Analysis of Brand Selection and Its Matrix Structure in an Automobile Case Study

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Abstract: It is often observed that consumers select upper class brand when they buy next time. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. Takeyasu et al. (2007) analyzed the brand selection and its matrix structure before. In this paper, a questionnaire investigation to automobile purchasing case is executed and above structure is analyzed. Multi-correspondence analysis is executed on these data. Some interesting results are obtained. Analyzing such structure provides useful applications. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

Key Words: brand selection, matrix structure, automobile industry

1. INTRODUCTION

It is often observed that consumers select the upper class brand when they buy the next time. Focusing the transition matrix structure of brand selection, their activities may be analyzed. In the past, there are many researches about brand selection [1-5]. But there are few papers concerning the analysis of the transition matrix structure of brand selection. In this paper, we make analysis of the preference shift of customer brand selection and confirm them by the questionnaire investigation for automobile purchasing case. If we can identify the feature of the matrix structure of brand selection, it can be utilized for the marketing strategy.

Suppose that the former buying data and the current buying data are gathered. Also suppose that the upper brand is located upper in the variable array. Then the transition

matrix becomes an upper triangular matrix under the supposition that the former buying variables are set input and the current buying variables are set output. If the top brand were selected from the lower brand in jumping way, corresponding part in the upper triangular matrix would be 0. These are verified by the numerical examples with simple models.

If the transition matrix is identified, a S-step forecasting can be executed. Generalized forecasting matrix components' equations are introduced. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

A quantitative analysis concerning brand selection has been executed by [4, 5]. [5] examined purchasing process by Markov Transition Probability with the input of advertising expense. [4] made analysis by the Brand Selection Probability model using logistics distribution.

In this paper, matrix structure is analyzed for the case the upper class brand is selected compared with the past purchasing case, and extensions for various applications are executed. Such research can not be found as long as searched.

Hereinafter, matrix structure is clarified for the selection of brand in section 2. A block matrix structure is analyzed when brands are handled in a group and a s-step forecasting is formulated in section 3. A questionnaire investigation to Automobile Purchasing case is examined and its numerical calculation is executed in section 4. The results by multi correspondence analysis are described in section 5. Application of this method is extended in section 6. Section 7 is a summary.

2. BRAND SELECTION AND ITS MATRIX STRUCTURE

(1) Upper shift of Brand selection

It is often observed that consumers select the upper class brand when they buy the next time. Now, suppose that x is the most upper class brand, y is the second upper class brand, and z is the lowest class brand.

Consumer's behavior of selecting brand might be $z \to y$, $y \to x$, $z \to x$ etc. $x \to z$ might be few. Suppose that x is the current buying variable, and x_b is the previous buying variable. Shift to x is executed from x_b, y_b, or z_b.

Therefore, x is stated in the following equation. a_{ij} represents transition probability from j-th to i-th brand.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

and

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \tag{1}$$

Set

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \qquad \mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{A}\mathbf{X_b} \tag{2}$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times 3}, \mathbf{X_b} \in \mathbf{R}^3$$

$\mathbf{A}$ is an upper triangular matrix.

To examine this, generating the following data, which are all consisted by the upper brand shift data,

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \cdots \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \cdots \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$$i = 1 \quad , \qquad 2 \qquad \cdots \qquad N$$

parameter can be estimated using least square method.

Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_\mathbf{b}^i + \boldsymbol{\varepsilon}^i \tag{5}$$

where $\varepsilon^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix}$ $\quad i = 1,2,\cdots,N$

and minimize following J

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT} \boldsymbol{\varepsilon}^i \rightarrow Min \tag{6}$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_\mathbf{b}^{iT} \right) \left(\sum_{i=1}^{N} \mathbf{X}_\mathbf{b}^i \mathbf{X}_\mathbf{b}^{iT} \right)^{-1} \tag{7}$$

In the data group of the upper shift brand, estimated value $\hat{\mathbf{A}}$ should be an upper triangular matrix. If the following data, that have the lower shift brand, are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{X}_\mathbf{b}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part of triangle.

(2) Sorting brand ranking by re-arranging row

In a general data, variables may not be in order as x, y, z. In that case, large and small value lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in an upper triangular matrix, and the small value parts are gathered in a lower triangular matrix.

$$\begin{matrix} & \hat{\mathbf{A}} & & & & \hat{\mathbf{A}} & \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \\ \varepsilon & \varepsilon & \bigcirc \end{pmatrix} & \text{Shifting row} \Leftarrow & & \begin{pmatrix} z \\ x \\ y \end{pmatrix} & \begin{pmatrix} \varepsilon & \varepsilon & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \end{pmatrix} \end{matrix} \tag{8}$$

(3) Matrix structure under the case skipping intermediate class brand is skipped

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the intermediate class brand.

We suppose v, w, x, y, z brands (suppose they are laid from the upper position to the lower position as $v > w > x > y > z$).

In the above case, the selection shifts would be

$$v \leftarrow z$$
$$v \leftarrow y$$

Suppose there is no shift from z to y, corresponding part of the transition matrix is 0(i.e. a_{45}=0) . Similarly, if there is no shift from z to y, from z to w, from y to x, form y to w, from x to w, then the matrix structure would be as follows.

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix} \tag{9}$$

3. BROCK MATRIX STRUCTURE IN BRAND GROUPS AND s-STEP FORECASTING

Next, we examine the case in brand groups. Matrices are composed by the Block Matrix.

(1) Brand Shift Group for The Case of Two Groups

Suppose the brand selection shifts from Corolla class to Mark Ⅱ class in a car. In this case, it does not matter which company's car they choose. Thus, selection of cars are executed in a group and the brand shift is considered to be done from group to group. Suppose brand groups at time n are as follows. $\mathbf{X}$ consists of p varieties of goods, and $\mathbf{Y}$ consists of q varieties of goods.

$$\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \qquad \mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}} \\ \mathbf{0}, & \mathbf{A_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \end{pmatrix} \tag{10}$$

Here,

$$\mathbf{X_n} \in \mathbf{R}^p \ (n = 1,2,\cdots),\ \mathbf{Y_n} \in \mathbf{R}^q \ (n = 1,2,\cdots),\ \mathbf{A_{11}} \in R^{p\times q},\quad \mathbf{A_{12}} \in R^{p\times q},\quad \mathbf{A_{22}} \in R^{q\times q}$$

Make one more step of shift, then we obtain the following equation.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^2, & \mathbf{A_{11}A_{12}} + \mathbf{A_{12}A_{22}} \\ \mathbf{0}, & \mathbf{A_{22}}^2 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-2}} \\ \mathbf{Y_{n-2}} \end{pmatrix} \tag{11}$$

Make one more step of shift again, then we obtain the following equation.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^3, & \mathbf{A_{11}}^2\mathbf{A_{12}} + \mathbf{A_{11}A_{12}A_{22}} + \mathbf{A_{12}A_{22}}^2 \\ \mathbf{0}, & \mathbf{A_{22}}^3 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-3}} \\ \mathbf{Y_{n-3}} \end{pmatrix} \tag{12}$$

Similarly,

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^4, & \mathbf{A_{11}}^3\mathbf{A_{12}} + \mathbf{A_{11}}^2\mathbf{A_{12}A_{22}} + \mathbf{A_{11}A_{12}A_{22}}^2 + \mathbf{A_{12}A_{22}}^3 \\ \mathbf{0}, & \mathbf{A_{22}}^4 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-4}} \\ \mathbf{Y_{n-4}} \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^5, & \mathbf{A_{11}}^4\mathbf{A_{12}} + \mathbf{A_{11}}^3\mathbf{A_{12}A_{22}} + \mathbf{A_{11}}^2\mathbf{A_{12}A_{22}}^2 + \mathbf{A_{11}A_{12}A_{22}}^3 + \mathbf{A_{12}A_{22}}^4 \\ \mathbf{0}, & \mathbf{A_{22}}^5 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-5}} \\ \mathbf{Y_{n-5}} \end{pmatrix} \tag{14}$$

Finally, we get the generalized equation for a s-step shift as follows.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^s, & \mathbf{A_{11}}^{s-1}\mathbf{A_{12}} + \sum_{k=2}^{s-1} \mathbf{A_{11}}^{s-k}\mathbf{A_{12}A_{22}}^{k-1} + \mathbf{A_{12}A_{22}}^{s-1} \\ \mathbf{0}, & \mathbf{A_{22}}^s \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-s}} \\ \mathbf{Y_{n-s}} \end{pmatrix} \tag{15}$$

If we replace $n - s \to n, n \to n + s$ in equation (15), we can make a s-step forecast.

(2) Brand Shift Group for The Case of Three Groups

Suppose the brand selection is executed in the same group or to the upper group, and

also suppose that the brand position is $x > y > z$ (x is upper position). Then the brand selection transition matrix would be expressed as :

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}}, & \mathbf{A_{13}} \\ \mathbf{0}, & \mathbf{A_{22}}, & \mathbf{A_{23}} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A_{33}} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix} \tag{16}$$

Where,

$$\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \quad \mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}, \quad \mathbf{Z_n} = \begin{pmatrix} \mathbf{z}_1^n \\ \mathbf{z}_2^n \\ \vdots \\ \mathbf{z}_r^n \end{pmatrix}$$

Here,

$$\mathbf{X_n} \in \mathbf{R}^p \ (n = 1,2,\cdots), \mathbf{Y_n} \in \mathbf{R}^q \ (n = 1,2,\cdots), \mathbf{Z_n} \in \mathbf{R}^r \ (n = 1,2,\cdots), \mathbf{A_{11}} \in R^{p\times p},$$

$$\mathbf{A_{12}} \in R^{p\times q}, \ \mathbf{A_{13}} \in R^{p\times r}, \ \mathbf{A_{22}} \in R^{q\times q}, \ \mathbf{A_{23}} \in R^{q\times r}, \mathbf{A_{33}} \in R^{r\times r}$$

These are re-stated as :

$$\mathbf{W_n} = \mathbf{A}\mathbf{W_{n-1}} \tag{17}$$

Where,

$$\mathbf{W_n} = \begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix}, \ \mathbf{A} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}}, & \mathbf{A_{13}} \\ \mathbf{0}, & \mathbf{A_{22}}, & \mathbf{A_{23}} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A_{33}} \end{pmatrix}, \ \mathbf{W_{n-1}} = \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix}$$

Hereinafter, we shift steps as is done in the previous section.
In the general description, we state as :

$$\mathbf{W_n} = \mathbf{A}^{(s)}\mathbf{W_{n-s}} \tag{18}$$

Here,

$$\mathbf{A}^{(s)} = \begin{pmatrix} \mathbf{A_{11}}^{(s)}, & \mathbf{A_{12}}^{(s)}, & \mathbf{A_{13}}^{(s)} \\ \mathbf{0}, & \mathbf{A_{22}}^{(s)}, & \mathbf{A_{23}}^{(s)} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A_{33}}^{(s)} \end{pmatrix}, \quad \mathbf{W_{n-s}} = \begin{pmatrix} \mathbf{X_{n-s}} \\ \mathbf{Y_{n-s}} \\ \mathbf{Z_{n-s}} \end{pmatrix}$$

From definition,

$$\mathbf{A}^{(1)} = \mathbf{A} \tag{19}$$

In the case $s = 2$, we obtain :

$$\begin{aligned}\mathbf{A}^{(2)} &= \begin{pmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33} \end{pmatrix}\begin{pmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{A}_{11}^2, & \mathbf{A}_{11}\mathbf{A}_{12} + \mathbf{A}_{12}\mathbf{A}_{22}, & \mathbf{A}_{11}\mathbf{A}_{13} + \mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{13}\mathbf{A}_{33} \\ \mathbf{0}, & \mathbf{A}_{22}^2, & \mathbf{A}_{22}\mathbf{A}_{23} + \mathbf{A}_{23}\mathbf{A}_{33} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^2 \end{pmatrix}\end{aligned} \tag{20}$$

Next, in the case $s = 3$, we obtain :

$$\mathbf{A}^{(3)} = \begin{pmatrix} \mathbf{A}_{11}^3, & \mathbf{A}_{11}^2\mathbf{A}_{12} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22} + \mathbf{A}_{12}\mathbf{A}_{22}^2, & \mathbf{A}_{11}^2\mathbf{A}_{13} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33} + \mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23} + \mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{13}\mathbf{A}_{33}^2 \\ \mathbf{0}, & \mathbf{A}_{22}^3, & \mathbf{A}_{22}^2\mathbf{A}_{23} + \mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{23}\mathbf{A}_{33}^2 \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^3 \end{pmatrix} \tag{21}$$

In the case $s = 4$, equations become wide-spread, so we express each Block Matrix as follows.

$$\mathbf{A}_{11}^{(4)} = \mathbf{A}_{11}^4$$

$$\mathbf{A}_{12}^{(4)} = \mathbf{A}_{11}^3\mathbf{A}_{12} + \mathbf{A}_{11}^2\mathbf{A}_{12}\mathbf{A}_{22} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^2 + \mathbf{A}_{12}\mathbf{A}_{22}^3$$

$$\begin{aligned}\mathbf{A}_{13}^{(4)} = {} & \mathbf{A}_{11}^3\mathbf{A}_{13} + \mathbf{A}_{11}^2\mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{11}^2\mathbf{A}_{13}\mathbf{A}_{33} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33}^2 \\ & + \mathbf{A}_{12}\mathbf{A}_{22}^2\mathbf{A}_{23} + \mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^2 + \mathbf{A}_{13}\mathbf{A}_{33}^3\end{aligned}$$

$$\mathbf{A}_{22}^{(4)} = \mathbf{A}_{22}^4$$

$$\mathbf{A}_{23}^{(4)} = \mathbf{A}_{22}^3\mathbf{A}_{23} + \mathbf{A}_{22}^2\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^2 + \mathbf{A}_{23}\mathbf{A}_{33}^3$$

$$\mathbf{A}_{33}^{(4)} = \mathbf{A}_{33}^4$$

In the case $s = 5$, we obtain the following equations similarly.

$$\mathbf{A}_{11}^{(5)} = \mathbf{A}_{11}^{5}$$
$$\mathbf{A}_{12}^{(5)} = \mathbf{A}_{11}^{4}\mathbf{A}_{12} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{22} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}^{2} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{3} + \mathbf{A}_{12}\mathbf{A}_{22}^{4}$$
$$\begin{aligned}\mathbf{A}_{13}^{(5)} = {} & \mathbf{A}_{11}^{4}\mathbf{A}_{13} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{11}^{3}\mathbf{A}_{13}\mathbf{A}_{33} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}^{2}\mathbf{A}_{13}\mathbf{A}_{33}^{2} \\ & + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33}^{3} \\ & + \mathbf{A}_{12}\mathbf{A}_{22}^{3}\mathbf{A}_{23} + \mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{13}\mathbf{A}_{33}^{4}\end{aligned}$$
$$\mathbf{A}_{22}^{(5)} = \mathbf{A}_{22}^{5}$$
$$\mathbf{A}_{23}^{(5)} = \mathbf{A}_{22}^{4}\mathbf{A}_{23} + \mathbf{A}_{22}^{3}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{23}\mathbf{A}_{33}^{4}$$
$$\mathbf{A}_{33}^{(5)} = \mathbf{A}_{33}^{5}$$

In the case $s = 6$, we obtain :

$$\mathbf{A}_{11}^{(6)} = \mathbf{A}_{11}^{6}$$
$$\mathbf{A}_{12}^{(6)} = \mathbf{A}_{11}^{5}\mathbf{A}_{12} + \mathbf{A}_{11}^{4}\mathbf{A}_{12}\mathbf{A}_{22} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{22}^{2} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}^{3} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{4} + \mathbf{A}_{12}\mathbf{A}_{22}^{5}$$
$$\begin{aligned}\mathbf{A}_{13}^{(6)} = {} & \mathbf{A}_{11}^{5}\mathbf{A}_{13} + \mathbf{A}_{11}^{4}\mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{11}^{4}\mathbf{A}_{13}\mathbf{A}_{33} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}^{3}\mathbf{A}_{13}\mathbf{A}_{33}^{2} \\ & + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}^{2}\mathbf{A}_{13}\mathbf{A}_{33}^{3} \\ & + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{3}\mathbf{A}_{23} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33}^{4} \\ & + \mathbf{A}_{12}\mathbf{A}_{22}^{4}\mathbf{A}_{23} + \mathbf{A}_{12}\mathbf{A}_{22}^{3}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{4} + \mathbf{A}_{13}\mathbf{A}_{33}^{5}\end{aligned}$$

We get the generalized equations for a *s*-step shift as follows.

$$\mathbf{A}_{11}^{(s)} = \mathbf{A}_{11}^{s}$$
$$\mathbf{A}_{12}^{(s)} = \mathbf{A}_{11}^{s-1}\mathbf{A}_{12} + \sum_{k=2}^{s-1}\mathbf{A}_{11}^{s-k}\mathbf{A}_{12}\mathbf{A}_{22}^{k-1} + \mathbf{A}_{12}\mathbf{A}_{22}^{s-1}$$
$$\mathbf{A}_{13}^{(s)} = \mathbf{A}_{11}^{s-1}\mathbf{A}_{13} + \mathbf{A}_{11}^{s-2}\left(\sum_{k=1}^{2}\mathbf{A}_{1(k+1)}\mathbf{A}_{(k+1)3}\right) + \sum_{j=1}^{s-3}\left[\mathbf{A}_{11}^{S-2-j}\left\{\mathbf{A}_{12}\left(\sum_{k=1}^{j+1}\mathbf{A}_{22}^{j+1-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}\right) + \mathbf{A}_{13}\mathbf{A}_{33}^{j+1}\right\}\right]$$
$$\mathbf{A}_{22}^{(s)} = \mathbf{A}_{22}^{s}$$
$$\mathbf{A}_{23}^{(s)} = \sum_{K-1}^{s}\mathbf{A}_{22}^{s-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}$$
$$\mathbf{A}_{33}^{(s)} = \mathbf{A}_{33}^{s}$$

Expressing them in matrix, it follows :

$$\mathbf{A}^{(S)}=\begin{pmatrix} \mathbf{A}_{11}^{s}, & \mathbf{A}_{11}^{s-1}\mathbf{A}_{12}+\sum_{k=2}^{s-1}\mathbf{A}_{11}^{s-k}\mathbf{A}_{12}\mathbf{A}_{22}^{k-1}+\mathbf{A}_{12}\mathbf{A}_{22}^{s-1}, & \mathbf{A}_{11}^{s-1}\mathbf{A}_{13}+\mathbf{A}_{11}^{s-2}\left(\sum_{k=1}^{2}\mathbf{A}_{1(k+1)}\mathbf{A}_{(K+1)3}\right)+\sum_{j=1}^{s-3}\left[\mathbf{A}_{11}^{s-2-j}\left\{\mathbf{A}_{12}\left(\sum_{k=1}^{j+1}\mathbf{A}_{22}^{j+1-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}\right)+\mathbf{A}_{13}\mathbf{A}_{33}^{j+1}\right\}\right] \\ \mathbf{0}, & \mathbf{A}_{22}^{s}, & \sum_{k=1}^{s}\mathbf{A}_{22}^{s-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^{s} \end{pmatrix} \tag{26}$$

Generalizing them to m groups, they are expressed as :

$$\begin{pmatrix} \mathbf{X}_{\mathbf{n}}^{(1)} \\ \mathbf{X}_{\mathbf{n}}^{(2)} \\ \vdots \\ \mathbf{X}_{\mathbf{n}}^{(m)} \end{pmatrix}=\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1m} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2m} \\ \vdots & \vdots & & \vdots \\ \mathbf{A}_{m1} & \mathbf{A}_{m2} & \cdots & \mathbf{A}_{mm} \end{pmatrix}\begin{pmatrix} \mathbf{X}_{\mathbf{n-1}}^{(1)} \\ \mathbf{X}_{\mathbf{n-1}}^{(2)} \\ \vdots \\ \mathbf{X}_{\mathbf{n-1}}^{(m)} \end{pmatrix} \tag{27}$$

$$\mathbf{X}_{\mathbf{n}}^{(1)} \in R^{k_1}$$
$$\mathbf{X}_{\mathbf{n}}^{(2)} \in R^{k_2}$$
$$\vdots$$
$$\mathbf{X}_{\mathbf{n}}^{(m)} \in R^{k_m}$$
$$\mathbf{A}_{\mathbf{ij}} \in R^{k_i \times k_j} \qquad (i=1,\cdots,m)(j=1,\cdots,m)$$

4. A QUESTIONNAIRE INVESTIGATION AND NUMERICAL CALCULATION

A questionnaire investigation for automobile purchasing case is executed.

<Delivery of Questionnaire Sheets>

- A questionnaire sheet : Appendix3
- Delivery Term : July 1 to 31 / 2009
- Delivery Place：Osaka, Hyogo, Tokyo in Japan
- Number of Delivered Questionnaire sheets: 500

<Result of collected Questionnaire Sheets>

- Collected Questionnaire Sheets: 199
- Collected data sets: 516

The questionnaire includes the question of the past purchasing history. Therefore the plural data may be gathered from one sheet. For example, we can get two data such as (before former automobile, former automobile), (former automobile, current automobile).

As a result, we obtained 516 data sets. 201 cases are the upper shifts, 192 cases are the same rank movement, and 123 cases are the lower shifts. Lower shift is consists of 1/5 in whole cases and the transition matrix corresponds to these facts on the whole.

Fundamental statistical result is exhibited in Table 1.

Table 1. Summary for 199 sheets

Age		Sex		Occupation		Annual income (Japanese Yen)		Marriage		Kids	
Teens	26	Male	173	Student	74	0-3 million	68	Single	100	0	117
Twenties	69	Female	26	Officer	13	3-5 million	17	Married	97	1	12
Thirties	31			Company employee	90	5-7.5 million	20	Not filled in	2	2	51
Forties	38			Clerk of Organization	1	7.5-10 million	10			3	17
Fifties	32			Independents	11	10-15 million	5			4	2
Sixties and over	3			Miscellaneous	9	15 million or more	1			5	0
Not filled in	0			Not filled in	1	Not filled in	78				
Sum	199		199		199		199		199		199

Analyzing collected sheets based on Model Ranking Table (Appendix1, Appendix2), we obtained the following 516 data sets. Appendix1 shows list of all cars and Appendix2 shows the ranking Table for this calculation.

1. Shift from 5th position to 5th position : 71
2. Shift from 5th position to 4th position : 31
3. Shift from 5th position to 3rd position : 26
4. Shift from 5th position to 2nd position : 29
5. Shift from 5th position to 1st position : 10
6. Shift from 4th position to 5st position : 19
7. Shift from 4th position to 4st position : 30
8. Shift from 4th position to 3st position : 24
9. Shift from 4th position to 2st position : 20
10. Shift from 4th position to 1st position : 4
11. Shift from 3th position to 5st position : 15
12. Shift from 3th position to 4st position : 14
13. Shift from 3th position to 3st position : 22
14. Shift from 3th position to 2st position : 28
15. Shift from 3th position to 1st position : 12
16. Shift from 2th position to 5st position : 15
17. Shift from 2th position to 4st position : 9
18. Shift from 2th position to 3st position : 30
19. Shift from 2th position to 2st position : 55
20. Shift from 2th position to 1st position : 17
21. Shift from 1th position to 5st position : 6
22. Shift from 1th position to 4st position : 5
23. Shift from 1th position to 3st position : 5
24. Shift from 1th position to 2st position : 5
25. Shift from 1th position to 1st position : 14

Total : 516

Using the description as is stated in 2.1, the vector $\mathbf{X}, \mathbf{X}_b$ in these cases are expressed as follows.

1. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ 2. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ 3. $\mathbf{X}=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$

4. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ 5. $\mathbf{X}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ 6. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$

7. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ 8. $\mathbf{X}=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ 9. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$

10. $\mathbf{X}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ 11. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ 12. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$

13. $\mathbf{X}=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ 14. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ 15. $\mathbf{X}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$

16. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ 17. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ 18. $\mathbf{X}=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$

19. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ 20. $\mathbf{X}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ 21. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$

22. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ 23. $\mathbf{X}=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ 24. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$

25. $\mathbf{X}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ $\mathbf{X}_b=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$

Substituting these to equation (7), we can get the transition matrix $\hat{\mathbf{A}}$ as follows.

$$\hat{\mathbf{A}} = \begin{pmatrix} 14 & 17 & 12 & 4 & 10 \\ 5 & 55 & 28 & 20 & 29 \\ 5 & 30 & 22 & 24 & 26 \\ 5 & 9 & 14 & 30 & 31 \\ 6 & 15 & 15 & 19 & 71 \end{pmatrix} \begin{pmatrix} 35 & 0 & 0 & 0 & 0 \\ 0 & 126 & 0 & 0 & 0 \\ 0 & 0 & 91 & 0 & 0 \\ 0 & 0 & 0 & 97 & 0 \\ 0 & 0 & 0 & 0 & 167 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{2}{5} & \frac{17}{126} & \frac{12}{91} & \frac{4}{97} & \frac{10}{167} \\ \frac{1}{7} & \frac{55}{126} & \frac{4}{13} & \frac{20}{97} & \frac{29}{167} \\ \frac{1}{7} & \frac{5}{21} & \frac{22}{91} & \frac{24}{97} & \frac{26}{167} \\ \frac{1}{7} & \frac{1}{14} & \frac{2}{13} & \frac{30}{97} & \frac{31}{167} \\ \frac{6}{35} & \frac{5}{42} & \frac{15}{91} & \frac{19}{97} & \frac{71}{167} \end{pmatrix} \quad (28)$$

A questionnaire investigation for automobile purchasing case is executed and the matrix structure stated in 2.1 can be confirmed on the whole.

Hearing results from the car dealers are as follows. There is a tendency to the shift to the upper brands. But some of them have the following features such as :

1. When young, they ride on the high ranked automobile. But when married, they ride on the automobile of the ordinary level.
2 Office workers are apt to buy the higher ranked automobile as they promote.
3. Recently the interior of automobile became upgraded. Therefore user can enjoy the higher ranked automobile in a rather lower grade automobile, which causes less need to upgrade.
4. People who ride on eco-car are increasing.

Therefore, there are cases that the shifts to the upper brands do not necessarily occur. Anyway, the results show that 76% (Upper shift: 201, Same shift: 192, Lower shift: 123) are the same rank movement or the upper shifts, and the transition matrix corresponds to these facts on the whole.

5. MULTI CORRESPONDENCE ANALYSIS

Multi Correspondence Analysis was executed based upon a questionnaire investigation for automobile purchasing case. Intimacy among questionnaire items is derived by this analysis.

5.1 Design and Sports

We made correspondence analysis between Q8-1 and Q9-1. From Chart 1, we can observe that those who esteem design of twenties have strong relationship with those who like sports.

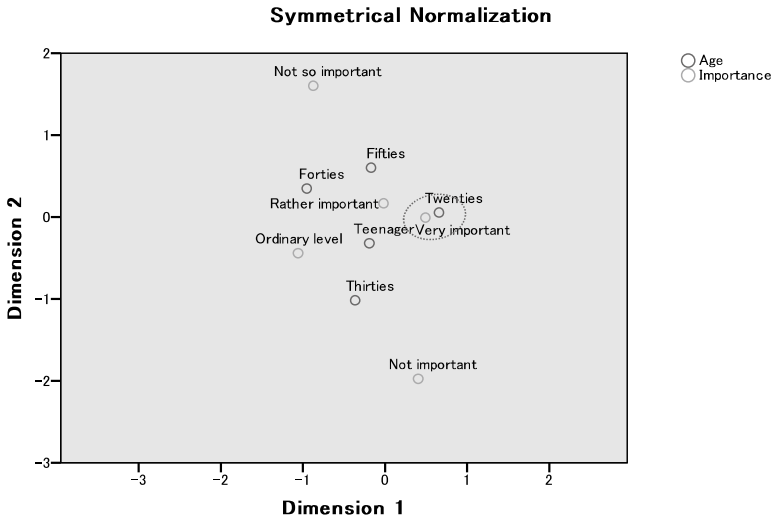

Chart 1：Q8-1 and Q9-1

5.2 Driving and Performance

We made correspondence analysis between Q8-3 and Q9-8. From Chart 2, we can observe that those who like driving of twenties have strong relationship with those who esteem performance of the car.

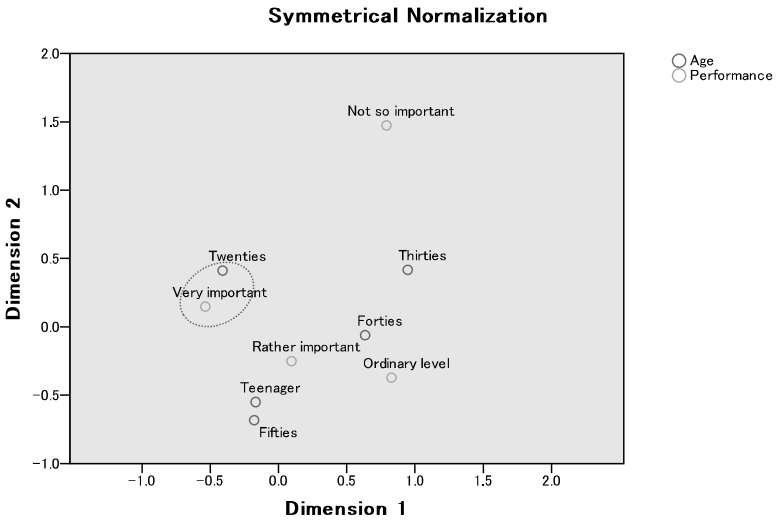

Chart 2：Q8-3 and Q9-8

5.3 Age and Price

From Chart 3, we can observe that those who are thirties and forties esteem price. This may be because they do not spend so much money on an automobile. They have children and house to spend a lot.

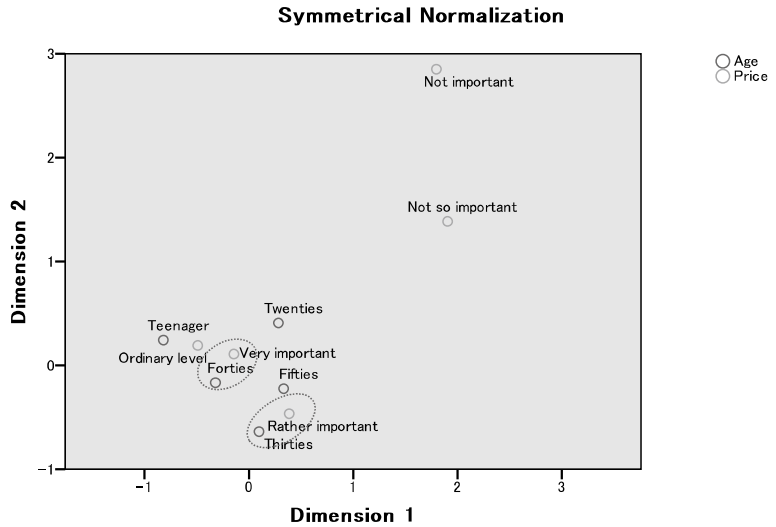

Chart 3：Q8-4

5.4 Object and Automobile Rank

From Chart 4, we can observe strong correlation between “Rank Ⅰ” and “Picking up and Dropping off”, “Shopping” and “Rank Ⅱ, Ⅴ”.

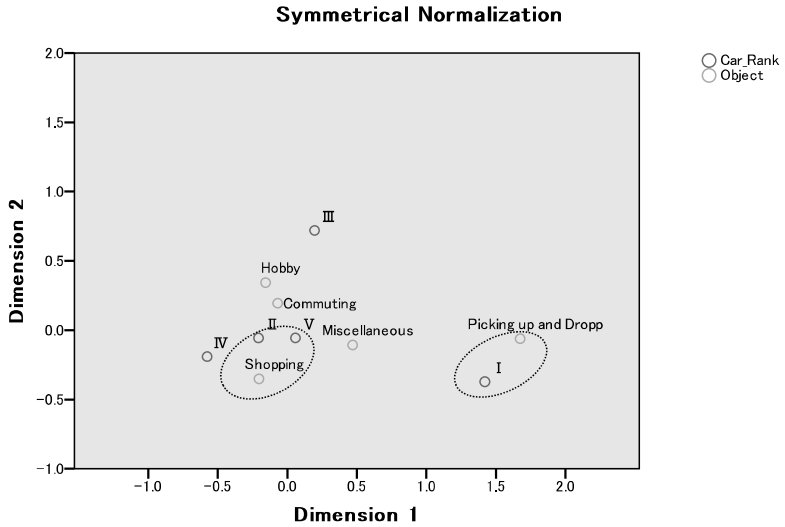

Chart 4：Q9-9

5.5 Object and Automobile (by Age)

Now, we see them by age in detail.

(1) Teenagers

From Chart 5, we can observe strong correlation between "Miscellaneous, Commuting" and "Rank Ⅲ". We can also observe those in "Hobby" and "Rank Ⅴ". We can assume that teenagers use automobile for hobby and playing. Questionnaire results show that they often use "Fit" and/or "Vitz", which are compact and easy to handle.

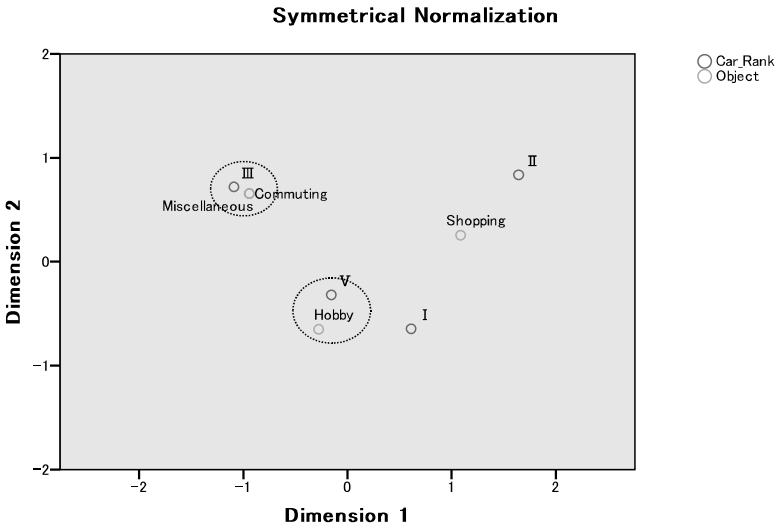

Chart 5：Q9-9 (Teenagers)

(2) Twenties

From Chart 6, we can observe strong correlation between "Hobby" and "Rank Ⅱ, Ⅴ" in twenties. These are similar to those of teenagers but as twenties have more money, they seek upper ranked automobiles.

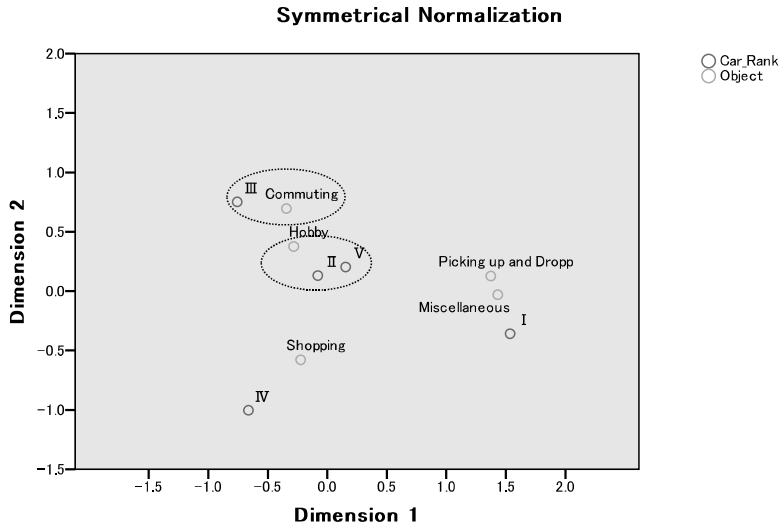

Chart 6：Q9-9（Twenties）

(3) Thirties

From Chart 7, we can observe strong correlation between "Commuting" and "Rank Ⅳ" in thirties. There is also a correlation between "Hobby" and "Rank Ⅱ". There was a correlation with "Rank Ⅴ" in twenties. This may be because they own higher ranked automobile as income increases.

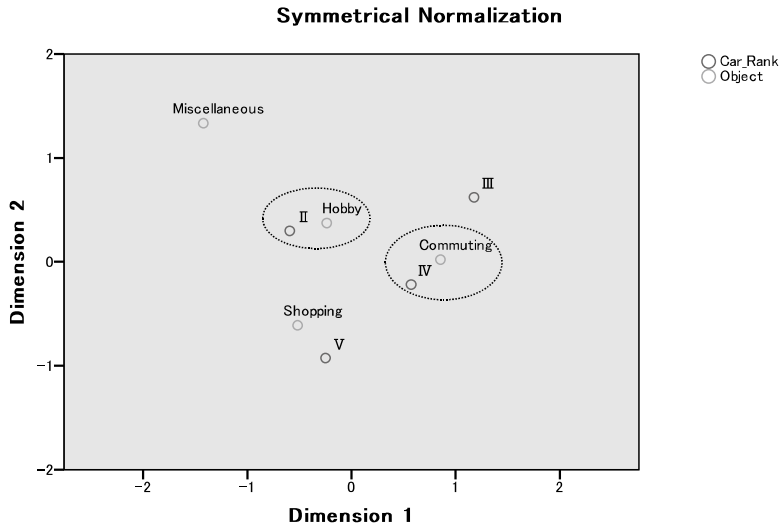

Chart 7：Q9-9（Thirties）

(4) Forties

From Chart 8, we can observe strong correlation between "Shopping" and "RankⅣ, Ⅴ" in forties. In the forties, using automobile for shopping was dominant from the result.

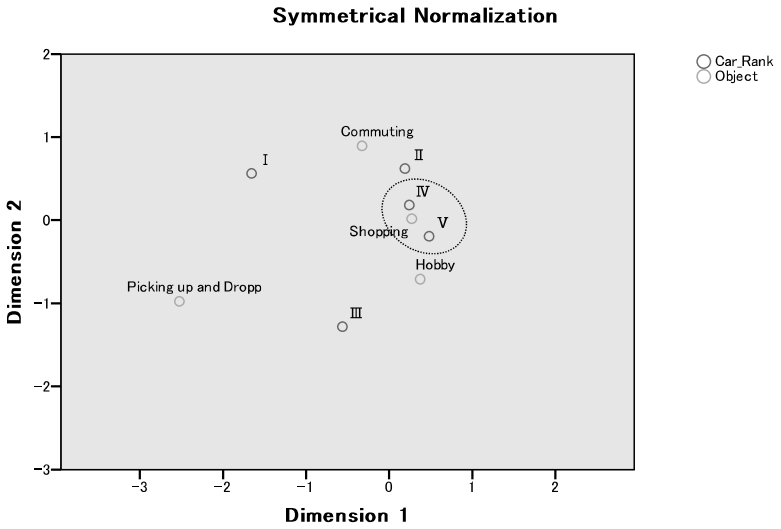

Chart 8：Q9-9（Fourties）

(5) Fifties

From Chart 9, we can observe strong relationship among "Commuting" and "Rank Ⅴ" in fifties. Generally, we can assume that they own plural automobiles and use cheaper one for commuting and higher one for hobby.

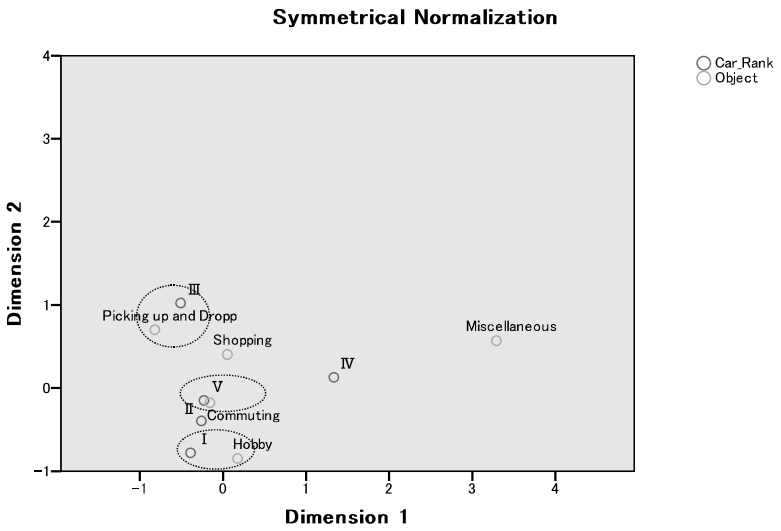

Chart 9：Q9-9（Fifties）

We can clearly see that Ⅴ Ⅳ ranked car is often used for commuting and Ⅱ for hobby. As ages grow up, upper shift for brand selection appears clearly, which verifies the obtained result in the previous section.

6. APPLICATIONS OF THIS METHOD

Applications of this method are considered to be as follows. Consumers' behavior may converge by repeating forecast under the above method and the total volume of sales of all brands may be reduced. Therefore, the analysis results suggest when and what to put the new brand into the market which contribute to the expansion of the market.

There may arise following cases. Consumers and producers do not recognize the brand position clearly. But the analysis of consumers' behavior let them know their brand position in the market. In such a case, strategic marketing guidance to select the brand would be introduced. Setting in order the brand position of various goods and taking suitable marketing policy, enhancement of sales would be enabled. By setting the higher ranked brand, consumption would be promoted.

7. CONCLUSION

It is often observed that consumers select the upper class brand when they buy the next time. Suppose that the former buying data and the current buying data are gathered. Also suppose that the upper brand is located upper in the variable array. Then the transition matrix becomes an upper triangle matrix under the supposition that former buying variables are set input and current buying variables are set output. If the top brand are selected from the lower brand in jumping way, corresponding part in an upper triangle matrix would be 0. A questionnaire investigation for automobile purchasing case was executed and the above structure was confirmed.

If the transition matrix is identified, a S-step forecasting can be executed. Generalized forecasting matrix components' equations were introduced. Furthermore, we executed multi correspondence analysis based upon a questionnaire investigation for automobile purchasing case. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach

enables to make effective marketing plan and/or establishing new brand. Various fields should be examined hereafter.

In the end, we appreciate Mr. Naoki IZUMI for his helpful support of work.

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Appendix1. List of all cars

	Toyota	Nissan	Honda	Subaru	Suzuki	Mitsubishi	Mazda	Daihatsu	Benz
Sedan	Opa WiLL Avalon Avensis Allion Aristo Windom Verossa Origin Camry Corolla Corolla axio Crown Crown-athlete Crown-comfort Crown sedan Crown hybrid Crown-majesta Crown-royalsaloon Classic Cressida Cresta Corsa Corona Comfort Sprinter cielo Century Tercel Chaser Duet Vista Platz Prius Brevis Premio Progres Pronard Belta Mark X Mark Ⅱ Lexus ES Lexus GS Lexus HS Lexus IS Lexus LS	Infiniti Q Auster Gloria Sunny Cima Stanza Cedric Cefiro Teana Tiida latio Pulsar Fuga Primera Bluebird-sylphy President Presea Maxima Langley Liberta villa Leopard Laurel	CR-X Accord Ascot Insight Inspire Integra Concerto City Civic Civic type-R Civic hybrid Saber Today Domani Torneo Ballade Vigor Fit aria Rafaga Legend Logo	Impreza Impreza-anesis Leone Legacy B4	SX4 sedan Aerio Cultus Chevrolet-optra Chevrolet-cruze	Aspire Eterna Emeraude Carisma Galant fortis Sigma Diamante Dignity Debonair Tredia Proudia Magna Mirage Lancer-evolution Lancer sedan		Applause Altis Charmant	
Coupe・Sports car	MR2 MR-S Curren Corolla levin Cynos Supra Starlet Sprinter Sera Celica Soarer Lexus SC	GT-R 180SX NX coupe Exa Gazelle Silvia Skyline-coupe Figaro Fairlady z Micra C+C Lucino	S2000 NSX Integra-type-R Prelude	Alcyone		FTO GTO Cordia Starion	RX-3 MX-6 RX-7 Etude Autozuma-AZ-3 Cosmo Familia-astina Eunos-presso Roadster		

One box・Minivan	Isis Alphard Ipsum Wish Vellfire Voxy Estima Gaia Corolla-spacio Granvia Sienta Sparky Nadia Noah Hiace Passo sette Masterace Regius	Elgrand Caravan-silkroad Serena Bassara Presage Lafesta Largo Liberty	Edix Elysion Elysion-prestige Odyssey Stepwgn Stepwgn-spada Stream Partner Freed Mobilio MObilio-spike	Exiga Domingo Traviq	Every landy Landy	Chariot Dion Delica D:5 Delica star-wagon Delica space-gear Lancer cargo	MPV Biante Premacy Bongo-friendee Eunos-cargo		
Wagon	Caldina Corolla-fielder Crown estate Succeed-wagon Sprinter carib Probox-wagon Mark x zio	Avenir Wingroad Expert Stagea R´nessa	Avancier Accord-tourer Airwave Orthia	Legacy-touring wagon		Lancer wagon Libero Legnum	Atenza-sportswagon Demio		
SUV	RAV4 Vanguard Cami Kluger Hilux surf Harrier Mega cruiser Rush Land cruiser Lexus GX Lexus LX Lexus RX	X-trail Safari Dualis Terrano Mistral Murano Rasheen	CR-V HR-V Z Crossroad Passport Horizon	Bighorn Forester Legacy-outback	X-90 Escudo Grand-escudo Jimny-sierra	Outlander Airtrek Pajero Pajero Jr	CX-7 Proceed Proceed-levante		
Compact car	bB iQ ist Vitz Auris Corolla-rumion Passo Fun cargo Blade Porte Raum Ractis	Cube Tiida Tino Note Pao March	S-MX Capa That's Fit	Justy Dex	SX4 Chevrolet-MW Swift Swift sports Splash Solio	RVR Colt Colt plus Mirage dingo	Infini MS-6 Verisa		

Light car		Otti Kix Clipper rio Pino Moco	Acty truck Acty van Street Zest Vamos Vamos-hobio Beat Life	R1 R2 Vivio Sambar Pleo Pleo van Rex	Kei Kei-works MR wagon MR wagon-wit Alto Alto lapin Every-wagon Cappuccino Cara Jimny Suzu light Cervo Cervo SR Twin Palette Fronte Wagon R Wagon R-stingray	eK sports eK wagon i Town box Toppo Toppo BJ Pajero mini Bravo Minica Minica van	AZ-offroad AZ-wagon R360 coupe Autozam-AZ-1 Carol Chantez Scrum-wagon Spiano Porter Laputa	MAX Atrai wagon Esse Esse-custom Opti Cuore Copen Sonica Tanto Tanto-custom Terioskid Naked Mira Mira custom Mira gino Move Move-custom Moveconte Moveconte-custom Leeza	

Appendix2. Model Ranking Table

	Sedan	Coupe・Sports car	One box・Minivan	Wagon	SUV	Compact car	Light car	Truck
Ⅰ	525i BMW Crown hybrid Crown majesta Celsior Benz Lexus Lexus ES Lexus LS	GTR M3 NSX Audi Countach Corvette Boxster Porsche Volvo Lexus SC			Hummer Land-cruiser Lexus GX Range rover			
Ⅱ	C4 MS-9 VW golf VW vento Accord Aristo Altezza Inspire Windom Camry Cadillac Crown Crown royalsaloon Gloria Cima Chanson Skyline Cedric Cedric cube Fuga Peugeot207 Bora Mark Ⅱ Mark Ⅱ blit Lancer evolution x Lexus IS	MR-S RX-7 RX-8 S2000 Integra type-R Cosmo Skyline coupe Fairlady Z	MPV Astro Alphard Alphard hybrid Vellfire Estima Elysion prestige Elgrand Odyssey Delica space gear Lucida	Accord tourer Mark x zio Airwave	Kluger Safari Bighorn Prado			
Ⅲ	Impreza Cresta Sigma Civic type-R Cefiro Diamante Beetle Vigor Prius Mark X Legacy Leopard Laurel	Levin	Ipsum Stepwgn Stepwgn- spada Spacio Serena Delica Hiace Regius	Accord wagon Golf wagon Stagea Primera wagon Legacy touring-wagon Legnum	CRV OUtlander X-trail Surf Terrano Hilux surf Pajero Harrier			

Ⅳ	SX4 seedan Ascot Insight Integra Impreza anesis Exiv Capella Carina ED Galant fortis Cronus Civic Chaser Vista Vista ardeo Primera Bluebird Bluebird sylphy	180SX Cavalier Silvia Supra Smart Celica Prelude	Isis Wish Voxy Edix Caravan Sienta Chariot Stream Noah Freed Premacy Bongo	Avenir Caldina Mark Ⅱ wagon	RAV-4 Airtrek Cami Tribute Forester	RVR Corolla rumion Mini cooper Raum Rumion		
Ⅴ	CR-X Axela Aerio Carina Corolla Corolla Ⅱ Cruze Corsa Corona Sunny Gemini City Tercel Tiida Pulsar Familia Festiva Platz Mirage Lancer sedan Leone Logo	MR2 Corolla levin Sprinter Sprinter- trueno	Acty van Every Chariot Townace Hijet Prairie Masterace Liteace	Sprinter carib Demio Lancer cedia- wagon Libero	Chevrolet Pajero Jr Rasheen	bB iQ Vitz Cube That's Swift Note Pyzar Passo Fun cargo Fit March	eK wagon MAX MR- wago Atrai- wagon Alto Vivio Every- wagon Otti Opti Jimny Scrum Stella Street Sonica Tanto Bistor Pino Minica Mira Move Moco Life Lapin Rex Wagon R	Acty truck High bit- truck Mighty boy

Appendix3. Questionnaire Sheet

About yourself
Q1. Age < 1.teenagers 2.twenties 3.thirties 4.forties 5.fifties 6.more than sixties >
Q2. Sex < 1.Male 2.Female >
Q3. Occupation < 1.Student 2.Officer 3.Company employee 4.Clerk of Organization 5.Independents 6.Miscellaneous >
Q4. Annual Income (Japanese Yen) < 1.0-3 million 2.3-5 million 3.5-7.5 million 4.7.5-10 million 5.10-15 million 6.More than 15million >
Q5. Address
Q6. Marriage < 1.Single 2.Married >
Q7. Number of children < Working ( ), University student ( ), High school student ( ), Junior high school student ( ), School child ( ), Child before entering school ( ) >
Important points in buying a car
Q8-1. Design < 1.Very important 2.Important 3.Ordinary level 4.Not so important 5.Not important >
Q8-2. Size of car
Q8-3. Performance
Q8-4. Price
Q8-5. Manufacturer's name (brand name)
Q8-6. Running cost
Q8-7. Purpose of use
Q8-8. Environmental care
Q8-9. Dealer's correspondence
Q8-10. Miscellaneous
About your hobby
Q9-1. Sports < 1.Very important 2.Important 3.Ordinary level 4.Not so important 5.Not important >
Q9-2. Reading <Same with above>
Q9-3. Shopping <Same with above>
Q9-4. Traveling <Same with above>
Q9-5. Music <Same with above>
Q9-6. Movie < 1. More than two movies in a month 2. One movie in a month 3. One movie in three months 4. One movie in half a year 5.Not at all >
Q9-7. Internet (except work) < 1.everyday 2.three days a week 3.one day a week 4. One time in two weeks 5.Not at all >
Q9-8. Driving
Q9-9. Purpose of car use < 1.commuting 2.shopping 3.transportation 4.hobby 5.others >
Q9-10. If you have any other hobby, please write down.
About your lifestyle
Q9-11. How do you spend a holiday? < 1.Outdoor 2.Indoor 3.Neither >
Q9-12. What is your occupational category? < 1.Deskwork 2. Going out for sales 3.Neither >
Please write the car that you own.

	Car name	Manufacturer's name	Car type	Reason of purchasing
Q10-1. Third Ahead				
Q10-2. Second Ahead				
Q10-3. First ahead				
Q10-4. Present				
Q10-5. Next time				
Q10-6. future				

【Reason of purchasing】

①Design ②Structure(freight a lot of luggage) ③Performance(zippy, good motor) ④Price ⑤Family structure ⑥Favorite manufacturer ⑦Lifestyle(hobby etc.) ⑧Environmental care ⑨Space of the garage ⑩Present from friends ⑪Low interest rate ⑫Running cost(tax etc.) ⑬Miscellaneous