A tuning strategy to avoid blocking and starving in a buffered production line

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Abstract

Blocking and starving of a buffered production line is investigated from the viewpoint of nonlinear dynamics. A strategy to avoid blocking and starving, which utilizes frequency response and $\mathcal{H}_\infty$-norm, is proposed. The proposed strategy does not need to tune all the work cells. Furthermore, we provide a systematic procedure for selecting and tuning the work cells. The numerical simulations are shown in order to verify the proposed strategy.

Key words: Buffered production flows, Traffic models, Dynamics of production systems

1 Introduction

Traffic flows have been a major subject in the field of nonlinear physics. Several phenomena in traffic flows are successfully explained from the viewpoint of nonlinear dynamics (Helbing, 2001). The optimal velocity (OV) traffic model proposed by Bando *et al.* has played an important role in this field (Bando et al., 1995). The mechanism of the traffic jam can be discussed in terms of the magnitude frequency response [1] of each vehicle transfer-function (Konishi et al., 1999a, 2000). A simple sufficient no-jam condition is that the $\mathcal{H}_\infty$-norm of every vehicle transfer-function is less than 1 (Konishi et al., 1999a, 2000). Furthermore, we know that the traffic jam is essentially the same phenomenon as a convective instability in a one-way coupled chaotic system (Kaneko, 1985; Willeboordse and Kaneko, 1995; Yamaguchi, 1997; Konishi et al., 1999b, 2001).

URL: http://www.eis.osakafu-u.ac.jp/~ecs/ (Keiji Konishi).

[1] This response is widely used in the field of control theory (Nise, 1995).
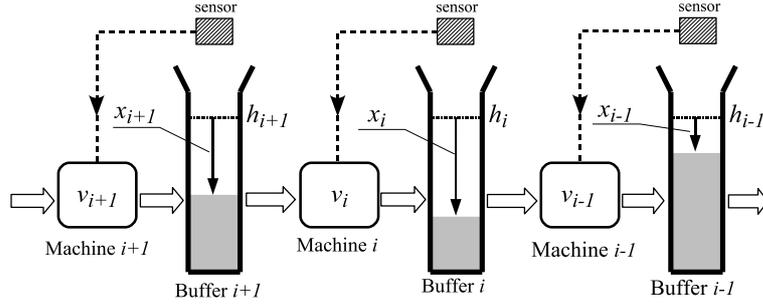
Fig. 1. Schematic illustration of a buffered production line.

It was recently reported that a buffered production line has the same dynamics as the OV traffic model (Radons and Neugebauer, 2003; Filliger and Hongler, 2005). Blocking and starving in the production line correspond to the stop-and-go phenomenon of the OV model. This analogy is useful for studying the production line dynamics. On the basis of the no-jam condition of a traffic model (Konishi et al., 1999a), Filliger and Hongler derived a sufficient condition under which blocking and starving never occurs in the production line (Filliger and Hongler, 2005), and Nagatani and Helbing provided a stability condition of linear supply chains and compared five strategies for stabilizing supply chains (Nagatani and Helbing, 2004).

Previous studies (Filliger and Hongler, 2005; Nagatani and Helbing, 2004) assumed work cells to be identical. Thus, the sufficient condition for smooth production flow (i.e., no-jam flow) was somewhat conservative. In other words, every cell has to be tuned so as to satisfy the condition. However, from a practical viewpoint, it is natural to assume that the work cells are not identical, and that some of the cells can be tuned while others may not.

The present paper investigates the blocking and starving phenomenon of a buffered production line from the viewpoint of nonlinear dynamics. The production line has one piecewise linear function and is a simplification of a model proposed in (Filliger and Hongler, 2005). The main purpose of the present paper is to provide a strategy to avoid blocking and starving in the production line. The proposed strategy tunes some of the work cells and utilizes the frequency response and $\mathcal{H}_\infty$-norm, which are well known in the field of control theory (Doyle et al., 1992). The proposed strategy can be easily applied because numerical calculation for tuning is realized using a well-known software package. Although the proposed strategy cannot avoid the oscillatory production flow, it is guaranteed that blocking and starving never occur.

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2   Buffered production line

2.1   Mathematical model

The present paper focuses on a buffered production line, as shown in Fig. 1, based on a previous study (Filliger and Hongler, 2005). This line consists of N work cells, where each cell has one machine and one buffer. The products handled by machine i are stocked in buffer i. The stocked products are then handled by the $(i-1)$-th machine. The production speed of machine i is denoted by $v_i > 0$. The capacity and empty level of buffer i are defined as $h_i > 0$ and $x_i \geq 0$, respectively. Throughout this paper, the following boundary conditions are assumed: completed products on the line are shipped from the buffer 0 at a constant speed v^*, and machine N never starves due to a sufficient source of products. These conditions can be written as

$$v_0 \equiv v^*, \quad x_{N+1} \equiv 0.$$

The following two reasonable situations for the machines are presumed: machine i always monitors x_i and controls itself, and machine i can obtain only information as to whether its upstream buffer is empty (i.e., $x_{i+1} = h_{i+1}$). Since each machine works independently of the other machines, the production line can be regarded as a distributed control system. Furthermore, the maximum speeds of all of the machines, V_i, are assumed to be greater than the shipping speed:

$$v^* < V_i, \ \forall i \in \{1, \ldots, N\}.$$

The i-th work-cell dynamics consists of the following two operations:

Normal operation   When the buffer i is not full $(x_i > 0)$ and its upstream buffer is not empty $(x_{i+1} < h_{i+1})$, the i-th machine dynamics is governed by

$$\dot{x}_i = v_{i-1} - v_i, \tag{1a}$$
$$\dot{v}_i = \alpha_i \left\{ F_i(x_i) - v_i \right\}. \tag{1b}$$

The function $F_i(x_i)$ [2] is given by

$$F_i(x_i) = \frac{V_i}{h_i} x_i \ \ (0 < x_i \leq h_i).$$

---

[2] This function corresponds to the optimal velocity function of the OV traffic model.

F_i denotes the ideal production-speed of machine i, which depends on the empty level x_i. $\alpha_i > 0$ is the reaction sensitivity of machine i. If buffer i is empty ($x_i = h_i$), then the goal speed is set to the maximum speed ($F_i(h_i) = V_i$).

Stop operation When the buffer i is full ($x_i = 0$) or when the upstream buffer is empty ($x_{i+1} = h_{i+1}$), then machine i has to stop due to the lack of buffer space and the lack of product resources. This stop operation is described by

$$v_i \rightarrow 0. \tag{2}$$

Although this operation was not employed previously (Filliger and Hongler, 2005), we use it here in order to describe more practical lines. We easily see that this operation can avoid overflow of the buffer and idle operation of machines.

Although the machines in existing production line models seem to operate only at two phases: "on" phase (run with maximal speed) and "off" phase (complete stop), the present paper considers a more advanced line. Each machine controls its production speed based on the signal from its sensor which measures the empty level of buffer. This type of line would be suitable for keeping production flow smoothly. However, if the reaction sensitivity of machine is not proper, blocking and starving occurs. Hence, the design of sensitivity is a crucial factor for this line.

2.2   *Dynamics of the production line*

In order to understand the line dynamics, we consider the phase portrait of the vector field. Figure 2 shows the behavior of the i-th machine and buffer on the phase plane (x_i vs. v_i). First, we find the steady state:

$$\begin{bmatrix} x_1 \ v_1 \ \cdots \ x_N \ v_N \end{bmatrix}^T = \begin{bmatrix} x_1^* \ v^* \cdots \ x_N^* \ v^* \end{bmatrix}^T, \tag{3}$$

where x_i^* satisfies $F_i(x_i^*) = v^*$ ($i = 1, \ldots, N$). A stability analysis guarantees that this state is stable. Second, we find the nullclines, which indicate points at which the vector flow is strictly horizontal or strictly vertical. From Eq. (1), the dash-dotted line **A** and the broken line **B** are the nullclines for $\dot{v}_i = 0$ and $\dot{x}_i = 0$, respectively. The flow directions on the lines can be easily estimated from Eq. (1) (see the arrows on these lines). The fixed point $\begin{bmatrix} x_i^* \ v^* \end{bmatrix}^T$, i.e., the intersection of the two lines, is moved according to v_{i-1}, because the line **B** depends on the variable v_{i-1} of the downstream machine. We shall consider separately the normal and stop operations for the i-th works-cell dynamics on the phase plane.
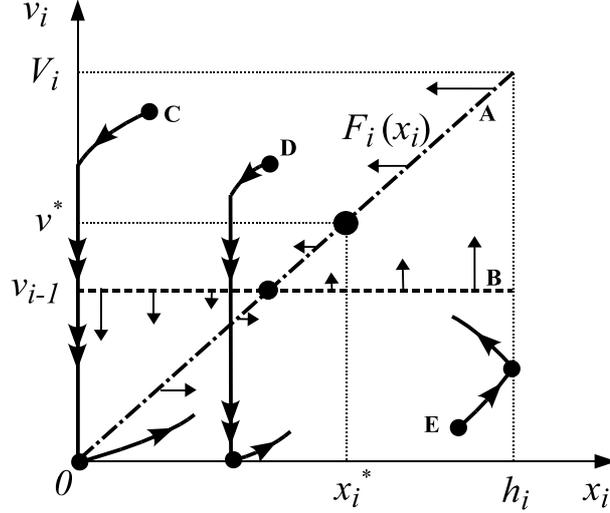
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Fig. 2. Illustration of machine and buffer behavior on the phase plane.

Normal operation As the flow near the upper (lower) bound $v_i = V_i$ ($v_i = 0$) is downward (upward), trajectories never pass through the bounds. Eventually, if v_{i-1} is constant, the trajectories spiral into the fixed point.

Stop operation If a trajectory, for example **C** in the portrait, is closed to the left bound ($x_i = 0$), then it hits the bound (i.e., the lack of buffer space) and jumps to the origin. After this jump, the trajectory is governed by the normal operation. If $x_{i+1} = h_{i+1}$ (i.e., the lack of resources), a trajectory (e.g., **D** in the portrait) jumps to the lower bound and moves upward by the normal operation. If $x_i = h_i$ (i.e., the buffer is empty), then $v_{i-1} = 0$ (i.e., $(i-1)$-th machine stops) and a trajectory (e.g., **E**) diverts at the right bound ($x_i = h_i$).

We next show the trajectories with the above two operations through numerical simulations.

2.3   Numerical example

Let the number of work-cells be $N = 10$, then the parameters are fixed at

$$
\begin{aligned}
&h_i = 1.0 \ (i = 1, \ldots, 10), \\
&\alpha_i =
\begin{cases}
0.6 \ (i = 1, 2, \ldots, 5) \\
1.2 \ (i = 6, 7, \ldots, 10)
\end{cases}, \\
&V_i =
\begin{cases}
1.0 \ (i = 1, 2, \ldots, 5) \\
1.2 \ (i = 6, 7, \ldots, 10)
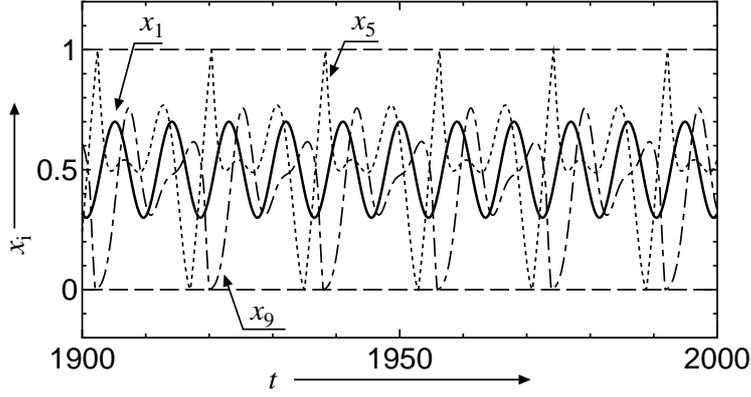\end{cases}.
\end{aligned}
\tag{4}
$$

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Fig. 3. Time series of the empty level x_i of the 1-st, 5-th, and 9-th buffers.
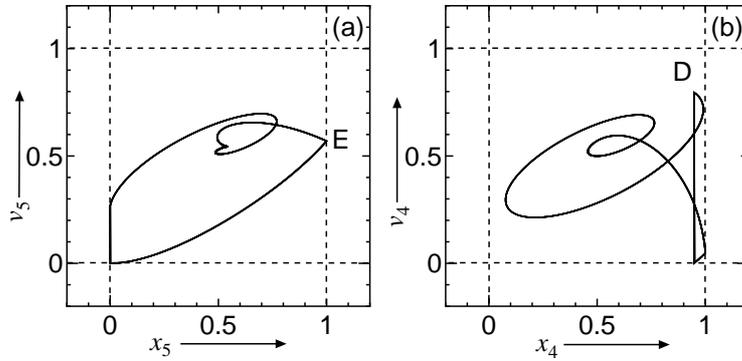

Fig. 4. Trajectories of the work cells on the phase plane. (a) 5-th cell, (b) 4-th cell.

We presume the following situation: the shipping speed v^* disturbed by the market or the following line affects the empty level of the buffer 1,

$$x_1 = x_1^* + 0.2 \sin 0.7t.$$

Every work cell is governed by Eqs. (1) and (2). Figure 3 shows x_i for the 1-st, 5-th, and 9-th buffers. The broken lines at $x_i = 1$ and 0 denote empty and full buffers, respectively. Note that the 5-th buffer becomes full and empty at intervals. This figure implies that blocking and starving occur on this production line.

Figure 4 shows the trajectories on the phase plane for the 5-th and 4-th work cells. At point $\mathbf{E}$ in Fig. 4(a), the 5-th buffer becomes empty, that is, the 4-th machine stops at point $\mathbf{D}$ in Fig. 4(b). This phenomenon occurs sporadically along the line.

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When all of the work cells normally operate without stop, the production line dynamics can be described by the frequency domain. Since this description enables us to grasp the relations between input and output signals, it has been used in the field of control theory and signal processing. We shall show a clear relation of the empty buffer levels x_i. This relation is useful in order to obtain the influence of the external disturbance on the production line dynamics.

Let us consider variations δx_i and δv_i from the steady state (3):

$$x_i = x_i^* + \delta x_i, \ v_i = v^* + \delta v_i. \tag{5}$$

Since $x_i \in [0, h_i]$ and $v_i \in [0, V_i]$ hold, the variations should be within the following ranges:

$$\delta x_i \in [-x_i^*, h_i - x_i^*], \ \delta v_i \in [-v^*, V_i - v^*].$$

These ranges define the normal operation regime. Substituting Eq. (5) into Eq. (1), the variational dynamics of the i-th work cell is given by

$$\dot{\delta x_i} = \delta v_{i-1} - \delta v_i, \tag{6a}$$

$$\dot{\delta v_i} = \alpha_i \left\{ \rho_i \delta x_i - \delta v_i \right\}, \tag{6b}$$

where $\rho_i := V_i / h_i$. The influence of the variation δx_i on δx_{i+1} can be described by the transfer function:

$$G_{i,i+1}(j\omega) = \frac{\alpha_i \rho_i}{\alpha_i + j\omega} \cdot \frac{\alpha_{i+1} + j\omega}{\alpha_{i+1}\rho_{i+1} - \omega^2 + j\omega\alpha_{i+1}}, \tag{7}$$

where $j = \sqrt{-1}$. Thus, the disturbance at the i-th buffer,

$$x_i = x_i^* + d_i \sin \omega t, \ (d_i \in [-x_i^*, h_i - x_i^*]),$$

influences the $(i+1)$-th buffer as

$$x_{i+1} = x_{i+1}^* + d_{i+1} \sin (\omega t + \theta_{i+1}), \ (d_{i+1} \in [-x_{i+1}^*, h_{i+1} - x_{i+1}^*]).$$

The amplitude d_{i+1} and the phase θ_{i+1} are estimated by the function (7),

$$d_{i+1} = |G_{i,i+1}(j\omega)| \, d_i, \ \theta_{i+1} = \arg\{G_{i,i+1}(j\omega)\}.$$

Note that $|G_{i,i+1}(j\omega)|$ can be regarded as an amplitude gain from buffer i to buffer $i + 1$ at frequency ω. Therefore, we can easily obtain the gain from the

first buffer to buffer i at ω:

$$|G_{1,i}(j\omega)| = \left|\prod_{q=1}^{i-1} G_{q,q+1}(j\omega)\right| = \prod_{q=1}^{i-1} |G_{q,q+1}(j\omega)| \tag{8}$$

$$= \sqrt{\frac{\alpha_i^2 + \omega^2}{\alpha_1^2 + \omega^2}} \cdot \prod_{q=1}^{i-1} \frac{\alpha_q \rho_q}{\sqrt{(\alpha_{q+1}\rho_{q+1} - \omega^2)^2 + \omega^2 \alpha_{q+1}^2}}. \tag{9}$$

3 Avoidance of blocking and starving

3.1 Problem statement

We know that if buffer i is full $(x_i = 0)$ or buffer $i+1$ is empty $(x_{i+1} = h_{i+1})$, machine i has to stop in order to prevent buffer overflow and idle operation of machines. This stop operation can induce a stop-operation in neighboring machines. Consequently, the induced stop-operation spreads to the other machines. This can be considered as a *production jam*. In a jam regime, since the machines repeat the on-off operation, machine energy is wasted and machine accidents tend to occur. Therefore, the jammed flow should be avoided.

The present paper assumes the following:

- Parameters (h_i, α_i, V_i) are known;
- Amplitude d_1 of the disturbance is known;
- Frequency w of the disturbance is unknown;
- Shipping speed v^* is known; and
- Empty level of the buffer at the steady state, x_i^*, can be estimated.

Our main purpose is to provide a strategy by which to avoid the stop-operation for all the machines:

$$x_i \in (0, h_i) \iff \delta x_i \in (-x_i^*, h_i - x_i^*) \tag{10}$$

for all $i = 1, 2, \ldots, N$.

Our strategy is assumed to tune only the reaction sensitivity α_i of the machines. We consider two cases:

(A) α_i is tunable for all machines; and
(B) α_i is tunable for some machines.

We shall consider the two cases below.

(A) α_i is tunable for all machines

We know that if all of the machines are tuned such that

$$\|G_{i,i+1}\|_\infty := \sup_{\omega \in \boldsymbol{R}} |G_{i,i+1}(j\omega)| < 1, \quad \forall i = 1, 2, \ldots, N-1, \tag{11}$$

then a jam never occurs (Filliger and Hongler, 2005). $\|G_{i,i+1}\|_\infty$ is $\mathcal{H}_\infty$ norm of $G_{i,i+1}$: this norm indicates a supremum of gain $|G_{i,i+1}(j\omega)|$ for all $\omega \in \boldsymbol{R}$ (Boyd et al., 1994). When a sinusoidal disturbance with amplitude d_i is added to buffer i, the amplitude d_{i+1} of the induced disturbance at buffer $i+1$ is less than $d_i \|G_{i,i+1}\|_\infty$ for any disturbance frequency $\omega \in \boldsymbol{R}$. Condition (11) shows that every work cell transmits its buffer disturbance to its upward buffer without amplification for any frequency. This can be realized by a simple tuning (Filliger and Hongler, 2005):

$$\alpha_i > \frac{2V_i}{h_i}, \quad \forall i = 1, 2, \ldots, N. \tag{12}$$

Therefore, if α_i is tunable for all machines based on condition (12), then the jam never occurs for any disturbance frequency.

(B) α_i is tunable for some machines

From a practical viewpoint, it is too difficult to tune all of the machines. Thus, we focus on the situation in which some of the machines are tunable. Let us assume that the first buffer is disturbed by $x_1 = x_1^* + d_1 \sin \omega t$. Then, the amplitude d_i of the induced disturbance at buffer i is

$$d_i = |G_{1,i}(j\omega)|d_1. \tag{13}$$

Hence, from condition (10), $d_i < \min(h_i - x_i^*, x_i^*)$ has to be satisfied in order to avoid blocking and starving at the i-th work cell. This inequality and Eq. (13) allows us to obtain

$$|G_{1,i}(j\omega)| < \frac{\min(h_i - x_i^*, x_i^*)}{d_1}. \tag{14}$$

If condition (14) holds for all of the work-cells ($i = 1, 2, \ldots, N$), blocking and starving never occur for the disturbance with frequency ω. In general, if ω is unavailable in advance, we have to derive the following condition:

$$\|G_{1,i}\|_\infty < \frac{\min(h_i - x_i^*, x_i^*)}{d_1}, \quad \forall i = 1, 2, \ldots, N, \tag{15}$$

which does not depend on ω. If condition (15) holds, then blocking and starving never occur in the production line. The left-hand side of (15) depends on
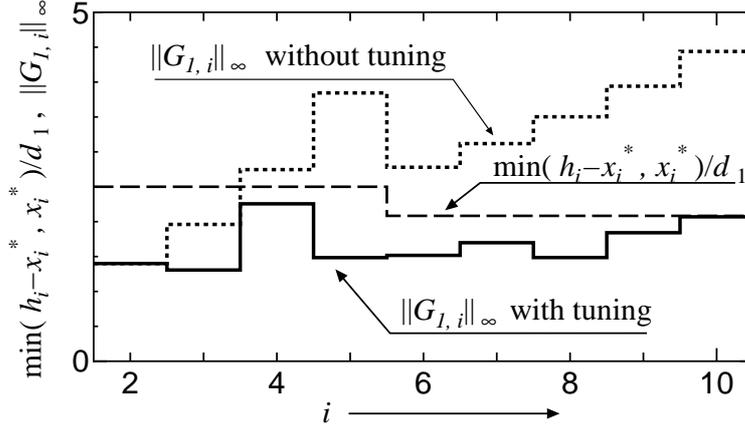
Fig. 5. Diagram of $\|G_{1,i}\|_\infty$ at the work cell i. The broken line represents the upper bound (15) of the no-jam flow. The dotted (solid) line shows $\|G_{1,i}\|_\infty$ without (with) tuning.

the downstream work-cells (i.e., $\|G_{1,i}\|_\infty = \left\|\prod_{q=1}^{i-1} G_{q,q+1}\right\|_\infty$). Therefore, we can avoid blocking and starving even if some of the cells do not satisfy (12). In other words, the tuning of some machines enables us to suppress the jam. Note that condition (15) is a more general condition, as compared to condition (12).

4   Numerical simulations

This section presents numerical simulations that verify the above tuning scheme. The line parameters are the same as in the previous example. The following situations are assumed: the parameters (4) are known; the amplitude $d_1 \leq 0.2$ of the disturbance is known; the frequency w of the disturbance is unknown; the shipping speed $v^* = 0.5$ is known; the empty level of the buffer at the steady state, x_i^*, is estimated; and machines 3, 5, and 7 can be tuned.

The right-hand side of inequality (15), that is the upper bound of the no-jam flow, is indicated by the broken line in Fig. 5. For parameter (4), $\|G_{1,i}\|_\infty$, which is given by the left-hand side of inequality (15), is plotted as the dotted line [3] . The dotted line is higher than the upper bound (broken line) for $i \geq 4$. Hence, the obtained analytical result indicates that blocking and starving can occur for $i \geq 4$. This result supports the previous examples shown in Figs. 3,4.

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[3] $\mathcal{H}_\infty$-norm can be easily estimated by well-known software packages such as Matlab, Octave, and Scilab. The present paper uses the Octave command `hinfnorm` to estimate $\mathcal{H}_\infty$-norm with tolerance 10^{-9}.
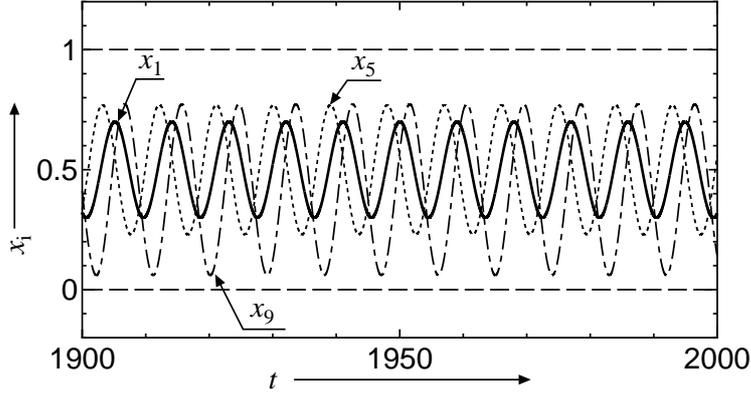
Fig. 6. Time series of the empty level x_i of the 1-st, 5-th, and 9-th buffers. The reaction sensitivity of the machines $i = 3, 5, 7$ are tuned as (16).
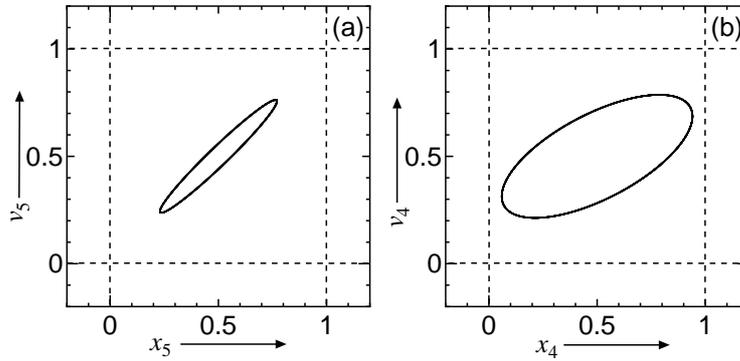

Fig. 7. Trajectories of the work cells on the phase plane. (a) 5-th cell, (b) 4-th cell. The reaction sensitivity of the machines $i = 3, 5, 7$ are tuned as (16).

Next, we tune the reaction sensitivity of the machines $i = 3, 5, 7$:

$$\alpha_3 = 1.0, \ \alpha_5 = \alpha_7 = 3.0. \tag{16}$$

$\|G_{1,i}\|_\infty$ with tuning (16) is plotted as the solid line in Fig. 5. This line does not exceed the upper bound (broken line). Therefore, condition (15) is always satisfied. This guarantees that blocking and starving never occur for the disturbance with $d_1 \leq 0.2$. Figure 6 shows x_i for the 1-st, 5-th, and 9-th buffers. Figure 7 shows the trajectories on the phase plane for the 4-th and 5-th work cells. All of the buffers behave in an oscillatory manner but do not reach the empty or full states. Thus, blocking and starving do not occur on the tuned production line.

5 Conclusion

The present paper investigated the dynamics of blocking and starving in a buffered production line, and proposed a strategy by which to avoid both

blocking and starving. The proposed strategy has the following features: some of the work cells are required to be tuned, the tuning is designed by a systematic procedure, the oscillatory production flow cannot be avoided, and blocking and starving can be avoided. The strategy is verified by numerical simulations.

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