Linear Induction Motor as Accelerator

Linear Induction Motor as Accelerator

Sadao ISHIKAWA*, Masatoshi KAJINO** and Takao HIRASA*

(Received June 15, 1971)

There is an accelerator in one of the application of Linear Induction Motor. It is easy to control the accelerating speed by the pole pitches for getting the maximum final speed within the limited condition. Such the optimum distribution of pole pitches can be determined by the analysis of the equivalent circuits of pole pitches. When the friction of secondary moving member is neglected one, the optimum pole pitch may be proportional to qubic root of the distance. Then the proportional coefficient is expressed by the power supply and the moving member condition and by only the equivalent circuit components at the standard pole pitch. The experimental results prove the above.

1. Introduction

One of the application of Linear Induction Motor (say LIM) is an accelerator,[1] such as catapult or impact testing machine etc. The accelerator has a construction as shown in Fig. 1. The secondary moving member runs to the end in the fixed primary windings being accelerated. This is to say the short secondary type of LIM.

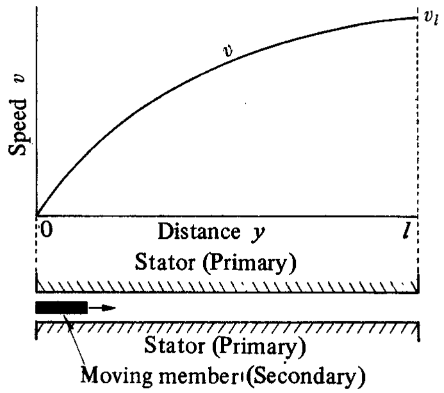

Fig. 1. LIM as accelerator.

Although the speed may be controlled by the primary voltage, the frequency or the pole pitches, it is proper that the speed is controlled by the pole pitches when the accelerating period is short enough or the characteristics of acceleration is determined beforehand. In this case it can be considered the optimum distribution of pole pitches for the maximum final speed in a certain stator length.[2),3),4)]

This paper deals with the optimum design of accelerator with graded pole pitch windings and its experimental characteristics on tubular motors.

* Department of Electrical Engineering, College of Engineering.
** Graduate Student, Department of Electrical Engineering, College of Engineering.

2. Accelerator with graded pole pitches[5)]

2.1. Optimum condition of accelerator

Consider an accelerator with a certain machine length. The kinetic energy W supplied to the moving member in the accelerating time t or the length l is given by

$$W=\frac{1}{2}Mv_l^2=\int_0^t p\mathrm{d}t=\int_0^t Fv\mathrm{d}t=\int_0^l F\mathrm{d}y \tag{1}$$

where M and v_l are the moving member mass and the final speed; and p, v and F are the instantaneous power, velocity and driving force on the moving member, respectively. It is seen from eq. (1) that the maximum final speed is in need of the maximum driving force under the limited condition at each point of the accelerator, as shown in Fig. 2.

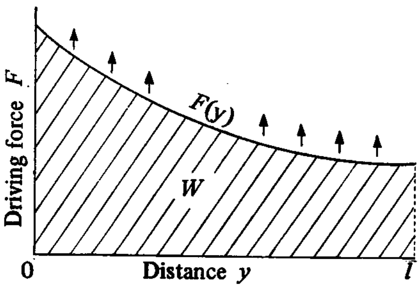

Fig. 2. Optimum condition of accelerator.

On the other hand, if the friction have a constant value R_f independently of the speed, the equation of motion for the moving member is shown by

$$M\frac{\mathrm{d}v}{\mathrm{d}t}=F-R_f \tag{2}$$

with zero initial condition, the integration of eq. (2) with respect to time t gives the velocity

$$v=\frac{F-R_f}{M}t \tag{3}$$

Integrating again, the distance y is

$$y=\frac{1}{2}\left(\frac{F-R_f}{M}\right)t^2 \tag{4}$$

From eq. (3) and eq. (4),

$$y=\frac{1}{2}\frac{M}{F-R_f}v^2 \tag{5}$$

with a synchronous speed v_s and a slip s, the speed of LIM used as an accelerator may be written

$$v = v_s(1-s) \tag{6}$$

Substituting eq. (6) in eq. (5)

$$y = \frac{1}{2}\frac{M}{F-R_f}v_s^2(1-s)^2 \tag{7}$$

If the friction R_f is negrigible, eq. (7) is given by

$$y = \frac{M}{2F}v_s^2(1-s)^2 \tag{8}$$

This equation expersses that the distance y is the function of the pole pitch τ because the driving force F and the synchronous speed v_s are relative to the pole pitch τ.

Under the maximum driving force condition, eq. (8) is as follows.

$$y = \frac{M}{2F_{\max}}v_s^2(1-s)^2 \tag{9}$$

2.2. Maximum driving force condition of LIM[6)]

The equivalent circuit for a phase of LIM may generally be shown in Fig. 3. Where r_1 and x_1 are the primary resistance and the leakage reactance; and x_m, r_2 and s are the magnetizing reactance, the secondary resistance referred to the primary side and the slip, respectively. The magnetizing resistance r_m and the secondary leakage reactance x_2 to be found in ordinary induction motor's equivalent circuit can be omitted because of its negrigible smallness.

The driving force F of the three phase LIM, from the equivalent circuit in Fig. 3, is

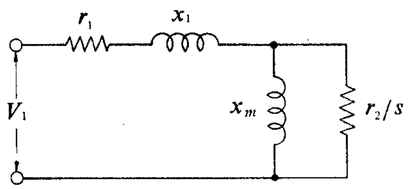

Fig. 3. Equivalent circuit of LIM.

$$F = \frac{3V_1^2}{2\tau f}\frac{\frac{r_2}{s}}{\left(r_1+\frac{r_2}{s}+\frac{x_1}{x_m}\frac{r_2}{s}\right)^2+\left(x_1-\frac{r_1}{x_m}-\frac{r_1}{x_m}\frac{r_2}{s}\right)^2} \text{ [N]} \tag{10}$$

where V_1: primary phase voltage [V]

τ: ploe pitch [m]

f: frequency [Hz]

The value of secondary resistance giving the maximum driving force can be derived by $\frac{\partial F}{\partial r_2}=0$.

Calculating above,

$$r_2 = K\ s\, x_m \quad [\Omega]$$

Then

$$s = r_2 / K\ x_m \tag{11}$$

$$K = \sqrt{\frac{r_1^2 + x_1^2}{r_1^2 + (x_1 + x_m)^2}} \tag{12}$$

Hence, the maximum driving force $F_{\max}$ is

$$F_{\max} = \frac{3V_1^2}{2\tau f} \frac{Kx_m}{(1+K)^2(r_1^2+x_1^2) + Kx_m(2r_1 + Kx_m + 2Kx_1)} \quad [\mathrm{N}] \tag{13}$$

2.3. Effect of pole pitch on eqivalent circuit

Consider a primary coil per pole per phase with N turns and the winding width L for tubular motor. r_1, r_2, x_1 and x_m have the following proportional relation on N, L and the magnetic circuit reluctance $\mathfrak{R}$,

$$\begin{aligned} & r_1 \propto N \\ & r_2 \propto N^2/L \\ & x_1 \propto N^2/L \\ & x_m \propto N^2/\mathfrak{R} \end{aligned} \tag{14}$$

For a standard pole pitch and an optional pole pitch multipled by α, add suffix o and α to each circuit component respectively. Hence, the former is represented as τ_0, r_{10}, x_{10}, etc. and the latter as, τ_α, $r_{1\alpha}$, $x_{1\alpha}$, etc. The circuit components per pole with the pole pitch $\tau_\alpha = \alpha\tau_0$ may be written by

$$\begin{aligned} r_{1\alpha} &= A\ r_{10} \\ r_{2\alpha} &= B\ r_{20} \\ x_{1\alpha} &= C\ x_{10} \\ x_{m\alpha} &= D\ x_{m0} \end{aligned} \tag{15}$$

where the coefficients A, B, C, D are expressed in Table 1, according to the condition of the numbers of primary coil turn and the iron core tooth width.

The eqivalent circuit diagrams for the cases of ③ and ④ in Table 1 are shown in Fig. 4. The calculation hereafter may be done for these cases of ③ and ④, because such windings are proper as constant ampere turn per unit length.

2.4. Slips at maximum driving force

When LIM has the graded pole pitches represented in the previous section, the slip s to give the maximum driving force may be derived as following.

For the case of ③;

From eq. (11) and eq. (12), the slip s_α for the pole pitch τ_α is

Table 1 Equivalent circuit components' coefficients.

No. of turn	Tooth width	No.	A	B	C	D
Constant	Constant	①	1	$1/\alpha$	$1/\alpha$	1
	Proportional to α	②	1	$1/\alpha$	$1/\alpha$	α
Proportional to α	Constant	③	α	α	α	α^2
	Proportional to α	④	α	α	α	α^3

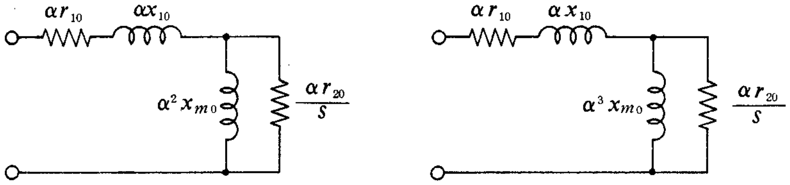

Fig. 4. Equivalent circuits for graded pole pitch.

$$s_\alpha = r_{2\alpha}/K_\alpha x_{m\alpha}$$

$$K_\alpha = \sqrt{\frac{r_{1\alpha}^2 + x_{1\alpha}^2}{r_{1\alpha}^2 + (x_{1\alpha} + x_{m\alpha})^2}} \tag{16}$$

Then

$$K_\alpha \doteqdot \frac{1}{\alpha}\sqrt{\frac{r_{10}^2 + x_{10}^2}{r_{10}^2 + (x_{10} + x_{m0})^2}} = \frac{K_0}{\alpha} \tag{17}$$

because $r_1 \ll x_m$, $x_1 \ll x_m$. Hence, s_α is given by

$$s_\alpha = \frac{r_{2\alpha}}{K_\alpha x_{m\alpha}} = \frac{r_{20}}{K_0 x_{m0}} = \text{const.} \tag{18}$$

where

$$K_0 = \sqrt{\frac{r_{10}^2 + x_{10}^2}{r_{10}^2 + (x_{10} + x_{m0})^2}} \tag{19}$$

For the case of ④;
Likewise above

$$K_\alpha \doteqdot \frac{K_0}{\alpha^2} \tag{20}$$

Then the slip s_α for the pole pitch τ_α is

$$s_\alpha = \frac{r_{20}}{K_0 x_{m0}} = \text{const.} \tag{21}$$

Therefoie it may be seen that the slip at the maximum driving force is constant and has no relation to the pole pitch.

2.5. Optimum distribution of pole pitch

Substituting the slip derived in the previous section into eq. (9), the optimum pole pitch at each point of the accelerator may be determined, from eq. (18) and $v_{s\alpha}=2\tau_\alpha f=2\alpha\tau_0 f$, eq. (9) may be written

$$y=\frac{M}{2F_{\max}}(2\alpha\tau_0 f)^2\left(1-\frac{r_{20}}{K_0 x_{m0}}\right)^2 \tag{22}$$

Whilst from eq. (13) the maximum driving force $F_{\max}$ on the moving member is

$$F_{\max}=\frac{l_2}{\alpha\tau_0}\frac{3\alpha^2 V_{10}^2}{2\alpha\tau_0 f}\frac{\frac{K_0}{\alpha}\alpha^2 x_{m0}}{\left(1+\frac{K_0}{\alpha}\right)^2(\alpha^2 r_{10}^2+\alpha^2 x_{10}^2)+\frac{K_0}{\alpha}\alpha^2 x_{m0}\times\left(2\alpha r_{10}+\frac{K_0}{\alpha}\alpha^2 x_{m0}+2\frac{K_0}{\alpha}\alpha x_{10}\right)} \tag{23}$$

where l_2 is the secondary (moving member) length.
Because of $K_0/\alpha \ll 1$, $x_{10} \ll \alpha$

$$F_{\max} \doteqdot \frac{l_2}{\tau_0}\frac{3V_{10}^2}{2\alpha\tau_0 f}\frac{K_0 x_{m0}}{r_{10}^2+x_{10}^2+K_0 x_{m0}(2r_{10}+K_0 x_{m0})} \tag{24}$$

From eq. (21) and eq. (24)

$$y=\frac{\tau_0}{l_2}\frac{(2\alpha\tau_0 f)^3 M}{6V_{10}^2}\frac{r_{10}^2+x_{10}^2+K_0 x_{m0}(2r_{10}+K_0 x_{m0})}{K_0 x_{m0}}\left(1-\frac{r_{20}}{K_0 x_{m0}}\right)^2 \tag{25}$$

This result is for the case of ③, and for the case of ④ same result may be derived, too.

The following points of interest can be deduced from eq. (25);

a) in the optimum condition, the pole pitch coefficient α is proportional to qubic root of the distance y.
b) referring to the circuit components, there is only that on the standard pole pitch.
c) the distance y is proportional to the moving member mass M and inversely proportional to the moving member length l_2.
d) the distance y is also proportional to f^3/V_{10}^2 for the power supply.

3. Experimental results[5)]

3.1. Tubular motor employed on experiments

Tubular motor is defind as the result of rolling an ordinary 'flat' linear motor for the axis of field travelling direction. The construction of motor employed on the experiments is shown in Fig. 5. The reason why the tubular motor is used in the experiments is that it easily can change its pole pitches. This tubular motor's circuit components may become to the case of ③ in Table 1, as the numbers of turn are proportional to the pole pitch and the tooth width are constant. The experimental values of each component per

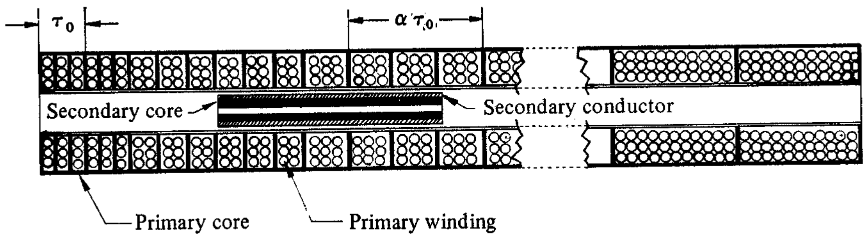

Fig. 5. Tubular motor.

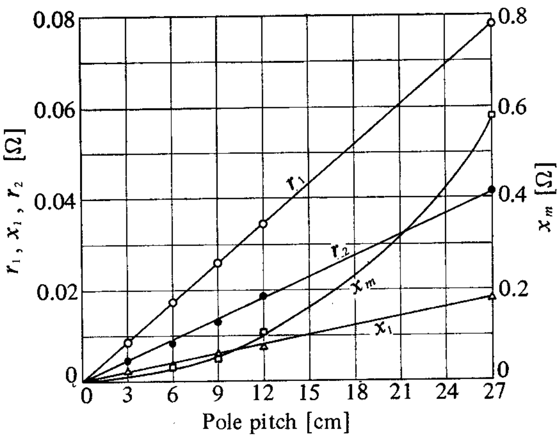

Fig. 6. Experimental equivalant circuit components of tubular motor.

pole are shown in Fig. 6. These results are agreeable to eq. (15) and to the case of ③ in Table 1.

3.2. Characteristics of accelerator

The optimum distribution of pole pitches is shown in eq. (25), but there are hardly such continuous distribution possible. On the experiments, therefore, two kinds of the graded pole pitch windings, Ⓐ and Ⓑ, shown in Fig. 7 are used. The curves in Fig. 7 show the optimum pole pitches (expressed in the synchronous speed) on the distance with the parameter V_1. Fig. 8 represents the speed versus the distance curves in the two distributions of Ⓐ and Ⓑ shown in Fig. 7. At the primary voltage $V_1 = 150$ [V], the speed in Ⓑ distribution is faster than that of Ⓐ, because, Ⓑ is nearer to the optimum curve at $V_1 = 150$ [V] than Ⓐ in Fig. 7. Likewise, as Ⓐ is nearer to the optimum curve than Ⓑ at $V_1 = 300$ [V], Fig. 8 shows that the speed of Ⓐ is faster than that of Ⓑ. But the opposite phenomenon may be seen at $V_1 = 220$[V]. This may be considered for the reason that the friction assume to be negrigible in eq. (8).

4. Conclusion

When Linear Induction Motor is used as an accelerator, it is easy to control the speed with the graded pole pitches. This paper represents the analysis and the experimental

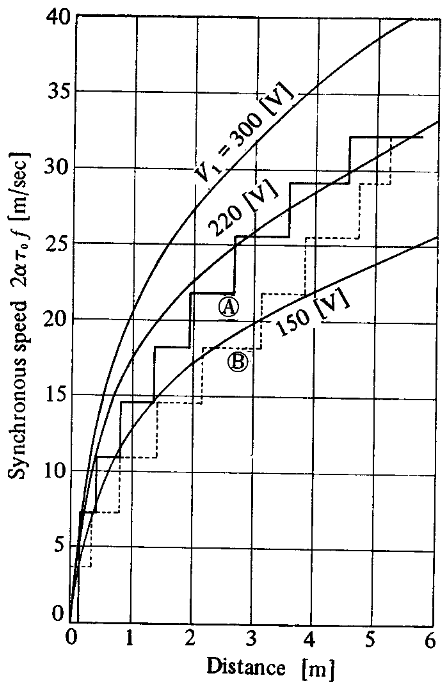

Fig. 7. Optimum and exeperimental distributions of pole pitches.

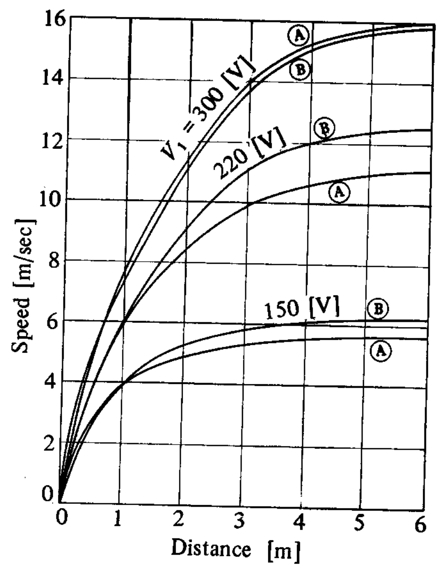

Fig. 8. Speed characteristics of accelerator.

results of the optimum distribution of pole pitch for the tubular motor with constant numbers of turn per unit length. As a result, it may be seen that the optimum pole pitch is proportional to qubic root of the distance, when the friction is negrigible; then the proportional coefficient is determined by the moving member components (mass and length), by the power supply components (voltage and frequency) and by only the equivalent circuit components at the standard pole pitch.

References

1) E.R. Laithwaite, Induction Machines for Special Purposes, p. 222, George Newnes, London (1966).
2) T. Onuki, H. Nomura, K. Miura, Lecture 506 in Annual Meeting of I.E.E. of Japan (Apr. 1970).
3) T. Onuki, T. Azukisawa, S. Morita, Lecture 507 in Annual Meeting of I.E.E. of Japan (Apr. 1970).
4) T. Onuki, Lecture S. 4–5 in Annual Meeting of I.E.E. of Japan (Apr. 1971).
5) M. Kajino, S. Ishikawa, T. Hirasa, Lecture 433 in Annual Meeting of I.E.E. of Japan (Apr. 1971).
6) M. Kajino, S. Ishikawa, T. Hirasa, Lecture 508 in Annual Meeting of I.E.E. of Japan (Apr. 1970).