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Amplitude death induced by a global dynamic coupling

Keiji Konishi *

Abstract

This paper presents a dynamic connection that can induce amplitude death in globally coupled oscillators. A linear analysis clarifies a local stability condition for global amplitude death. The analysis also indicates that the odd-number property, which is known in delayed feedback control, exists in global dynamic coupled oscillators. Furthermore, global amplitude death is experimentally observed in Chua's circuits coupled by an RC line.

1 Introduction

Amplitude death, an oscillation stops in diffusive coupled oscillators, has been studied in the field of nonlinear physics [Yamaguchi & Shimizu, 1984; Bar-Eli, 1985; Aronson *et al.*; Pikovsky *et al.*, 2001, 1990; Mirollo & Strogatz, 1990]. It is known that death never occurs for a pair of identical oscillators [Bar-Eli, 1985; Aronson *et al.*, 1990; Konishi, 2003a; Konishi, 2005]. However, Reddy *et al.* [1998] found that a *time delay* connection can induce amplitude death in coupled identical oscillators. Time-delay induced death has created considerable interest [Strogatz, 1998]: it was theoretically investigated in detail [Reddy *et al.*, 1999] and was experimentally observed in electronic circuits [Reddy *et al.*, 2000] and thermo-optical oscillators [Herrero *et al.*, 2000]. Furthermore, a sufficient condition under which death never occurs was derived [Konishi, 2003a; Konishi, 2004a; Konishi, 2005].

It was recently reported that amplitude death in two coupled identical oscillators can be induced by incorporating a dynamic coupling without a time delay [Konishi, 2003b; Konishi, 2004b]. These reports provided the following results: the death was observed in both numerical simulations and electronic circuit experiments [Konishi, 2003b; Konishi, 2004b]; a sufficient condition under which death never occurs was derived [Konishi, 2003b]; and a necessary and sufficient condition for death in van der Pol oscillators was obtained [Konishi, 2004b]. However, these results are exclusive to two identical oscillators. It would be advantageous to extend these results to an arbitrary number of identical oscillators and to confirm the experimental feasibility of death.

In this paper, a dynamic coupling is proposed that can induce amplitude death in globally coupled systems with an arbitrary number of identical oscillators. This system can be realized by electronic oscillators coupled with an RC line connection. It is proven that death induced by the dynamic coupling never occurs if the Jacobi matrix evaluated at fixed point of an

*Department of Electrical and Information Systems, Osaka Prefecture University, 1-1 Gakuen-cho, Nakaku, Sakai, Osaka 599-8531 Japan, e-mail: konishi@eis.osakafu-u.ac.jp

isolated oscillator satisfies an odd-number property. The circuit experiments demonstrate that amplitude death occurs in Chua's oscillators coupled by an RC line connection.

2 Coupled Oscillators

Consider N identical oscillators that are globally coupled by a coupling unit as illustrated in Fig. 1. The identical oscillators are described by

$$\Sigma_j : \begin{cases} \dot{\mathbf{x}}_j &= \mathbf{F}(\mathbf{x}_j) + \mathbf{b}u_j \\ y_j &= \mathbf{c}\mathbf{x}_j, \end{cases} \quad (j = 1, 2, \dots, N)$$

where $\mathbf{x}_j \in \mathbf{R}^m$ is the m -dimensional system variable and $u_j \in \mathbf{R}$ and $y_j \in \mathbf{R}$ are the coupling signals. $\mathbf{F} : \mathbf{R}^m \rightarrow \mathbf{R}^m$ is a continuously differentiable nonlinear function. The input and output vectors are denoted by $\mathbf{b} \in \mathbf{R}^m$ and $\mathbf{c} \in \mathbf{R}^{1 \times m}$. Each individual oscillator is assumed to have an unstable fixed point \mathbf{x}_f (i.e., $\mathbf{F}(\mathbf{x}_f) = \mathbf{0}$).

Two types of globally diffusive coupling are considered: static and dynamic. Static coupling is denoted as

$$u_j = k_s \left\{ \left(\frac{1}{N-1} \sum_{\substack{l=1 \\ l \neq j}}^N y_l \right) - y_j \right\}, \quad (1)$$

where $k_s \in \mathbf{R}$ is the coupling strength. The input signal u_j is proportional to the difference between its own signal y_j and the mean of the other output signals. The steady state of the static coupled system is described by

$$[\mathbf{x}_1^T \quad \mathbf{x}_2^T \quad \dots \quad \mathbf{x}_N^T]^T = [\mathbf{x}_f^T \quad \mathbf{x}_f^T \quad \dots \quad \mathbf{x}_f^T]^T. \quad (2)$$

On the other hand, for dynamic coupling, the oscillators Σ_j ($j = 1, 2, \dots, N$) are coupled by

$$\dot{z} = \gamma \left\{ \left(\sum_{l=1}^N y_l \right) - Nz \right\}, \quad (3a)$$

$$u_j = k_d(z - y_j), \quad (3b)$$

where $k_d \in \mathbf{R}$ is the coupling strength. The coupling signal u_j is proportional to the difference between $z \in \mathbf{R}$ and y_j , where z is an additional variable governed by dynamical equation (3a) and $\gamma > 0$ is a parameter. It should be noted that this coupling differs from that described in a previous paper [Konishi, 2003b] (see Appendix A). The steady state of the dynamic coupled system is described by

$$[\mathbf{x}_1^T \quad \mathbf{x}_2^T \quad \dots \quad \mathbf{x}_N^T \quad z]^T = [\mathbf{x}_f^T \quad \mathbf{x}_f^T \quad \dots \quad \mathbf{x}_f^T \quad \mathbf{c}\mathbf{x}_f]^T. \quad (4)$$

Since static coupling (1) and dynamic coupling (3) are diffusive, they both exhibit the following common features: coupling signals u_j ($j = 1, 2, \dots, N$) become zero even if all of the system variables \mathbf{x}_j ($j = 1, 2, \dots, N$) are synchronized; the coupling does not change the location of fixed point \mathbf{x}_f .

Wu [2000] provided a generalized form of the coupled systems from the viewpoint of chaotic synchronization. The coupled system proposed in this paper is a special case of Wu's form.

3 Linear Stability Analysis

This section presents a linear stability analysis of steady states (2) and (4). Let $\mathbf{x}_j := \mathbf{x}_f + \mathbf{X}_j$ ($j = 1, 2, \dots, N$), where \mathbf{X}_j is assumed to be small. The linearized subsystems at \mathbf{x}_f ,

$$\Delta\Sigma_j : \begin{cases} \dot{\mathbf{X}}_j &= \mathbf{A}\mathbf{X}_j + \mathbf{b}U_j \\ Y_j &= \mathbf{c}\mathbf{X}_j, \end{cases}$$

are obtained by substituting \mathbf{x}_j into oscillators Σ_j , where $Y_j := y_j - \mathbf{c}\mathbf{x}_f$. The Jacobi matrix of the nonlinear function \mathbf{F} is given by $\mathbf{A} := \{\partial\mathbf{F}(\mathbf{x})/\partial\mathbf{x}\}_{\mathbf{x}=\mathbf{x}_f}$. \mathbf{A} is assumed not to have an eigenvalue on the origin throughout this paper. The linearized subsystems $\Delta\Sigma_j$ are then coupled by

$$U_j = k_s \left\{ \left(\frac{1}{N-1} \sum_{\substack{l=1 \\ l \neq j}}^N Y_l \right) - Y_j \right\}, \quad (5)$$

for static coupling. On the other hand, $\Delta\Sigma_j$ are coupled with

$$\dot{Z} = \gamma \left\{ \left(\sum_{l=1}^N Y_l \right) - NZ \right\}, \quad (6a)$$

$$U_j = k_d(Z - Y_j), \quad (6b)$$

for dynamic coupling, where $Z := z - \mathbf{c}\mathbf{x}_f$.

3.1 Static Coupling

The linear stability of steady state (2) for the static coupled system is equivalent to that in the linearized systems $\Delta\Sigma_j$ with connection (5). Hence, the closed loop system consisting of $\Delta\Sigma_j$ and (5) is investigated,

$$\dot{\mathbf{X}}_j = \mathbf{A}\mathbf{X}_j + \mathbf{b}k_s\mathbf{c} \left\{ \left(\frac{1}{N-1} \sum_{\substack{l=1 \\ l \neq j}}^N \mathbf{X}_l \right) - \mathbf{X}_j \right\}, \quad (7)$$

for $j = 1, 2, \dots, N$. Appendix B provides the characteristic function of linear system (7),

$$f(\lambda) = f_1(\lambda)f_2(\lambda)^{N-1}, \quad (8)$$

where

$$f_1(\lambda) := \det[\lambda\mathbf{I}_m - \mathbf{A}], \quad f_2(\lambda) := \det \left[\lambda\mathbf{I}_m - \mathbf{A} + \frac{N}{N-1}\mathbf{b}k_s\mathbf{c} \right]. \quad (9)$$

It is obvious that $f_1(\lambda)$ is the characteristic function of $\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}$. Since \mathbf{A} is assumed to be unstable (i.e., \mathbf{x}_f is unstable), $f_1(\lambda) = 0$ has at least one root in the open right-half of the complex plane. This implies that steady-state stabilization never occurs for any $\mathbf{b}, k_s, \mathbf{c}, N$. The above analysis can be summarized as follows: *static coupling (1) never induces amplitude death for any $\mathbf{b}, k_s, \mathbf{c}, N$.*

3.2 Dynamic Coupling

The linear stability of steady state (4) for the dynamic coupled system is equivalent to that in the linearized systems $\Delta\Sigma_j$ with connection (6). The closed loop system consisting of $\Delta\Sigma_j$ and (6) is

$$\dot{\mathbf{X}}_j = \mathbf{A}\mathbf{X}_j + \mathbf{b}k_d(Z - \mathbf{c}\mathbf{X}_j), \quad (10a)$$

$$\dot{Z} = \gamma \left\{ \left(\mathbf{c} \sum_{l=1}^N \mathbf{X}_l \right) - NZ \right\}. \quad (10b)$$

The characteristic function of linear system (10) can be simplified to

$$g(\lambda) = g_1(\lambda)^{N-1}g_2(\lambda), \quad (11)$$

where

$$g_1(\lambda) := \det [\lambda\mathbf{I}_m - (\mathbf{A} - \mathbf{b}k_d\mathbf{c})], \quad g_2(\lambda) := \det \begin{bmatrix} \lambda\mathbf{I}_m - (\mathbf{A} - \mathbf{b}k_d\mathbf{c}) & -N\mathbf{b}k_d \\ -\gamma\mathbf{c} & \lambda + \gamma N \end{bmatrix}. \quad (12)$$

The derivation of (11) is provided in Appendix B. $g_1(\lambda)$ and $g_2(\lambda)$ are the characteristic functions of the matrices,

$$\mathbf{A} - \mathbf{b}k_d\mathbf{c}, \quad \begin{bmatrix} \mathbf{A} - \mathbf{b}k_d\mathbf{c} & N\mathbf{b}k_d \\ \gamma\mathbf{c} & -\gamma N \end{bmatrix}, \quad (13)$$

respectively. Therefore, the necessary and sufficient condition for system (10) to be stable can be derived as follows: *steady state (4) for the dynamic coupled system is stable if and only if both matrices in (13) are stable matrices.* It should be noted that there is no guarantee that death occurs when steady state (4) is stable, because the stability analysis is valid only in the neighborhood of steady state (4).

Furthermore, a simple sufficient condition under which death never occurs is provided. If the following two conditions are held; i) $\lim_{\lambda \rightarrow \infty} g_2(\lambda) = \infty$ for real positive λ : ii) $g_2(0) < 0$, then at least one root of $g_2(\lambda) = 0$ is in the open right-half of the complex plane (i.e., steady state (4) is unstable). Condition i) is obviously held. Condition ii) is described by

$$g_2(0) = N\gamma \det [-\mathbf{A}] = N\gamma \prod_{q=1}^m (-\sigma_q) < 0,$$

where σ_q ($q = 1, 2, \dots, m$) are the eigenvalues of \mathbf{A} . Hence, if \mathbf{A} has an odd-number of real positive eigenvalues (*odd-number property*), then $g_2(0) < 0$ is satisfied. This analysis can be summarized as follows: *steady state (4) for the dynamic coupled system is unstable, that is, amplitude death never occurs for any $\mathbf{b}, k_d, \mathbf{c}, N$, if \mathbf{A} has an odd-number of real positive eigenvalues.*

The odd-number property is well known in the field of delayed feedback control of chaos. A similar stability analysis can be found in [Ushio,1996; Konishi, 1999; Nakajima, 1997; Kokame *et al.*, 2001]. Namajūnas, Pyragas, and Tamaševičius [1995] proposed the tracking filter technique for stabilizing an unstable steady state in the Mackey-Glass system described by a delay differential equation. Since this technique is similar to dynamic coupling (3), the dynamic coupling in this paper can be considered an extension of this technique.

4 Experiments

This paper employs the well-known Chua's circuit as the oscillator in order to confirm the stability analysis. The Chua's circuit is a third order autonomous chaotic oscillator that can be easily constructed with simple electronic components [Matsumoto *et al.*, 1985; Kennedy, 1992; Chua, 1993]. Since this circuit exhibits various nonlinear phenomena, it has been typically used to investigate nonlinear dynamics and its applications [Wu, 2002].

4.1 Stability Analysis

Consider N identical Chua's oscillators as shown in Fig. 2. $v_j^{(a)}$, $v_j^{(b)}$, and i_j denote the voltage across C_a , C_b , and the current through L of the j -th oscillator respectively. The coupled Chua's circuits are governed by

$$\begin{cases} C_a \frac{dv_j^{(a)}}{dt} &= \frac{1}{r} (v_j^{(b)} - v_j^{(a)}) - h_c(v_j^{(a)}) \\ C_b \frac{dv_j^{(b)}}{dt} &= \frac{1}{r} (v_j^{(a)} - v_j^{(b)}) + i_j + \frac{1}{R} (v_0 - v_j^{(b)}) \\ L \frac{di_j}{dt} &= -v_j^{(b)} \end{cases} \quad (j = 1, 2, \dots, N). \quad (14)$$

Current $h_c(v_j^{(a)})$ flows through the nonlinear resistor:

$$h_c(v) = m_0 v + \frac{1}{2}(m_1 - m_0)|v + B_p| + \frac{1}{2}(m_0 - m_1)|v - B_p|.$$

The parameters are set to

$$\begin{aligned} C_a &= 0.01[\mu\text{F}], \quad C_b = 0.1[\mu\text{F}], \quad L = 18[\text{mH}], \\ B_p &= 1.0[\text{V}], \quad r = 1800[\Omega], \quad m_0 = -0.42 \times 10^{-3}, \quad m_1 = -0.75 \times 10^{-3}, \end{aligned} \quad (15)$$

and the coupling parameters R and C_0 are varied as the accessible parameters.

For open S (i.e., static coupling), the potential in the coupling unit,

$$v_0 = \frac{1}{N} \sum_{j=1}^N v_j^{(b)}, \quad (16)$$

is the mean of $v_j^{(b)}$ ($j = 1, 2, \dots, N$). Using the following dimensionless variables and parameters,

$$\begin{aligned} x_{j1} &:= v_j^{(a)}/B_p, \quad x_{j2} := v_j^{(b)}/B_p, \quad x_{j3} := ri_j/B_p, \quad \tau := t/(rC_b), \quad \hat{h}_c(x) := rh_c(x)/B_p, \\ \alpha &:= C_b/C_a, \quad \beta := C_b r^2/L, \quad k_s := r(N-1)/(RN), \end{aligned}$$

the oscillators (14) coupled by (16) are transformed into Σ_j coupled by (1), where

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \alpha \left\{ x_2 - x_1 - \hat{h}_c(x_1) \right\} \\ x_1 - x_2 + x_3 \\ -\beta x_2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T. \quad (17)$$

\mathbf{F} has three fixed points,

$$\mathbf{x}_{f+} = [\eta \ 0 \ -\eta]^T, \mathbf{x}_{f0} = [0 \ 0 \ 0]^T, \mathbf{x}_{f-} = [-\eta \ 0 \ \eta]^T, \quad (18)$$

where $\eta := (m_0 - m_1)/(m_0 + 1/r)$. The Jacobi matrix evaluated at \mathbf{x}_{f+} , \mathbf{x}_{f0} , and \mathbf{x}_{f-} is given by

$$\mathbf{A} = \begin{bmatrix} -\alpha(1 + \hat{m}r) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix},$$

where \hat{m} is m_1 for \mathbf{x}_{f0} and m_0 for \mathbf{x}_{f+} and \mathbf{x}_{f-} . Substituting parameter values (15) into \mathbf{A} , the eigenvalues of \mathbf{A} are estimated as $\lambda_1 = 4.55$, $\lambda_{2,3} = -1.03 \pm i3.58$ at \mathbf{x}_{f0} and are $\lambda_1 = -3.77$ and $\lambda_{2,3} = 0.16 \pm i3.41$ at \mathbf{x}_{f+} and \mathbf{x}_{f-} . Therefore, the Jacobi matrices \mathbf{A} at \mathbf{x}_{f+} , \mathbf{x}_{f0} , and \mathbf{x}_{f-} are unstable.

The coupled Chua's oscillators contain two types of steady states.

- Type (I): $[\mathbf{x}_1^T \ \mathbf{x}_2^T \ \cdots \ \mathbf{x}_N^T]^T = [\mathbf{x}_{f\pm}^T \ \mathbf{x}_{f\pm}^T \ \cdots \ \mathbf{x}_{f\pm}^T]^T$
- Type (II): $[\mathbf{x}_1^T \ \mathbf{x}_2^T \ \cdots \ \mathbf{x}_N^T]^T = [\mathbf{x}_{f0}^T \ \mathbf{x}_{f0}^T \ \cdots \ \mathbf{x}_{f0}^T]^T$

$\mathbf{x}_{f\pm}$ describes each individual oscillator staying at \mathbf{x}_{f+} or \mathbf{x}_{f-} . Therefore, the type (I) has 2^N steady states. On the other hand, type (II) simply denotes all the oscillators at \mathbf{x}_{f0} . From the previous section, we notice that if oscillators (14) are coupled only by the resistors R (S is open), then amplitude death never occurs (i.e., neither type (I) nor (II) states are stable) for any R and N .

For closed S (i.e., dynamic coupling), v_0 is governed by

$$C_0 \frac{dv_0}{dt} = \frac{1}{R} \left\{ \left(\sum_{l=1}^N v_l^{(b)} \right) - Nv_0 \right\}. \quad (19)$$

Oscillators (14) coupled by connection (19) correspond to the dynamic coupled system consisting of oscillators Σ_j and coupling (3), where $z := v_0/B_p$, $\gamma := rC_b/(RC_0)$, $k_d = r/R$ and (17). The dynamic coupled system also contains two types of steady states.

- Type (I): $[\mathbf{x}_1^T \ \mathbf{x}_2^T \ \cdots \ \mathbf{x}_N^T \ z]^T = [\mathbf{x}_{f\pm}^T \ \cdots \ \mathbf{x}_{f\pm}^T \ 0]^T$
- Type (II): $[\mathbf{x}_1^T \ \mathbf{x}_2^T \ \cdots \ \mathbf{x}_N^T \ z]^T = [\mathbf{x}_{f0}^T \ \cdots \ \mathbf{x}_{f0}^T \ 0]^T$

Type (I) has 2^N steady states and type (II) has one steady state.

First, the type (II) steady state is considered. It is noticed that the Jacobi matrix \mathbf{A} at \mathbf{x}_{f0} estimated above satisfies the odd-number property. Hence, amplitude death never occurs in the type (II) steady state for any R , C_0 , and N . Next, the stability of the type (I) steady state is examined. Since \mathbf{A} at \mathbf{x}_{f+} and \mathbf{x}_{f-} estimated above does not satisfy the odd-number property, $\mathbf{b}, k_d, \mathbf{c}, N, \gamma$ must be specified. The matrices (13) including $\mathbf{b}, k_d, \mathbf{c}, N, \gamma$ are estimated. If they are stable, then amplitude death may occur in the type (I) steady state. If not, death never occurs.

4.2 Implementation

The coupled circuits shown in Fig. 2 were constructed. The nonlinear resistor has the same structure as in [Kennedy, 1992]. The inductor L was realized by a general impedance converter [Itoh, 2001] consisting of four resistors and one capacitor. Figure 3 shows the double scroll attractor in each individual circuit without coupling. Three unstable fixed points, \mathbf{x}_{f+} , \mathbf{x}_{f-} , \mathbf{x}_{f0} , coexist with the attractor.

The stable region for the R – C_0 parameter space, in which the type (I) steady state is judged to be stable from stability condition (13), was estimated. The region for the twelve coupled circuits ($N = 12$) is shown as the gray region in Fig. 4. The dots indicate the parameter set (R, C_0) where death is experimentally observed in the coupled electronic oscillators. It can be seen that the theoretical gray region is consistent with the experimental dots.

Figure 5 (a) shows the time series data of $v_1^{(a)}$ and $v_2^{(a)}$ at the parameter set P ($R = 6.8[\text{k}\Omega]$ and $C_0 = 1[\mu\text{F}]$) in Fig. 4. The switch S is closed at the center of the figure. It can be seen that the circuits behave chaotically while S is open. After S is closed, however, the chaotic behavior is changed to a periodic one, but death is not observed. For parameter set Q ($R = 2.2[\text{k}\Omega]$ and $C_0 = 1[\mu\text{F}]$), the time series data are shown in Fig. 5 (b). After S is closed, $v_1^{(a)}$ and $v_2^{(a)}$ converge on η and $-\eta$, respectively. This fact implies that \mathbf{x}_1 and \mathbf{x}_2 converge on \mathbf{x}_{f+} and \mathbf{x}_{f-} , respectively. The circuits experiments for $N = 1, 2, \dots, 11$ have also been demonstrated by the above procedure, and similar results were obtained.

The circuit elements used in these experiments are low in cost and can be easily found, although they have an error of several percent. Therefore, these experiments demonstrated that death induced by dynamic coupling is a robust phenomenon for external noise and parameter mismatch.

5 Conclusion

In this study, a dynamic connection that can induce amplitude death in globally coupled oscillators is proposed. The linear stability analysis provides the following results: death never occurs in a static coupling system and death never occurs when the odd-number property is satisfied. It should be noted that the analysis can be applied to general oscillators. Furthermore, amplitude death was experimentally observed in global-dynamic coupled Chua's oscillators.

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A Type of dynamic coupling

The previous paper [Konishi, 2003b] considered two oscillators, α - and β -oscillators coupled by

$$\begin{cases} \dot{z}_\alpha = y_\beta - z_\alpha \\ u_\alpha = k(z_\alpha - y_\alpha), \end{cases} \quad \begin{cases} \dot{z}_\beta = y_\alpha - z_\beta \\ u_\beta = k(z_\beta - y_\beta), \end{cases} \quad (20)$$

where $y_{\alpha,\beta}$, $u_{\alpha,\beta}$, and $z_{\alpha,\beta}$ are the coupling signals and additional variables, respectively. Equation (20) is a diffusive dynamic coupling; however, there is a difference between coupling (3) and (20). If coupling (20) is applied to N oscillators, N additional variables z_i ($i = 1, 2, 3, \dots, N$) are required. On the contrary, one additional variable z is used for coupling (3). For example, consider five oscillators coupled by a dynamic connection where each oscillator has three variables. The dimension of the coupled system with (20) is twenty; on the other hand, that with (3) is sixteen.

B Derivations of Eqs. (8) and (11)

System (7) is rewritten in a matrix form:

$$\begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \\ \vdots \\ \dot{\mathbf{X}}_N \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{A}}_s & \overline{\mathbf{b}}_s & \cdots & \overline{\mathbf{b}}_s \\ \overline{\mathbf{b}}_s & \overline{\mathbf{A}}_s & \cdots & \overline{\mathbf{b}}_s \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathbf{b}}_s & \overline{\mathbf{b}}_s & \cdots & \overline{\mathbf{A}}_s \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_N \end{bmatrix}, \quad (21)$$

where $\overline{\mathbf{A}}_s := \mathbf{A} - \mathbf{b}k_s\mathbf{c}$ and $\overline{\mathbf{b}}_s := \frac{1}{N-1}\mathbf{b}k_s\mathbf{c}$. A property of the determinant states that it is invariant under the addition of a scalar multiple of a row (column) to another row (column). This property can simplify the characteristic function of linear system (21):

$$\begin{aligned} f(\lambda) &= \det \begin{bmatrix} \lambda\mathbf{I}_m - \overline{\mathbf{A}}_s & -\overline{\mathbf{b}}_s & \cdots & -\overline{\mathbf{b}}_s \\ -\overline{\mathbf{b}}_s & \lambda\mathbf{I}_m - \overline{\mathbf{A}}_s & \cdots & -\overline{\mathbf{b}}_s \\ \vdots & \vdots & \ddots & \vdots \\ -\overline{\mathbf{b}}_s & -\overline{\mathbf{b}}_s & \cdots & \lambda\mathbf{I}_m - \overline{\mathbf{A}}_s \end{bmatrix} \\ &= \det [\lambda\mathbf{I}_m - \overline{\mathbf{A}}_s - (N-1)\overline{\mathbf{b}}_s] \det \begin{bmatrix} \lambda\mathbf{I}_m - \overline{\mathbf{A}}_s + \overline{\mathbf{b}}_s & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \lambda\mathbf{I}_m - \overline{\mathbf{A}}_s + \overline{\mathbf{b}}_s & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \lambda\mathbf{I}_m - \overline{\mathbf{A}}_s + \overline{\mathbf{b}}_s \end{bmatrix} \\ &= f_1(\lambda)f_2(\lambda)^{N-1}, \end{aligned} \quad (22)$$

where $f_1(\lambda)$ and $f_2(\lambda)$ are given by Eq. (9).

System (10) is rewritten as

$$\begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \\ \vdots \\ \dot{\mathbf{X}}_N \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{A}}_d & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{b}k_d \\ \mathbf{0} & \overline{\mathbf{A}}_d & \cdots & \mathbf{0} & \mathbf{b}k_d \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \overline{\mathbf{A}}_d & \mathbf{b}k_d \\ \gamma\mathbf{c} & \gamma\mathbf{c} & \cdots & \gamma\mathbf{c} & -\gamma N \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_N \\ Z \end{bmatrix}, \quad (23)$$

where $\overline{\mathbf{A}}_d := \mathbf{A} - \mathbf{b}k_d\mathbf{c}$. The characteristic function of linear system (23) can be simplified as

follows:

$$\begin{aligned}
g(\lambda) &= \det \begin{bmatrix} \lambda \mathbf{I}_m - \overline{\mathbf{A}}_d & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{b}k_d \\ \mathbf{0} & \lambda \mathbf{I}_m - \overline{\mathbf{A}}_d & \cdots & \mathbf{0} & -\mathbf{b}k_d \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \lambda \mathbf{I}_m - \overline{\mathbf{A}}_d & -\mathbf{b}k_d \\ -\gamma \mathbf{c} & -\gamma \mathbf{c} & \cdots & -\gamma \mathbf{c} & \lambda + \gamma N \end{bmatrix} \\
&= \det \begin{bmatrix} \lambda \mathbf{I}_m - \overline{\mathbf{A}}_d & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{b}k_d \\ \mathbf{0} & \lambda \mathbf{I}_m - \overline{\mathbf{A}}_d & \cdots & \mathbf{0} & -\mathbf{b}k_d \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \lambda \mathbf{I}_m - \overline{\mathbf{A}}_d & -N\mathbf{b}k_d \\ \mathbf{0} & \mathbf{0} & \cdots & -\gamma \mathbf{c} & \lambda + \gamma N \end{bmatrix} \\
&= g_1(\lambda)^{N-1} g_2(\lambda),
\end{aligned} \tag{24}$$

where $g_1(\lambda)$ and $g_2(\lambda)$ are given by Eq. (12).

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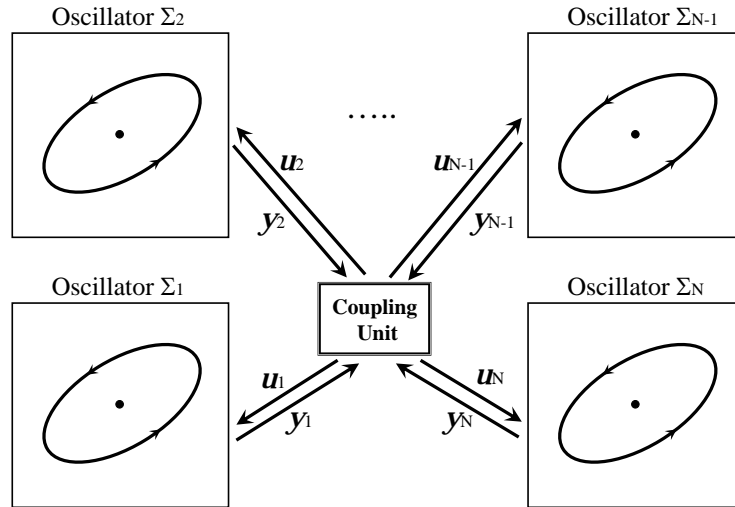


Figure 1: Globally coupled oscillators.

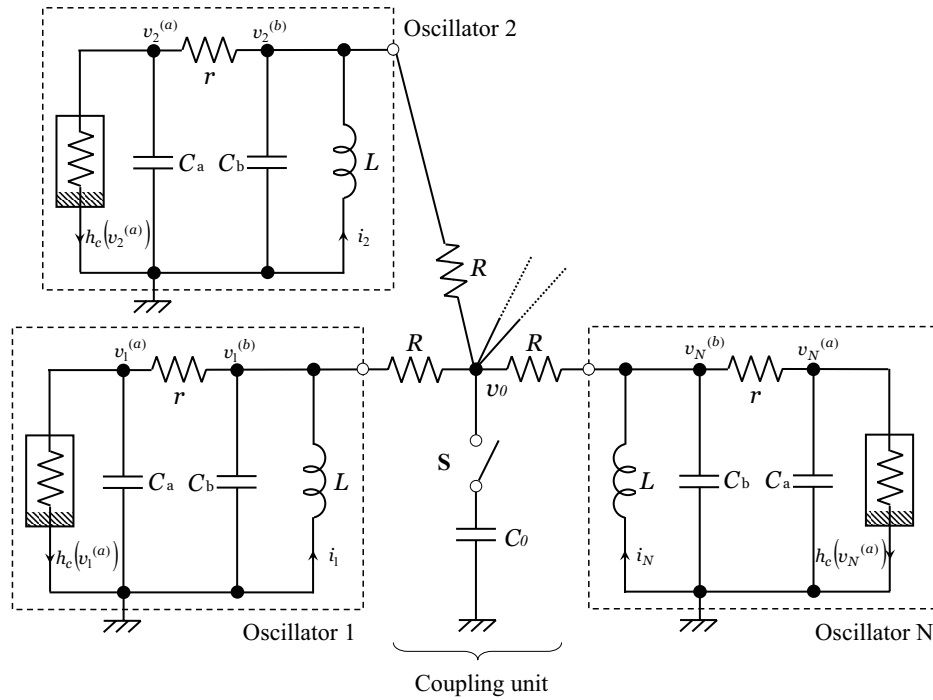


Figure 2: Globally coupled Chua's oscillators

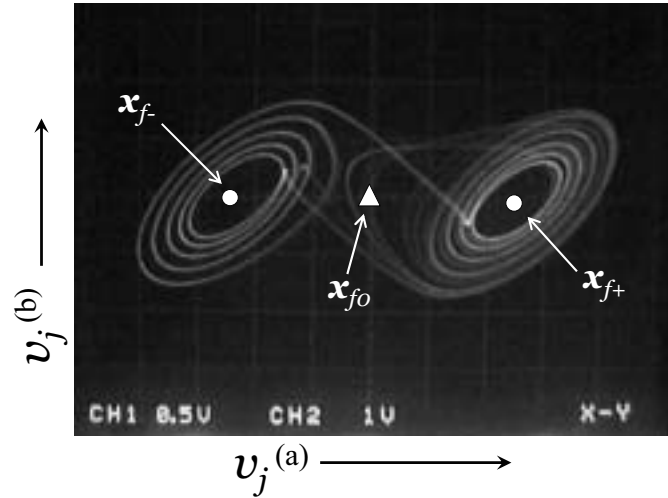


Figure 3: Double scroll attractor and three unstable fixed points in the circuit without coupling

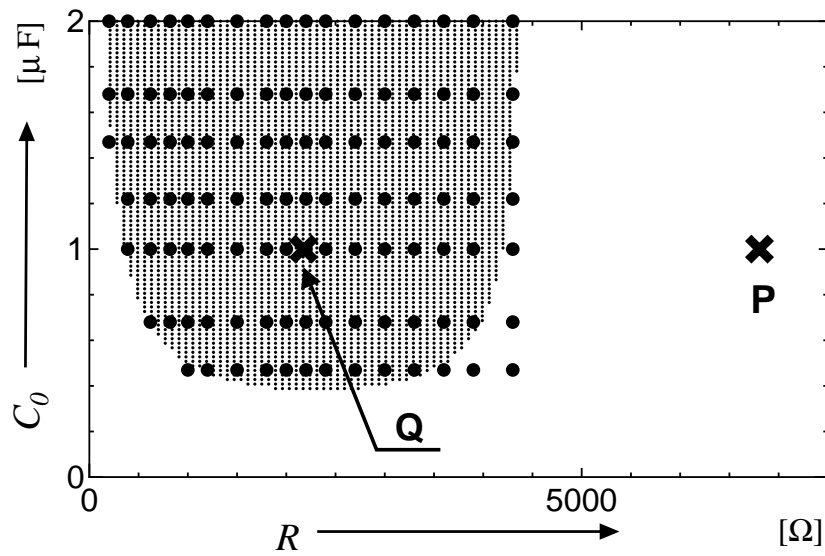


Figure 4: $R - C_0$ parameter region for the twelve coupled circuits ($N = 12$). Death is observed experimentally at the parameter described by the dot. The death region estimated analytically is presented.

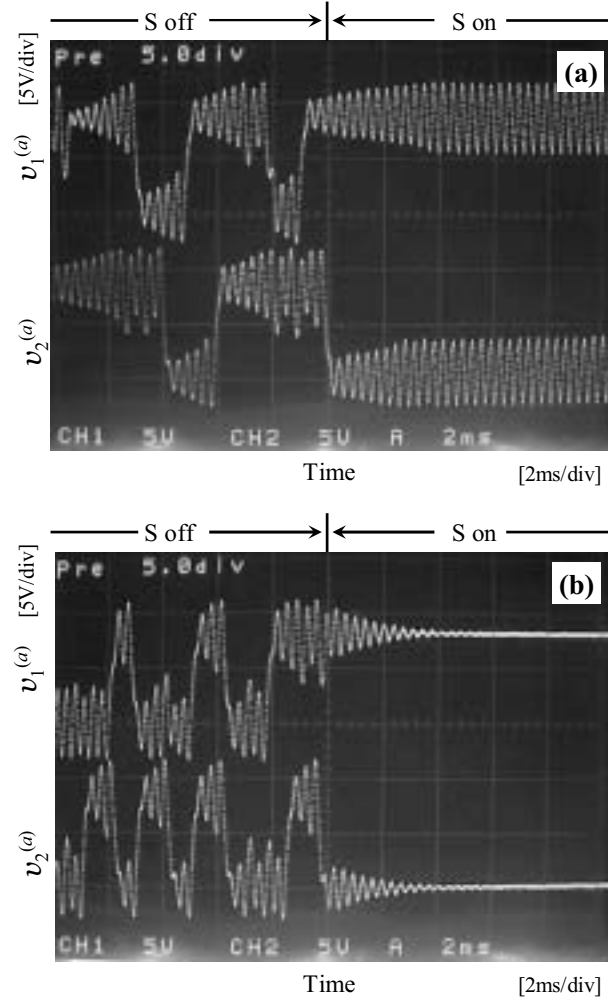


Figure 5: Time series data ($v_1^{(a)}$ and $v_2^{(a)}$) of Chua's circuits connected by a globally dynamic coupling. (a) parameter set P ($R = 6.8 \text{ k}\Omega$ and $C_0 = 1 \text{ }\mu\text{F}$). (b) parameter set Q ($R = 2.2 \text{ k}\Omega$ and $C_0 = 1 \text{ }\mu\text{F}$).