (i.e., $x_i^{(1)}(n)$, $x_i^{(2)}(n)$, \dots , $x_i^{(m)}(n)$) are observable, the controller criterion should be changed as $\|\mathbf{x}_i(n) - \mathbf{x}_f\| < \mu$.

3.2 Design of controller

System (14) is linearized at steady state (3):

$$\begin{cases} \xi_i(n+1) = \mathbf{A}\xi_i(n) + \mathbf{b}_w w_i(n) + \mathbf{b}_u u_i(n) \\ \Delta z_i(n) = \mathbf{c}_z \xi_i(n) \\ \Delta y_i(n) = \mathbf{c}_y \xi_i(n) \end{cases} \quad (i = 1, 2, \dots, N),$$

where $\Delta y_i(n) := y_i(n) - \mathbf{c}_y \mathbf{x}_f$. The dynamics of map (14) controlled by (15) is governed by

$$\begin{cases} \xi_i(n+1) = (\mathbf{A} + \mathbf{b}_u k \mathbf{c}_y) \xi_i(n) + \mathbf{b}_w w_i(n) \\ \Delta z_i(n) = \mathbf{c}_z \xi_i(n) \end{cases} \quad (i = 1, 2, \dots, N). \quad (16)$$

Linearized system (16) corresponds to system (5). From Theorem 2, it is known that if $\mathbf{A}' := \mathbf{A} + \mathbf{b}_u k \mathbf{c}_y$ satisfies the odd number property, then steady state (3) is not stabilized for any $\mathbf{b}_w, \mathbf{c}_z, \tau, \varepsilon, N$. Accordingly, death never occurs.

Now, attention is focused on a controller design problem how to determine the gain k such that \mathbf{A}' satisfies the odd number property. Theorem 5 can solve this problem.

Theorem 5 *Steady state (3) is not stabilized for any $\mathbf{b}_w, \mathbf{c}_z, \varepsilon, \tau, N$ if the gain k satisfies the inequality:*

$$1 - k \mathbf{c}_y (\mathbf{I}_m - \mathbf{A})^{-1} \mathbf{b}_u < 0. \quad (17)$$

Accordingly, amplitude death never occurs.

PROOF. Let us show that if inequality (17) is held, then \mathbf{A}' satisfies the odd number property. The characteristic polynomial of \mathbf{A}' is $g(\lambda) = \det [\lambda \mathbf{I}_m - \mathbf{A}']$. It is obvious that the roots $\lambda_{1, \dots, m}$ of $g(\lambda) = 0$ are the eigenvalues of \mathbf{A}' . The characteristic polynomial $g(\lambda)$ at $\lambda = 1$ can be described by

$$\begin{aligned} g(1) &= \det [\mathbf{I}_m - \mathbf{A} - \mathbf{b}_u k \mathbf{c}_y] \\ &= \{1 - k \mathbf{c}_y (\mathbf{I}_m - \mathbf{A})^{-1} \mathbf{b}_u\} \det [\mathbf{I}_m - \mathbf{A}]. \end{aligned}$$

Since \mathbf{A} does not satisfy the odd number property, $\det [\mathbf{I}_m - \mathbf{A}] > 0$ holds. $g(1) < 0$ is equivalent to that \mathbf{A}' satisfies the odd number property. Therefore,

if inequality (17) is held (i.e., $g(1) < 0$), then \mathbf{A}' satisfies the odd number property. \square

From this theorem, it is observed that, in order to determine gain k , the only requirement of the controller design is knowledge of \mathbf{A} , \mathbf{b}_u , and \mathbf{c}_y . In addition, the controller does not require the map state $\mathbf{x}_i(n)$: even a part of state information, $y_i(n)$, is observable. This situation would be convenient for practical applications.

A similar result was reported in [34]. This report, however, restricts the case of $N = 2$, and does not consider a large-scale coupled chaotic maps. Furthermore, since the proposed controller is designed to destabilize steady state (3), it can be classified into the field of anti-control of chaos [35].

Martí, Ponce, and Masoller studied a steady-state stabilization in coupled map lattices with random delays connection [32]: they show that an instability scenario depends on whether all the delays are odd or even. On the other hand, our main results (Theorems 2 and 5) are valid for any delay; then, the results do not depend on odd-or-even.

3.3 Numerical examples

The controller will be designed according to Theorem 5 and the controller performance will be confirmed using numerical simulations. The function \mathbf{f} , input vector \mathbf{b}_w , and output vector \mathbf{c}_z for the coupling are the same as previous numerical examples. The input and output vectors for the control signals are set to $\mathbf{b}_u = [0 \ 1 \ 0]^T$, $\mathbf{c}_y = [1 \ 0 \ 0]$, respectively. From these values, $\mathbf{c}_y(\mathbf{I}_m - \mathbf{A})^{-1}\mathbf{b}_u = -0.5619$ can be estimated; hence, k is designed such that $1 + 0.5619k < 0$. In this numerical example, the gain is fixed at $k = -2$. \mathbf{A}' has the eigenvalues $\lambda_{1,2,3}(\mathbf{A}') = 1.1275, -1.0424, -0.0851$, so \mathbf{A}' satisfies the odd number property. The threshold of the controllers is set to $\mu = 0.1$ and y_f is given by $y_f = \mathbf{c}_y\mathbf{x}_{f(1)}$.

In the case of $N = 2$, the bifurcation diagrams of the first map state $x_1^{(1)}(n)$ and the input control signal $u_1(n)$ are shown in Figs. 5 (a) and (b), respectively. From these diagrams, it is seen that death does not occur in the region $\varepsilon \in [0.27, 0.50]$ where death occurs in Fig. 2(a). Figure 6 shows the average distances $\delta_{(1)}$ and $\delta_{(2)}$ for a large scale chaotic-map-lattice $N = 50$. $\delta_{(1)}$ in Fig. 3 becomes zero in the region $\varepsilon \in [0.27, 0.50]$; on the contrary, in Fig. 6, there is no region where $\delta_{(1)}$ becomes zero. The above results demonstrate that the proposed controller works well independent of the number of maps, N .

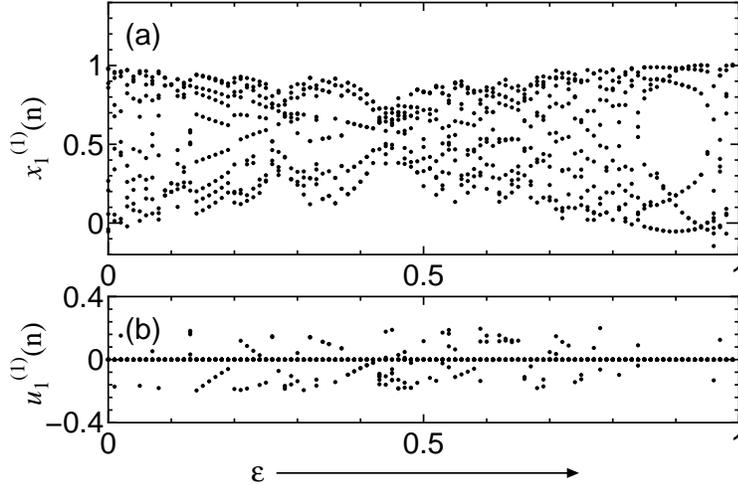


Fig. 5. Bifurcation diagram (with control) of generalized Henon map (4) ($N = 2$): (a) map state $x_1^{(1)}(n)$; (b) input signal $u_1(n)$ for control.

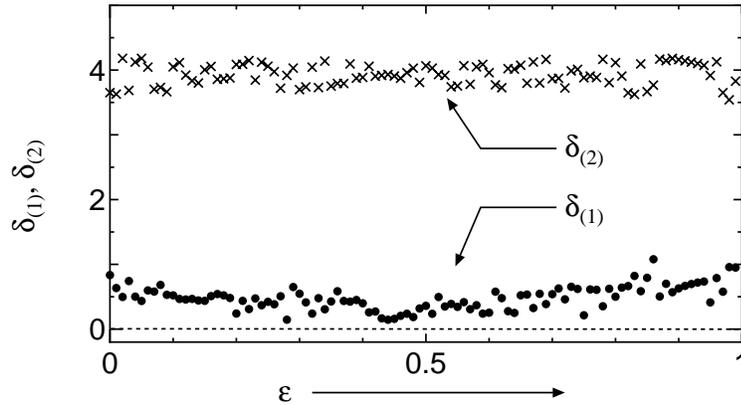


Fig. 6. Avoiding death in a large scale coupled map lattice (generalized Henon map (4) ($N = 50$))

4 Conclusion

This paper extends the odd-number property for small scale maps (i.e. $N = 2$) to large scale chaotic-map lattices (i.e. $N \geq 2$). A controller is proposed to avoid amplitude death in large scale chaotic map lattices. The extended odd-number property allows for the systematic design of the controller. The numerical simulations show that the designed controller successfully avoids amplitude death.

Our results are useful when oscillatory behavior in coupled oscillators is used

for applications. If the structure of oscillator is allowed to be changed, then one should modify it on the basis of Theorem 2. Otherwise one should use feedback signal (15) and design the feedback gain k according to Theorem 5. This procedure could be worthwhile to avoid death in physical systems, such as electronic oscillators and mechanical systems.

A Circulant matrix

Consider an N -square circulant matrix,

$$\mathbf{V} = \begin{bmatrix} v_0 & v_1 & 0 & \cdots & v_1 \\ v_1 & v_0 & v_1 & \cdots & 0 \\ 0 & v_1 & v_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_1 & 0 & 0 & \cdots & v_0 \end{bmatrix}.$$

From Theorem 4.8 in [36], the determinant of \mathbf{V} can be given as

$$\det \mathbf{V} = \prod_{q=0}^{N-1} f(\omega^q).$$

Since $f(\eta) := v_0 + v_1\eta + v_1\eta^{N-1}$ and $\omega := e^{j\frac{1}{N}2\pi}$, then $\det \mathbf{V}$ is simplified as follows:

$$\det \mathbf{V} = \prod_{q=0}^{N-1} \left[v_0 + v_1 e^{j\frac{q}{N}2\pi} + v_1 e^{j\frac{(N-1)q}{N}2\pi} \right] \quad (\text{A.1})$$

$$= \prod_{q=0}^{N-1} \left[v_0 + 2v_1 \cos\left(\frac{2\pi q}{N}\right) \right]. \quad (\text{A.2})$$

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