Model reduction by time aggregation for optimal design of energy supply systems by an MILP hierarchical branch and bound method

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Abstract

Mixed-integer linear programming (MILP) methods have been applied widely to optimal design of energy supply systems in consideration of multi-period operation. A hierarchical MILP method has been proposed to solve such optimal design problems efficiently. An original problem has been solved by dividing it into a relaxed optimal design problem at the upper level and optimal operation problems which are independent of one another at the lower level. In addition, some strategies have been proposed to enhance the computation efficiency furthermore. In this paper, a method of reducing model by time aggregation is proposed as a novel strategy to search design candidates efficiently in the relaxed optimal design problem at the upper level. In

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addition, the previous strategies are modified in accordance with the novel strategy. This method is realized only by clustering periods and averaging energy demands for clustered periods, while it guarantees to derive the optimal solution. Thus, it may decrease the computation time at the upper level. Through a case study on the optimal design of a gas turbine cogeneration system, it is clarified how the model reduction is effective to enhance the computation efficiency in comparison and combination with the modified previous strategies.

Keywords: Energy supply, Optimal design, Mixed-integer linear programming, Hierarchical optimization, Model reduction, Time aggregation

1. Introduction

To attain the highest performance of energy supply systems, it is important to rationally determine their structures by selecting energy producing and conversion equipment from many alternatives so that they match energy demand requirements. It is also important to rationally determine capacities and numbers of selected equipment in consideration of their operational strategies such as on/off status of operation and load allocation corresponding to seasonal and hourly variations in energy demands. One of the ways to rationally determine the aforementioned design and operation items of energy supply systems is to use mathematical programming methods, and they have been applied increasingly with the development of computation hardware and software. Especially, the mixed-integer linear programming (MILP) method has been utilized widely. This is because it can consider discrete characteristics for selection and on/off status of operation of equipment, and can also treat nonlinear performance characteristics of equipment by piecewise linear approximations.

However, the difficulty of optimal design problems depends on the formulation using binary and integer variables. For example, the selection, capacities, numbers, and on/off status of operation of equipment can be expressed by binary and integer variables. In this case, the number of binary and integer variables increases with the complexity of energy supply systems and the number of periods set to consider seasonal and hourly variations in energy demands. In recent years, since commercial MILP solvers have become more efficient, many applications to the optimal design have been conducted in consideration of multi-period operation for a large number of periods. In many cases, however, equipment capacities have been treated as parameters or continuous variables to solve the optimal design problems relatively easily, and this treatment cannot express real situations regarding performance characteristics and capital costs of equipment. For example, only the types of equipment have been determined under fixed capacities [1, 2]; the types and numbers of equipment have been determined under fixed capacities [3–5]; the types and capacities of equipment have been determined, but the capacities have been treated as continuous variables [6–10]; similar models have been used, but the dependence of performance characteristics of equipment on their capacities or part load levels have not been taken into account [11, 12]. On the other hand, optimal design methods have been proposed in consideration of discreteness of equipment capacities to resolve the aforementioned insufficiency of equipment models for performance characteristics and capital costs [13–15]. However, since these methods make the optimal design problem more complex, even commercial MILP solvers which are recently available may not derive the optimal solution in a practical computation time.

One approach to solve optimal design problems with large numbers of periods efficiently is to derive approximate optimal solutions by reducing the numbers of periods as much as possible. Some approximate solution methods for reducing the numbers of periods have been proposed by selecting representative days and aggregating periods based on energy demands. For example, representative days have been selected using the k-medoids clustering method [16]; typical periods have been selected using the k-means clustering method assisted by the ε-constraint optimization technique [17]; periods have been aggregated using a hierarchical clustering method [18]; several aggregation methods including the aforementioned clustering ones have been compared [19, 20]. On the other hand, a method of bounding the error in the optimal value of the objective function by aggregating periods has been proposed [21], and a decomposition method of synthesizing energy systems by evaluating upper and lower bounds for the optimal value of the objective function has also been proposed [22]. Although these methods are applicable to any optimal design problems regardless of MILP solvers, they sacrifice solution exactness and thus affect both design and operation solutions. Thus, it is necessary to investigate how the optimal solutions of the optimal design problems with reduced numbers of periods are close to those with the original ones. However, this approach is used because it is difficult to derive the optimal solutions of the optimal design problems with the original numbers of periods.

Another approach to solve optimal design problems with large numbers of periods efficiently is to utilize structural features of the optimization problems. An MILP method utilizing the hierarchical relationship between design and operation variables has been proposed to solve optimal design problems efficiently, and has been implemented into a commercial MILP solver utilizing its callback functions to solve large-scale problems with large numbers of variables [23]. This method has been extended to derive not only the optimal design but also suboptimal ones which follow the optimal one without any omissions, what are called K-best solutions [24]. For the purpose of enhancing the computation efficiency, some strategies have been proposed to reduce the number of design candidates generated at the upper level and the number of optimal operation problems solved at the lower level as much as possible, and the method with these strategies has been applied to the multiobjective optimal design [25]. Furthermore, for the purpose of enhancing the computation efficiency at the lower level, parallel computing is adopted to solve multiple optimal operation problems in parallel [26]. Especially, the computation efficiency at the lower level has been enhanced drastically by the series of work.

In this paper, as a novel strategy to enhance the computation efficiency at the upper level in the aforementioned hierarchical MILP method, a method of reducing model by time aggregation is proposed to search design candidates efficiently at the upper level. In addition, the previous strategies to enhance the computation efficiency are modified in accordance with the novel strategy. This method is realized only by clustering periods and averaging energy demands for clustered periods, while it guarantees to derive the optimal solution. This is because a lower bound for the optimal value of the objective function in the reduced problem is smaller than or equal to that in the original problem. This is the most important feature in comparison with the aforementioned approximate solution methods for reducing the numbers of periods. On the one hand, the method can decrease the number of design variables and constraints at the upper level, and thus may decrease the computation time at the upper level. On the other hand, the method may increase the numbers of design candidates generated at the upper level and optimal operation problems solved at the lower level, and thus the computation time at both the levels. Through a case study on the optimal design of a gas turbine cogeneration system, it is investigated how the model reduction is effective to enhance the computation efficiency in comparison and combination with the modified previous strategies.

2. Formulation of optimal design problem

A formulation for the optimal design of an energy supply system in consideration of discrete equipment capacities proposed previously is used in this paper [13]. A typical year is divided into M periods to consider seasonal and hourly variations in energy demands, and each period is identified by the subscript or argument m $(m = 1, 2, \cdots, M)$. Energy demands $\boldsymbol{y}(m)$ are estimated certainly at each period. First, a super structure for an energy supply system, which is composed of all the units of equipment considered as candidates for selection, is created to match energy demand requirements. Here, it is assumed that energy storage units are not included in the system. Then, a real structure is designed by selecting some units of equipment from the candidates. Furthermore, some units of equipment are operated to satisfy energy demands at each period. Here, it is assumed that transient characteristics of equipment are not considered. The selection, capacities, and numbers of equipment as well as the maximum demands of utilities such as purchased electricity and city gas are considered as binary and integer design variables $\boldsymbol{\eta}$. The number of equipment at on status of operation, and the load allocation of equipment and consumptions of utilities are considered as integer and continuous operation variables, $\boldsymbol{\delta}(m)$ and $\boldsymbol{x}(m)$, respectively. The annual total cost is adopted as the objective function to be minimized. It is evaluated as the sum of the annual capital cost of equipment and the annual operational cost of utilities.

Under the aforementioned conditions, the optimal design problem is formulated as follows:

$$\left.\begin{array}{ll}\text{min.} & z = f_0(\boldsymbol{\eta}) + \sum_{m=1}^{M} f_m(\boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m))\Delta t(m) \\ \text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\eta}) \leq \mathbf{0} \\ & \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \leq \mathbf{0} \quad (m = 1, 2, \cdots, M) \\ & \boldsymbol{h}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) = \mathbf{0} \quad (m = 1, 2, \cdots, M) \\ & \boldsymbol{\eta} \in \mathbb{Z}^{n_1} \\ & \boldsymbol{\delta}(m) \in \mathbb{Z}^{n_2} \quad (m = 1, 2, \cdots, M) \\ & \boldsymbol{x}(m) \in \mathbb{R}^{n_3} \quad (m = 1, 2, \cdots, M) \end{array}\right\} \quad (1)$$

where f_0 and f_m denote the annual capital cost of equipment plus the annual demand charge of utilities and the annual energy charge of utilities per hour at each period, which are functions with respect to the design and operation variables, respectively, in the objective function z, and $\Delta t(m)$ is the duration per year of each period. $\boldsymbol{g}_0$ denotes the inequality constraints for restrictions concerning the selection, capacities, and numbers of equipment, which relate the design variables. $\boldsymbol{g}_m$ and $\boldsymbol{h}_m$ denote the inequality and equality constraints, respectively, for performance characteristics of equipment, relationships between maximum demands and consumptions of utilities, and energy balance relationships, which relate the design and operation variables. In addition, n_1, n_2, and n_3 are the numbers of the variables in $\boldsymbol{\eta}$, $\boldsymbol{\delta}(m)$, and $\boldsymbol{x}(m)$, respectively.

Both the design and operation variables are mixed in the optimal design problem of Eq. (1), and the enumeration tree is wide and deep. This structure of the enumeration tree means that a large computation time must be required to obtain the optimal solution.

3. Solution by hierarchical MILP method

As shown in Fig. 1, the optimal design problem of Eq. (1) has the hierarchical relationship between the design and operation variables. Namely, if the values of the design variables $\boldsymbol{\eta}$ are assumed tentatively, the constraints $\boldsymbol{g}_m$ and $\boldsymbol{h}_m$ become independent at each period, and the values of the operation variables $\boldsymbol{\delta}(m)$ and $\boldsymbol{x}(m)$ can be optimized independently at each period. Thus, a hierarchical MILP method proposed previously is used in this paper [23], as shown in Fig. 2.

At the upper level, the binary and integer design variables $\boldsymbol{\eta}$ are selected as branching variables in prior to the integer operation variables $\boldsymbol{\delta}(m)$, and their values are assumed tentatively. Then, a design candidate is generated. This process is conducted by searching a feasible solution in the following relaxed optimal design problem:

$$\left.\begin{array}{ll}
\text{min.} & z = f_0(\boldsymbol{\eta}) + \displaystyle\sum_{m=1}^{M} f_m(\boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m))\Delta t(m) \\
\text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\eta}) \leq \mathbf{0} \\
 & \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \leq \mathbf{0} \quad (m = 1, 2, \cdots, M) \\
 & \boldsymbol{h}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) = \mathbf{0} \quad (m = 1, 2, \cdots, M) \\
 & \boldsymbol{\eta} \in \mathbb{Z}^{n_1} \\
 & \boldsymbol{\delta}(m) \in \mathbb{R}^{n_2} \quad (m = 1, 2, \cdots, M) \\
 & \boldsymbol{x}(m) \in \mathbb{R}^{n_3} \quad (m = 1, 2, \cdots, M)
\end{array}\right\} \quad (2)$$

where the integer operation variables $\boldsymbol{\delta}(m)$ are relaxed to continuous ones. Under the values of the design variables $\boldsymbol{\eta}$, the values of the operation variables $\boldsymbol{\delta}(m)$ and $\boldsymbol{x}(m)$ can be determined at the lower level by solving the following optimal operation problems:

$$\left.\begin{array}{ll}\text{min.} & f_m(\boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \\ \text{sub. to} & \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \leq \boldsymbol{0} \\ & \boldsymbol{h}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) = \boldsymbol{0} \\ & \boldsymbol{\delta}(m) \in \mathbb{Z}^{n_2} \\ & \boldsymbol{x}(m) \in \mathbb{R}^{n_3}\end{array}\right\} (m = 1, 2, \cdots, M) \tag{3}$$

The value of the objective function z is assessed based on the values of f_0 and f_m, which are evaluated based on the values of the design and operation variables assumed tentatively and determined optimally, respectively. A design candidate can be an incumbent solution, and the corresponding value of the objective function can be an upper bound for the optimal value of the objective function. Thus, it is used for the bounding procedure in searching other design candidates.

Figure 3 shows the enumeration tree for these optimization problems related hierarchically. The branches show the selection of values of the binary and integer design variables at the upper level as well as the integer operation variables at the lower level. The design and operation variables are separated, and the enumeration tree is narrow and shallow. This structure of the enumeration tree means that a short computation time may be required to obtain the optimal solution.

4. Model reduction by time aggregation

A method of reducing the optimization model by aggregating periods and the corresponding optimal operation problems is proposed here. Generally, the periods may be divided into clusters in any way. However, the periods are divided into clusters with the same number of periods in the order of time series here, for simplicity. Concretely, M periods are divided into L clusters each of which includes N periods, i.e., $L = M/N$. Then, the set which includes the indices for periods in each cluster is defined as follows:

$$A_l = \left\{(l-1)N+1,\ (l-1)N+2,\ \cdots,\ lN\right\} \quad (l = 1,\ 2,\ \cdots,\ L) \tag{4}$$

In addition, energy demands in each cluster are averaged as follows:

$$\boldsymbol{y}'(l) = \sum_{m \in A_l} \boldsymbol{y}(m) \Delta t(m) \Bigg/ \sum_{m \in A_l} \Delta t(m) \quad (l = 1, 2, \cdots, L) \tag{5}$$

In accordance with this clustering, only the relaxed optimal design problem of Eq. (2) at the upper level is reduced to

$$\left.\begin{array}{ll} \text{min.} & z = f_0(\boldsymbol{\eta}) + \displaystyle\sum_{l=1}^{L} f'_l(\boldsymbol{\delta}'(l), \boldsymbol{x}'(l), \boldsymbol{y}'(l)) \sum_{m \in A_l} \Delta t(m) \\ \text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\eta}) \leq \mathbf{0} \\ & \boldsymbol{g}'_l(\boldsymbol{\eta}, \boldsymbol{\delta}'(l), \boldsymbol{x}'(l), \boldsymbol{y}'(l)) \leq \mathbf{0} \quad (l = 1, 2, \cdots, L) \\ & \boldsymbol{h}'_l(\boldsymbol{\eta}, \boldsymbol{\delta}'(l), \boldsymbol{x}'(l), \boldsymbol{y}'(l)) = \mathbf{0} \quad (l = 1, 2, \cdots, L) \\ & \boldsymbol{\eta} \in \mathbb{Z}^{n_1} \\ & \boldsymbol{\delta}'(l) \in \mathbb{R}^{n_2} \quad (l = 1, 2, \cdots, L) \\ & \boldsymbol{x}'(l) \in \mathbb{R}^{n_3} \quad (l = 1, 2, \cdots, L) \end{array}\right\} \tag{6}$$

where $(\ )'$ denotes the variables, objective function, and constraints after model reduction by time aggregation. The optimal operation problems of Eq. (3) at the lower level are used in combination with Eq. (6) in place of Eq. (2). By this model reduction, the numbers of the operation variables $\boldsymbol{\delta}'(l)$ and $\boldsymbol{x}'(l)$, and the constraints $\boldsymbol{g}'_l$ and

$\boldsymbol{h}'_l$ decrease with an increase in N or a decrease in L. Thus, this may enhance the computation efficiency at the upper level.

The purpose of the relaxed optimal design problem of Eq. (2) is not to find the optimal solution and evaluate the optimal value of the objective function, but to search design candidates and evaluate lower bounds for the optimal value of the objective function. Even the reduced optimal design problem of Eq. (6) is effective for this purpose as follows. At each branching node in the branch and bound method, where part of the values of the design variables $\boldsymbol{\eta}$ are fixed, a continuous relaxation or linear programming (LP) problem of Eq. (2) or (6) is considered. Parameters such as energy demands on the right side of constraints in a LP problem affect an objective function as shown in Fig. 4, i.e., the objective function changes piecewise linearly and its gradient increases with an increase in the value of a parameter. Thus, even if energy demands with small differences are averaged, Eq. (6) may give the same lower bound as Eq. (2). If energy demands with large differences are averaged, Eq. (6) may give a lower bound smaller than Eq. (2). However, this smaller lower bound obtained by Eq. (6) is also used as a lower bound for Eq. (2). This feature is proved mathematically in appendix A. Thus, the reduced optimal design problem of Eq. (6) never cut the optimal design solution off, and the proposed method guarantees to derive the optimal solution. Smaller lower bounds increase the numbers of design candidates generated at the upper level and optimal operation problems solved at the lower level. Thus, this may deteriorate the computation efficiency at both the levels.

It is difficult to judge whether the computation efficiency at the upper level is enhanced or deteriorated as a result of the model reduction by time aggregation. It should be judged through case studies.

5. Modification of previous strategies

Several strategies have been proposed previously to enhance the computation efficiency at both the levels [23, 25]. However, they have to be modified in accordance with the aforementioned model reduction at the upper level.

5.1. Bounding at lower level

The first strategy to enhance the computation efficiency is bounding at the lower level. The bounding procedure is conducted automatically in solving each optimal operation problem by an MILP solver. However, since each optimal operation problem is solved independently, the bounding procedure cannot be conducted automatically in solving all the optimal operation problems sequentially. Thus, the self-made bounding procedure is introduced outside the MILP solver. It is assumed here that optimal operation problems are solved sequentially in a specified order of clusters set for the model reduction at the upper level, and that they are also solved sequentially in their specified order in each cluster. Before solving an optimal operation problem in a target cluster, a lower bound for the optimal value of the objective function is evaluated as follows:

$$\begin{aligned} \underset{\sim}{z} = f_0 + \sum_{l \in C_{\mathrm{S}}} \sum_{m \in A_l} f_m \mathit{\Delta} t(m) + \sum_{l \in C_{\mathrm{U}}} \max\Big(\underset{\sim}{f}{}'^{\mathrm{C}}_l \sum_{m \in A_l} \mathit{\Delta} t(m), \sum_{m \in A_l} \underset{\sim}{f}{}^{\mathrm{O}}_m \mathit{\Delta} t(m)\Big) \\ + \sum_{m \in P_{\mathrm{S}}} f_m \mathit{\Delta} t(m) + \sum_{m \in P_{\mathrm{U}}} \underset{\sim}{f}{}^{\mathrm{O}}_m \mathit{\Delta} t(m) \end{aligned} \tag{7}$$

where C_{S} and C_{U} are the sets of indices for the clusters where all the optimal operation problems are solved and not solved, respectively, P_{S} and P_{U} are the sets of indices for the optimal operation problems which are solved and not solved, respectively, in the target cluster, $\underset{\sim}{f}{}'^{\mathrm{C}}_{l}$ is the value of f'_l evaluated in the reduced optimal design problem at the upper level, and $\underset{\sim}{f}{}^{\mathrm{O}}_{m}$ is the value of f_m evaluated in the critical operation problems, which have been proposed to evaluate an alternative lower bound [23]. This lower bound is used for the bounding procedure. In accordance with Eq. (7), an upper bound for the optimal value of f_k for the kth optimal operation problem can be evaluated as follows:

$$\tilde{f}_k = \left\{ \tilde{z} - \left(f_0 + \sum_{l \in C_{\mathrm{S}}} \sum_{m \in A_l} f_m \Delta t(m) + \sum_{l \in C_{\mathrm{U}}} \max\left(\underset{\sim}{f}{}'^{\mathrm{C}}_{l} \sum_{m \in A_l} \Delta t(m), \sum_{m \in A_l} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m) \right) + \sum_{m \in P_{\mathrm{S}}} f_m \Delta t(m) + \sum_{m \in P_{\mathrm{U}} \setminus \{k\}} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m) \right) \right\} \Big/ \Delta t(k) \tag{8}$$

This strategy is expected to reduce the number of solved optimal operation problems and enhance the computation efficiency at the lower level.

5.2. Bounding at upper level

The second strategy to enhance the computation efficiency is bounding at the upper level. The bounding procedure is conducted automatically in solving the reduced optimal design problem by an MILP solver. If the lower bounds obtained not only by the reduced optimal design problem but also by the critical design and operation problems are used, a lower bound for the optimal value of the objective function is evaluated as follows:

$$\underset{\sim}{z} = \max\left(\underset{\sim}{f}{}^{\mathrm{C}}_{0}, \underset{\sim}{f}{}^{\mathrm{D}}_{0}\right) + \sum_{l=1}^{L} \max\left(\underset{\sim}{f}{}'^{\mathrm{C}}_{l} \sum_{m \in A_l} \Delta t(m), \sum_{m \in A_l} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m) \right) \tag{9}$$

where $\underset{\sim}{f}_0^{\mathrm{C}}$ and $\underset{\sim}{f}{}_l'^{\mathrm{C}}$ are the values of f_0 and f'_l, respectively, evaluated by the optimal solution of the continuous relaxation problem at each branching node in the reduced optimal design problem at the upper level, and $\underset{\sim}{f}_0^{\mathrm{D}}$ is the value of f_0 evaluated in the critical design problems, which have also been proposed to evaluate an alternative lower bound [23]. This strategy is expected to reduce the number of generated design candidates and enhance the computation efficiency at the upper level.

5.3. Ordering of optimal operation problems

The third strategy to enhance the computation efficiency is ordering of optimal operation problems at the lower level. As the optimal operation problems are solved sequentially in the specified order of clusters, the lower bound of Eq. (7) increases. This is because after the optimal operation problems in the *l*th cluster are solved, $\max(\underset{\sim}{f}{}_l'^{\mathrm{C}} \sum_{m \in A_l} \Delta t(m), \sum_{m \in A_l} \underset{\sim}{f}_m^{\mathrm{O}} \Delta t(m))$ of the third term is replaced with $\sum_{m \in A_l} f_m \Delta t(m)$ of the second term on the right side of Eq. (7). This means that the value of $\underset{\sim}{z}$ increases by $\sum_{m \in A_l} f_m \Delta t(m) - \max(\underset{\sim}{f}{}_l'^{\mathrm{C}} \sum_{m \in A_l} \Delta t(m), \sum_{m \in A_l} \underset{\sim}{f}_m^{\mathrm{O}} \Delta t(m))$. On the other hand, as the optimal operation problems are solved sequentially in their specified order in each cluster, the lower bound of Eq. (7) also increases. This is because after the *m*th optimal operation problem in the cluster is solved, $\underset{\sim}{f}_m^{\mathrm{O}} \Delta t(m)$ of the fifth term is replaced with $f_m \Delta t(m)$ of the fourth term on the right side of Eq. (7). This means that the value of $\underset{\sim}{z}$ increases by $(f_m - \underset{\sim}{f}_m^{\mathrm{O}}) \Delta t(m)$. Thus, as the optimal operation problems are solved sequentially, the possibility of conducting the bounding procedure increases. Therefore, if the clusters for optimal operation problems and the optimal operation problems in each cluster are solved in the descending order of $\sum_{m \in A_l} f_m \Delta t(m) - \max(\underset{\sim}{f}{}_l'^{\mathrm{C}} \sum_{m \in A_l} \Delta t(m), \sum_{m \in A_l} \underset{\sim}{f}_m^{\mathrm{O}} \Delta t(m))$ and $(f_m - \underset{\sim}{f}_m^{\mathrm{O}}) \Delta t(m)$, respectively, the possibility of conducting the bounding procedure increases faster. The best order of the clusters for the optimal operation problems to be solved $l_1 \rightarrow l_2 \rightarrow \cdots \rightarrow l_L$ is expressed as

$$\begin{aligned} &\sum_{m \in A_{l_1}} f_m \Delta t(m) - \max(\underset{\sim}{f}{}'^{\mathrm{C}}_{l_1} \sum_{m \in A_{l_1}} \Delta t(m), \sum_{m \in A_{l_1}} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m)) \\ &\quad \geq \sum_{m \in A_{l_2}} f_m \Delta t(m) - \max(\underset{\sim}{f}{}'^{\mathrm{C}}_{l_2} \sum_{m \in A_{l_2}} \Delta t(m), \sum_{m \in A_{l_2}} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m)) \geq \cdots \\ &\quad \geq \sum_{m \in A_{l_L}} f_m \Delta t(m) - \max(\underset{\sim}{f}{}'^{\mathrm{C}}_{l_L} \sum_{m \in A_{l_L}} \Delta t(m), \sum_{m \in A_{l_L}} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m)) \end{aligned} \tag{10}$$

In addition, the best order of the optimal operation problems to be solved in each claster $m_1 \rightarrow m_2 \rightarrow \cdots \rightarrow m_N$ is expressed as

$$(f_{m_1} - \underset{\sim}{f}{}^{\mathrm{O}}_{m_1}) \Delta t(m_1) \geq (f_{m_2} - \underset{\sim}{f}{}^{\mathrm{O}}_{m_2}) \Delta t(m_2) \geq \cdots \geq (f_{m_N} - \underset{\sim}{f}{}^{\mathrm{O}}_{m_N}) \Delta t(m_N) \tag{11}$$

However, the values of $\sum_{m \in A_l} f_m \Delta t(m) - \max(\underset{\sim}{f}{}'^{\mathrm{C}}_{l} \sum_{m \in A_l} \Delta t(m), \sum_{m \in A_l} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m))$ and $(f_m - \underset{\sim}{f}{}^{\mathrm{O}}_{m}) \Delta t(m)$ cannot be compared with one another until all the optimal operation problems are solved. In this paper, approximate values of $\sum_{m \in A_l} f_m \Delta t(m) - \max(\underset{\sim}{f}{}'^{\mathrm{C}}_{l} \sum_{m \in A_l} \Delta t(m), \sum_{m \in A_l} \underset{\sim}{f}{}^{\mathrm{O}}_{m} \Delta t(m))$ and $(f_m - \underset{\sim}{f}{}^{\mathrm{O}}_{m}) \Delta t(m)$ are evaluated based on their values of the newest incumbent solution. This strategy is expected to reduce the number of solved optimal operation problems and enhance the computation efficiency at the lower level.

6. Case study

6.1. Input data

A gas turbine cogeneration system for district energy supply shown in Fig. 5 is investigated in a case study. The super structure for the cogeneration system is defined for the optimal design. It is composed of four gas turbine generators (GT), four waste heat recovery boilers (BW), four gas-fired auxiliary boilers (BG), four electric compression refrigerators (RE), four steam absorption refrigerators (RS), a device for receiving electricity (EP). Pumps for supplying cold water (PC) are common to all the possible structures, and only their power consumption is considered. Data on the capacities, performance characteristic values, and capital costs of candidates of equipment for selection as well as the unit costs for demand and energy charges of electricity and city gas are shown in Tables 1 and 2, respectively [13].

Two hotels and four office buildings with the total floor area of 383.7×10^3 m^2 are selected as the buildings which are supplied with electricity, cold water, and steam by the cogeneration system. To take account of seasonal and hourly variations in energy demands, a typical year is divided into three representative days in winter, mid-season, and summer whose numbers of days per year are set at 122, 121, and 122 d/y, respectively, and each day is further divided into 24 sampling time intervals of 1 h. Thus, the year is divided into M = 72 periods. Figure 6 shows the hourly variations in electricity, cold water, and steam demands on the representative day in summer as an example. Figure 6 (a) shows the original energy demands, and Figs. 6 (b) to (d) show the energy demands averaged for model reduction by setting the number of periods per cluster at N = 2, 3, and 4, respectively. As a result, the original optimal design problem has 22 binary design variables, 24 integer design variables, 288 integer operation variables, 4476 continuous operation variables, and 14820 constraints.

The optimization calculations by the hierarchical MILP method are conducted using each and combinations of the modified previous strategies and the novel strategy.

These strategies are defined as follows:

Strategy A: bounding at lower level in section 5.1

Strategy B: bounding at upper level in section 5.2

Strategy C: ordering of optimal operation problems in section 5.3

Strategy D: model reduction by time aggregation in chapter 4

The names of cases mean combinations of these strategies, and case R denotes the reference case without strategies A to D. When strategy D is used, the number of periods per cluster N is changed as a parameter.

All the optimization calculations are conducted using a commercial MILP solver IBM ILOG CPLEX Optimization Studio Ver. 12.6.1 on a MacBook Pro with Intel Core i7 processor (4 cores and 2.4 GHz) [27]. To confirm the effectiveness of the hierarchical MILP method, the conventional optimization calculation is also conducted using a commercial MILP solver GAMS/CPLEX Ver. 12.6.0 on the same computer [28].

6.2. Results and discussion

Figures 7 (a) and (b) show changes in upper and lower bounds for the optimal value of the objective function in relation to the computation time obtained by the conventional optimization calculation, and the hierarchical MILP method in cases ABC and ABCD, respectively. In the latter case, the number of periods per cluster is set at N = 4. In the conventional optimization calculation, the upper and lower bounds approach each other quickly, but a relative gap of 0.73 % is remaining even if 1000 s are used for computation, and it seems that the upper and lower bounds do not coincide with each other even with a longer computation time. Thus, the direct use of the commercial MILP solver is not so efficient to solve the optimal design problem under consideration. In the hierarchical MILP method, the strategies for enhancing the computation efficiency and the number of periods per cluster affect the computation efficiency. The combination of strategies A to C completes the optimization calculation in 63.4 s. The addition of strategy D to strategies A to C reduces the computation time drastically, and completes the optimization calculation only in 12.6 s. Thus, it turns out that strategy D is very effective to enhance the computation efficiency in the hierarchical MILP method.

Figure 8 shows the effect of the number of periods per cluster on the computation time in cases D, AD, BD, CD, and ABCD ($N \geq 2$) in comparison with cases R, A, B, C, and ABC (N = 1), respectively. The computation time tends to decrease with an increase in the number of periods per cluster N. When the number of periods per cluster is set at N = 3, the computation time tends to increase in cases D, AD, BD, and CD. It seems that the division of periods into clusters is not suitable. When only strategy D is used, it hardly affects the computation time and is hardly effective to enhance the computation efficiency. However, when strategy D is combined with strategy A, B, or C, it comes to affect the computation time. Especially, when strategy D is combined with strategy A or B, it affects the computation time more effectively. As aforementioned, when strategy D is combined with strategies A to C, it affects the computation time drastically and is very effective to enhance the computation efficiency. However, when the number of periods per cluster is set at N = 6, the computation time increases drastically. For example, the computation time is 646.9 s in case ABCD. This is because energy demands with much larger differences are averaged, Eq. (6)

gives a lower bound much smaller than Eq. (2), and the numbers of design candidates generated at the upper level and optimal operation problems solved at the lower level increase drastically.

Figure 9 shows the difference between the lower bounds for the value of objective function evaluated for design candidates generated in solving the following two problems: one is the original relaxed optimal design problem of Eq. (2) in case ABC, and the other is the relaxed optimal design problem of Eq. (6) reduced by time aggregation in case ABCD with $N = 2$, 3, 4, and 6. Since design candidates of both the problems do not necessarily coincide with each other, only the design candidates common to both the problems are selected here. Although the differences in the lower bound with $N = 2$ and 4 are small, those with $N = 3$ are larger, and those with $N = 6$ are much larger. These results correspond to the results shown in Fig. 8. Thus, when $N = 2$ and 4, the computation time decreases significantly with an increase in N. However, when $N = 3$, as aforementioned, the computation time tends to increase in cases D, AD, BD, and CD. In addition, when $N = 6$, as aforementioned, the computation time becomes too long. When $N = 3$ and 6, it turns out the division of periods into clusters is not suitable in terms of the differences in the lower bound. From these results, it turns out that the differences in the lower bound are the important criteria to evaluate the computation efficiency. In other words, it should be necessary to divide periods into clusters so that the differences in the lower bound are minimized to enhance the computation efficiency.

Figures 10 (a) to (c) show the effect of each and combinations of the strategies for enhancing the computation efficiency on some evaluation criteria, i.e., the number of design candidates generated at the upper level, the number of optimal operation problems solved at the lower level, and the computation time. Here, the number of periods per cluster is set at $N = 4$. The generated design candidates are composed of those rejected before the lower level, those rejected at the lower level, and incumbent solutions. The computation time is composed of those at the upper and lower levels. According to Fig. 10 (a), strategies A and C do not affect the number of generated design candidates, because they are strategies to enhance the computation efficiency only at the lower level. Strategy B is effective to reduce the number of generated design candidates, because it increases lower bounds evaluated by Eq. (9), while strategy D increases the number of generated design candidates, because it decreases lower bounds evaluated by Eq. (9). As a result, the number of generated design candidates in case ABCD is larger than that in case ABC. According to Fig. 10 (b), strategies A and C are effective to reduce the number of solved optimal operation problems directly. Especially, strategy A is very effective. In addition, strategy B is effective to reduce the number of solved optimal operation problems indirectly by reducing the number of generated design candidates. However, strategy D increases the number of solved optimal operation problems by increasing the number of generated design candidates. As a result, the number of solved optimal operation problems in case ABCD is also larger than that in case ABC. According to Fig. 10 (c), strategies A and C are effective to reduce the computation time only at the lower level. Although strategy B is effective to reduce the computation times at both the upper and lower levels, the effectiveness is very limited. Although strategy D increases the computation time at the lower level, it is very effective to reduce the computation time at the upper level. These results show that the combination of strategies A and D is very effective to reduce the total computation time. As a result, the total computation time in case ABCD is much shorter than that in case ABC.

The values of the design and operation variables for the optimal design obtained by the hierarchical MILP method are the same in all the cases regardless of using each or any combination of strategies A to D, and the values of the design variables are shown in Table 3.

7. Conclusions

For the purpose of enhancing the computation efficiency in the hierarchical MILP method for the optimal design of energy supply systems, a method of reducing model by time aggregation has been proposed as a novel strategy to search design candidates efficiently in the relaxed optimal design problem at the upper level. In addition, the strategies proposed previously to enhance the computation efficiency have been modified in accordance with the novel strategy. This method has been realized only by clustering periods and averaging energy demands for clustered periods. The method has the advantages that it guarantees to derive the optimal solution, and that it can decrease the number of design variables and constraints at the upper level, and thus may lead to a decrease in the computation time at the upper level. On the other hand, the method has the disadvantages that it increases the numbers of design candidates generated at the upper level and optimal operation problems solved at the lower level, and thus may lead to an increase in the computation time at both the levels. Through a case study on the optimal design of a gas turbine cogeneration system, it has been investigated how the model reduction is effective to enhance the computation efficiency in comparison and combination with the modified previous strategies. The following are the main results obtained through the case study.

- Although the novel strategy increases the numbers of design candidates generated at the upper level and optimal operation problems solved at the lower level, it decreases the computation time at the upper level drastically.
- Even if the novel strategy is used alone, it is not effective to enhance the computation efficiency. However, if it is used in combination with the modified previous strategies, it is very effective to enhance the computation efficiency.
- The computation efficiency is enhanced with a moderate increase in the number of periods per cluster. However, an excessive increase in the number of periods per cluster may deteriorate the computation efficiency drastically.

Although the periods have been divided into clusters with the same number of periods in the order of time series in this paper, for simplicity, the periods may generally be divided into clusters in any way. Thus, it is necessary to establish a method of clustering the periods so that the computation efficiency is enhanced as much as possible. In addition, although the proposed method has been applied only to a limited optimal design problem in this paper, it is necessary to apply it to optimal design problems of different types and scales. These are subsequent important subjects.

Acknowledgements

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Appendix A

It is proved here that the optimal value of the objective function in Eq. (6) is smaller or equal to that in Eq. (2) for arbitrary values of the design variables $\boldsymbol{\eta}$. The optimization problem at each period composed of the second term of the objective function and the second and third constraints in Eq. (2) or Eq. (6) is a LP problem.

Let us consider the following LP problems at two periods for Eq. (2):

$$(\mathrm{P}_1) \quad \left.\begin{array}{ll} \text{min.} & d_1 = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{u}_1 \Delta t_1 \\ \text{sub. to} & \boldsymbol{A}\boldsymbol{u}_1 = \boldsymbol{b}_1 \\ & \boldsymbol{u}_1 \geq \boldsymbol{0} \end{array}\right\} \tag{A1}$$

$$(\mathrm{P}_2) \quad \left.\begin{array}{ll} \text{min.} & d_2 = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{u}_2 \Delta t_2 \\ \text{sub. to} & \boldsymbol{A}\boldsymbol{u}_2 = \boldsymbol{b}_2 \\ & \boldsymbol{u}_2 \geq \boldsymbol{0} \end{array}\right\} \tag{A2}$$

where d is the objective function to be minimized, $\boldsymbol{u} \in \mathbb{R}^{n_5}$ is a vector composed of variables, $\boldsymbol{A} \in \mathbb{R}^{n_4 \times n_5}$, $\boldsymbol{b} \in \mathbb{R}^{n_4}$, and $\boldsymbol{c} \in \mathbb{R}^{n_5}$ are matrix and vectors composed of coefficients, Δt is the duration of period, and subscripts 1 and 2 denote different periods. Here, n_4 and n_5 are the numbers of constraints and variables, respectively.

Let us also consider the following LP problem at the aggregated period for Eq. (6):

$$(\mathrm{P}_3) \quad \left.\begin{array}{ll} \text{min.} & d_3 = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{u}_3 (\Delta t_1 + \Delta t_2) \\ \text{sub. to} & \boldsymbol{A}\boldsymbol{u}_3 = (\boldsymbol{b}_1 \Delta t_1 + \boldsymbol{b}_2 \Delta t_2)/(\Delta t_1 + \Delta t_2) \\ & \boldsymbol{u}_3 \geq \boldsymbol{0} \end{array}\right\} \tag{A3}$$

where $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ in Eqs. (A1) and (A2), respectively, are averaged in consideration of the durations of periods.

The dual problem of (P_1) is expressed as follows:

$$(\mathrm{D}_1)\quad \left.\begin{array}{ll} \max. & e_1 = \boldsymbol{b}_1^{\mathrm{T}} \boldsymbol{v}_1 \Delta t_1 \\ \text{sub. to} & \boldsymbol{A}^{\mathrm{T}} \boldsymbol{v}_1 \leq \boldsymbol{c} \end{array}\right\} \tag{A4}$$

where e is the objective function to be maximized, and $\boldsymbol{v} \in \mathbb{R}^{n_4}$ is a vector composed of dual variables. Introducing slack variables and converting free variables into nonnegative ones, Eq. (A4) is transformed into

$$(\hat{\mathrm{D}}_1)\quad \left.\begin{array}{ll} \max. & e_1 = \hat{\boldsymbol{b}}_1^{\mathrm{T}} \boldsymbol{w}_1 \Delta t_1 \\ \text{sub. to} & \hat{\boldsymbol{A}} \boldsymbol{w}_1 = \boldsymbol{c} \\ & \boldsymbol{w}_1 \geq \boldsymbol{0} \end{array}\right\} \tag{A5}$$

where $\hat{\boldsymbol{A}} \in \mathbb{R}^{n_5 \times (2n_4 + n_5)}$ is a matrix composed of $\boldsymbol{A}$ and a unit matrix, $\hat{\boldsymbol{b}} \in \mathbb{R}^{(2n_4 + n_5)}$ is a vector composed of $\boldsymbol{b}$ and a zero vector, and $\boldsymbol{w} \in \mathbb{R}^{(2n_4 + n_5)}$ is a vector composed of the slack variables and nonnegative variables. The value of the objective function for a basic feasible solution B is expressed as follows:

$$e_1 = \hat{\boldsymbol{b}}_{1B}^{\mathrm{T}} \hat{\boldsymbol{A}}_B^{-1} \boldsymbol{c} \Delta t_1 \tag{A6}$$

where $\hat{\boldsymbol{A}}_B \in \mathbb{R}^{n_5 \times n_5}$ is a basic matrix, and $\hat{\boldsymbol{b}}_B \in \mathbb{R}^{n_5}$ is a part for the basis of $\hat{\boldsymbol{b}}$. Thus, the optimal value of the objective function is obtained as follows:

$$e_1^* = \hat{\boldsymbol{b}}_{1B_1^*}^{\mathrm{T}} \hat{\boldsymbol{A}}_{B_1^*}^{-1} \boldsymbol{c} \Delta t_1 = \max_B \hat{\boldsymbol{b}}_{1B}^{\mathrm{T}} \hat{\boldsymbol{A}}_B^{-1} \boldsymbol{c} \Delta t_1 \tag{A7}$$

where $(\ )^*$ means the optimal solution.

The same procedure is applied to (P_2) and (P_3). The feasible region for the dual problems of (P_2) and (P_3) are the same with that for $(\hat{\mathrm{D}}_1)$. Thus, the optimal value of the objective function for the dual problem of (P_2) is obtained as follows:

$$e_2^* = \hat{\boldsymbol{b}}_{2B_2^*}^{\mathrm{T}} \hat{\boldsymbol{A}}_{B_2^*}^{-1} \boldsymbol{c} \Delta t_2 = \max_B \hat{\boldsymbol{b}}_{2B}^{\mathrm{T}} \hat{\boldsymbol{A}}_B^{-1} \boldsymbol{c} \Delta t_2 \tag{A8}$$

Similarly, the optimal value of the objective function for the dual problems of (P_3) is obtained as follows:

$$\begin{aligned} e_3^* &= \left\{ (\hat{\boldsymbol{b}}_{1B_3^*}^{\mathrm{T}} \Delta t_1 + \hat{\boldsymbol{b}}_{2B_3^*}^{\mathrm{T}} \Delta t_2) / (\Delta t_1 + \Delta t_2) \right\} \hat{\boldsymbol{A}}_{B_3^*}^{-1} \boldsymbol{c} (\Delta t_1 + \Delta t_2) \\ &= \hat{\boldsymbol{b}}_{1B_3^*}^{\mathrm{T}} \hat{\boldsymbol{A}}_{B_3^*}^{-1} \boldsymbol{c} \Delta t_1 + \hat{\boldsymbol{b}}_{2B_3^*}^{\mathrm{T}} \hat{\boldsymbol{A}}_{B_3^*}^{-1} \boldsymbol{c} \Delta t_2 \\ &\leq \hat{\boldsymbol{b}}_{1B_1^*}^{\mathrm{T}} \hat{\boldsymbol{A}}_{B_1^*}^{-1} \boldsymbol{c} \Delta t_1 + \hat{\boldsymbol{b}}_{2B_2^*}^{\mathrm{T}} \hat{\boldsymbol{A}}_{B_2^*}^{-1} \boldsymbol{c} \Delta t_2) \\ &= e_1^* + e_2^* \end{aligned} \tag{A9}$$

where the equality is satisfied in case that $B_3^* = B_1^* = B_2^*$. According to the duality theorem, $d_1^* = e_1^*$, $d_2^* = e_2^*$, and $d_3^* = e_3^*$. Therefore, it is concluded that $d_3^* \leq d_1^* + d_2^*$.

Nomenclature

A : set for indices of periods in cluster

$\boldsymbol{A}$: matrix for coefficients of constraints in primary problem

B : basic solution

$\boldsymbol{b}$: vector for coefficients of constraints in primary problem

C : set for indices of clusters for optimal operation problems

$\boldsymbol{c}$: vector for coefficients of objective function in primary problem

(D) : dual problem

d : objective function in primary problem

e : objective function in dual problem

f : part of objective function yen/y, yen/h

$\boldsymbol{g}$: vector for inequality constraints

$\boldsymbol{h}$: vector for equality constraints

L : number of clusters for periods

M : total number of periods

N : number of periods per cluster

P : set for indices of optimal operation problems in cluster

(P) : primary problem

Δt : duration per year of period h/y

$\boldsymbol{u}$: vector for variables in primary problem

$\boldsymbol{v}$: vector for variables in dual problem

$\boldsymbol{w}$: vector for variables in transformed dual problem

$\boldsymbol{x}$: vector for continuous operation variables

$\boldsymbol{y}$: vector for energy demands

z : objective function, or annual total cost yen/y

$\boldsymbol{\delta}$: vector for integer operation variables

$\boldsymbol{\eta}$: vector for binary and integer design variables

$\tilde{(\ )}$: upper bound

$\underset{\sim}{(\ )}$: lower bound

$\hat{(\ )}$: transformed dual problem

$(\ )'$: reduced optimal design problem

$(\ )^*$: optimal solution

Subscripts and arguments

k, m : indices of periods in part related with operation

l : index of clusters for periods

S : solved optimal operation problems

U : unsolved optimal operation problems

0 : part related with design

1, 2 : periods 1 and 2

3 : aggregated period

Superscripts

C : continuous relaxation problem

D : critical design problem

n_1, n_2, n_3 : numbers of variables in $\boldsymbol{\eta}$, $\boldsymbol{\delta}$, and $\boldsymbol{x}$, respectively

n_4, n_5 : numbers of constraints and variables in LP problem, respectively

O : critical operation problem

Equipment symbols

BG : gas-fired auxiliary boiler

BW : waste heat recovery boiler

EP : device for receiving electricity

GT : gas turbine generator

PC : pump

RE : electric compression refrigerator

RS : steam absorption refrigerator

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Captions for tables and figures

criteria ($N = 4$)

(a) Number of generated design candidates

(b) Number of solved optimal operation problems

(c) Computation time

Table 1 Capacities and performance characteristic values of candidates of equipment for selection

Equipment	Capacity/performance*	Candidate			
Gas turbine cogeneration unit	#	1	2	3	4
	Max. power output MW	1.29	1.60	2.00	2.40
	Max. steam output MW	5.69	3.34	4.10	4.57
	Power generating efficiency	0.140	0.173	0.169	0.179
	Heat recovery efficiency	0.617	0.362	0.347	0.341
	#	5	6	7	8
	Max. power output MW	2.93	3.50	3.54	4.36
	Max. steam output MW	6.44	6.97	6.89	8.92
	Power generating efficiency	0.256	0.271	0.273	0.273
	Heat recovery efficiency	0.563	0.540	0.531	0.559
	#	9	10		
	Max. power output MW	5.23	5.32		
	Max. steam output MW	8.91	9.05		
	Power generating efficiency	0.301	0.306		
	Heat recovery efficiency	0.513	0.521		
Gas-fired auxiliary boiler	#	1	2	3	4
	Max. steam output MW	5.24	6.55	7.86	9.82
	Thermal efficiency	0.92	0.92	0.92	0.92
Electric compression refrigerator	#	1	2	3	4
	Max. cooling output MW	2.82	3.52	4.22	5.28
	Coefficient of performance	4.57	4.73	4.76	5.04
Steam absorption refrigerator	#	1	2	3	4
	Max. cooling output MW	3.46	5.18	6.91	8.64
	Coefficient of performance	1.20	1.20	1.20	1.20

* At rated load status

Table 2 Capital unit costs of equipment, and unit costs for demand and energy charges of utilities

Equipment/utility		Unit cost
Gas turbine generator		230.0×10^3 yen/kW
Waste heat recovery boiler		9.6×10^3 yen/kW
Gas-fired auxiliary boiler		6.6×10^3 yen/kW
Electric compression refrigerator		34.4×10^3 yen/kW
Steam absorption refrigerator		30.1×10^3 yen/kW
Receiving device		56.3×10^3 yen/kW
Electricity	Demand charge	1740 yen/(kW·month)
	Energy charge	10.77 yen/kWh (Summer) 9.79 yen/kWh (Other seasons)
Natural gas	Demand charge	2033 yen/(m^3/h·month)
	Energy charge	30.88 yen/m^3

Table 3 Optimal values of capacities and numbers of equipment, and maximum demands of utilities

Equipment/utility	Candidate	Number	Capacity
Gas turbine cogeneration unit	#10	3	15.96 MW
Gas-fired auxiliary boiler	#2	1	6.55 MW
Electric compression refrigerator	#1	1	2.82 MW
Steam absorption refrigerator	#4	4	34.56 MW
Receiving device	–	–	9.00 MW
Electricity maximum demand	–	–	9.00 MW
City gas maximum demand	–	–	5.00×10^3 m^3/h

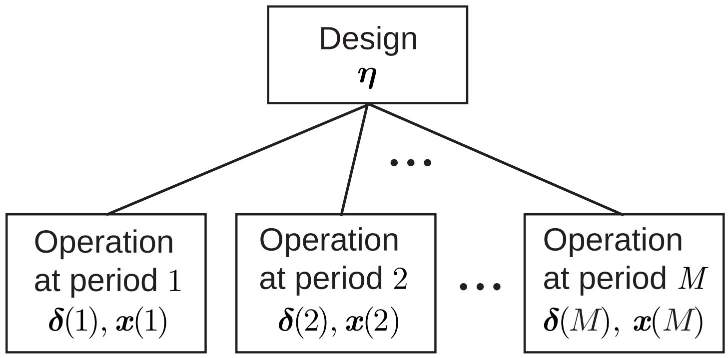


Fig. 1 Hierarchical relationship between design and operation of energy supply system

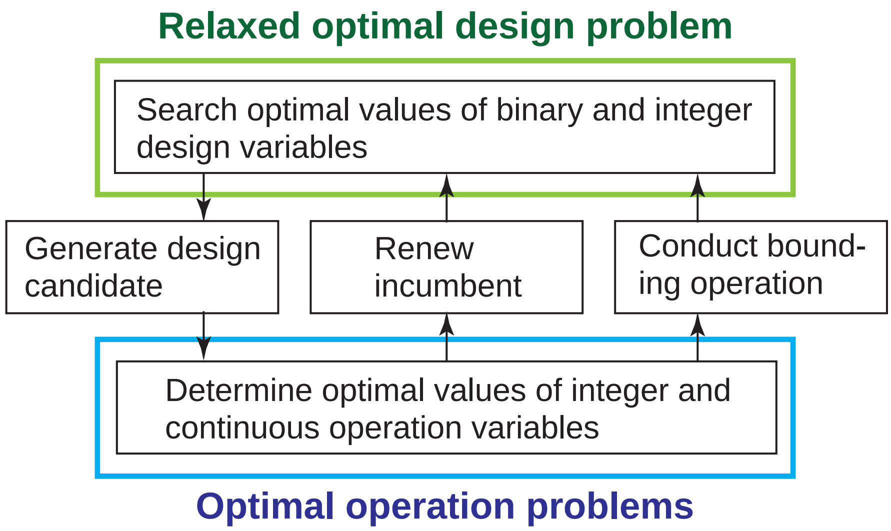


Fig. 2　Solution process by hierarchical MILP method

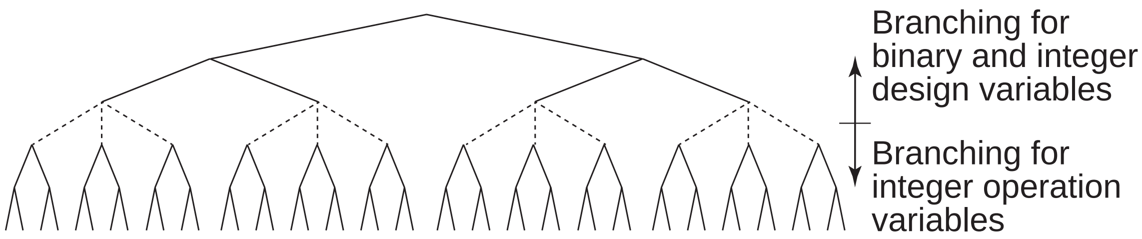


Fig. 3 Enumeration tree for optimization problems related hierarchically

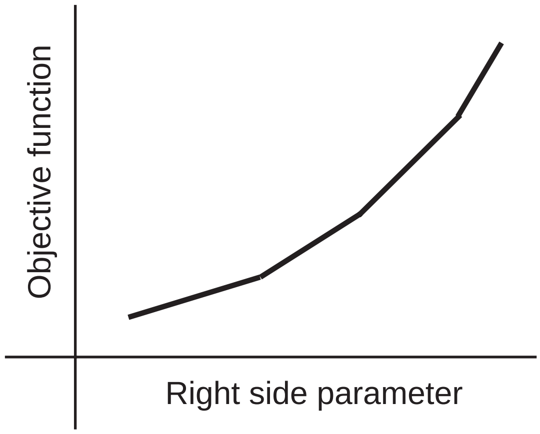


Fig. 4 Influence of right side parameter on objective function in LP problem

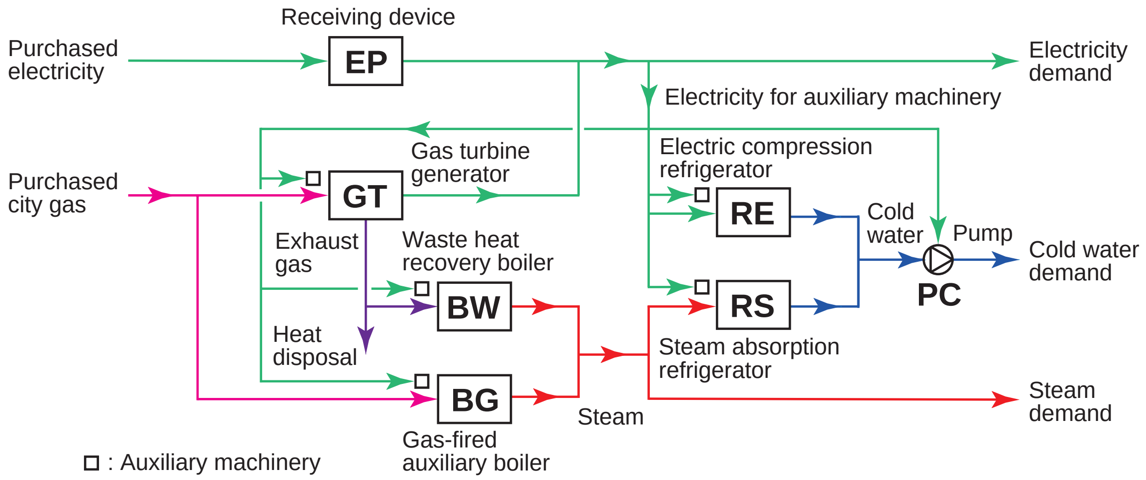
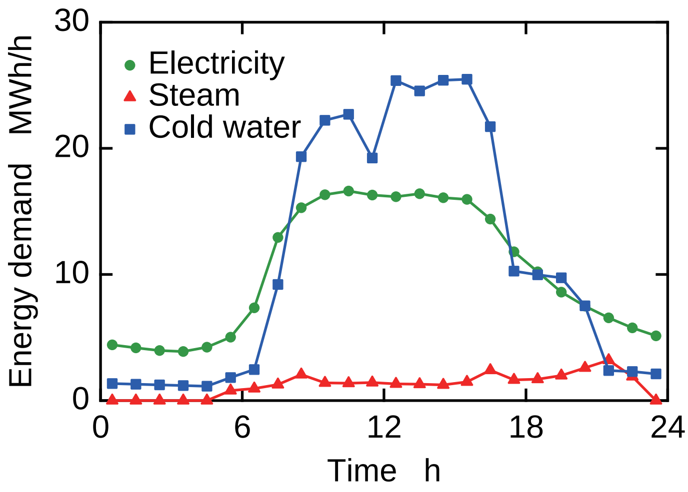
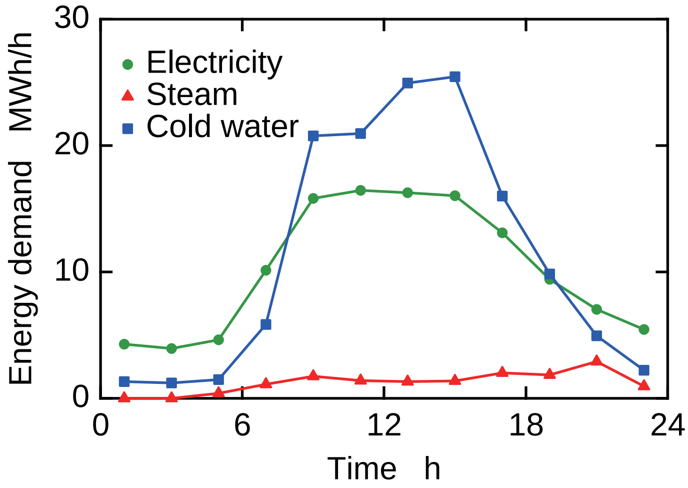


Fig. 5　Configuration of gas turbine cogeneration system

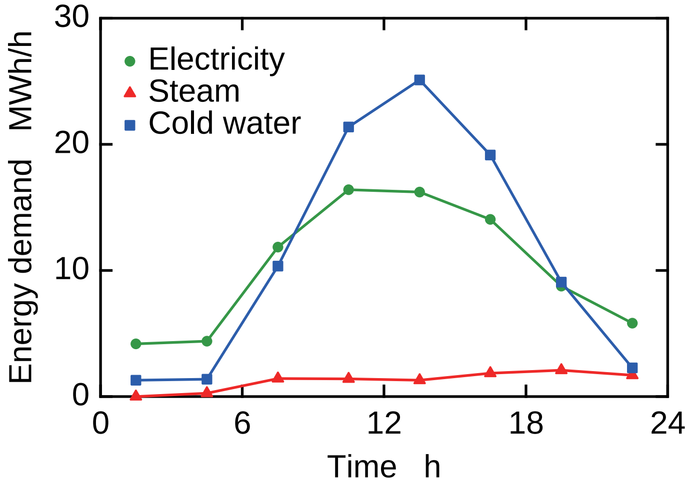


(a) Original

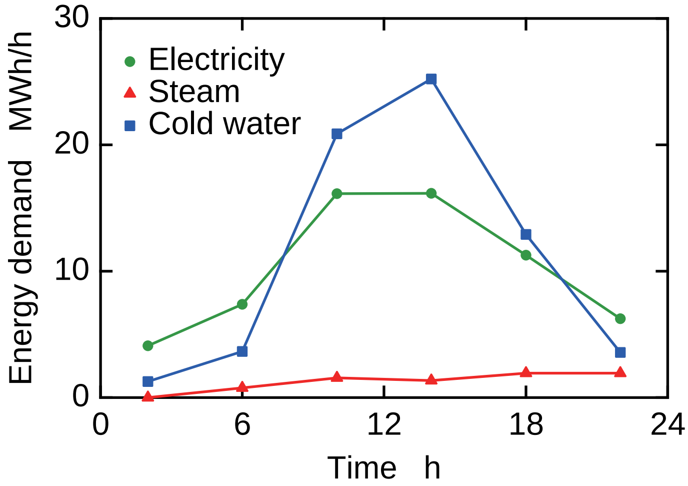


(b) Averaged ($N = 2$)

Fig. 6 Energy demands in summer

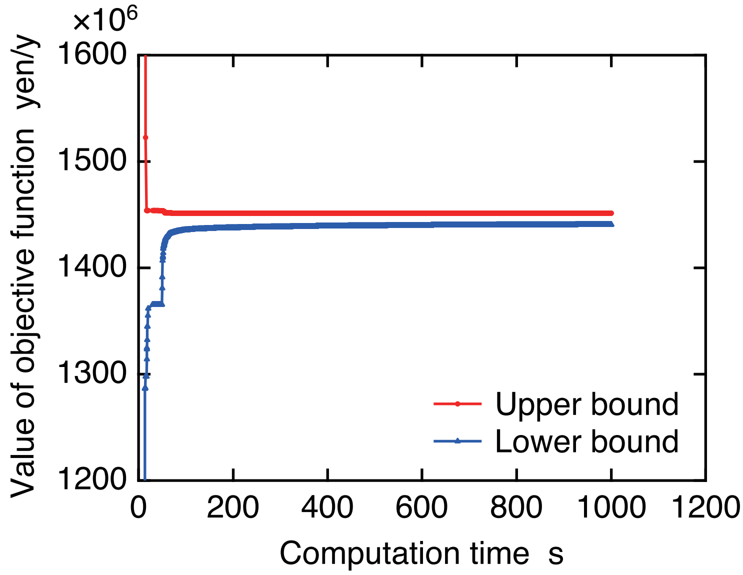


(c) Averaged ($N = 3$)

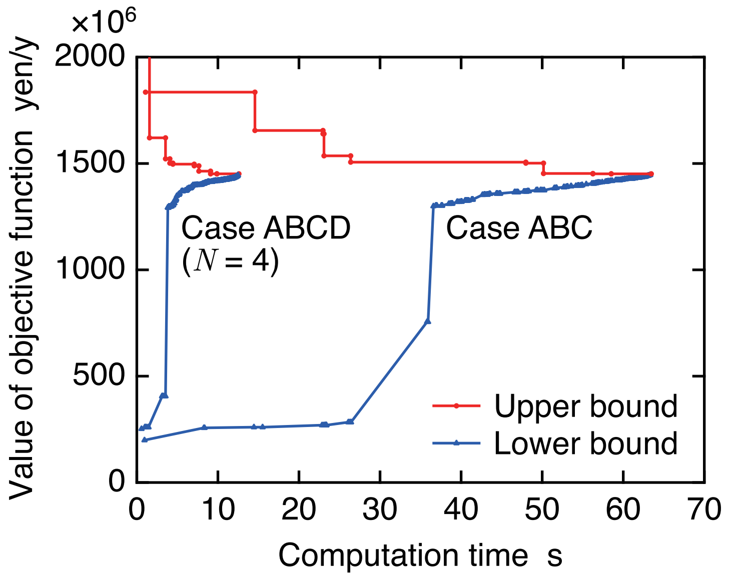


(d) Averaged ($N = 4$)

Fig. 6 Energy demands in summer



(a) Conventional optimization calculation



(b) Hierarchical MILP method

Fig. 7 Changes in upper and lower bounds for optimal value of objective function

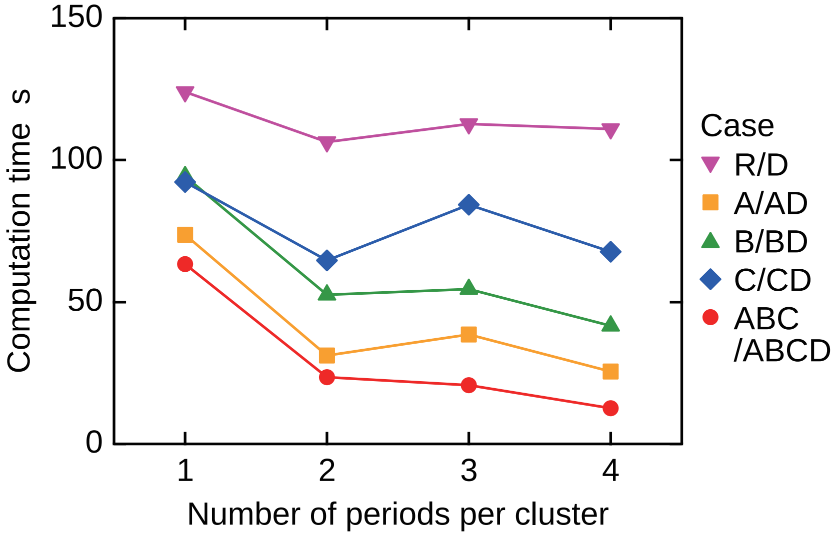


Fig. 8　Effect of number of periods per cluster on computation time

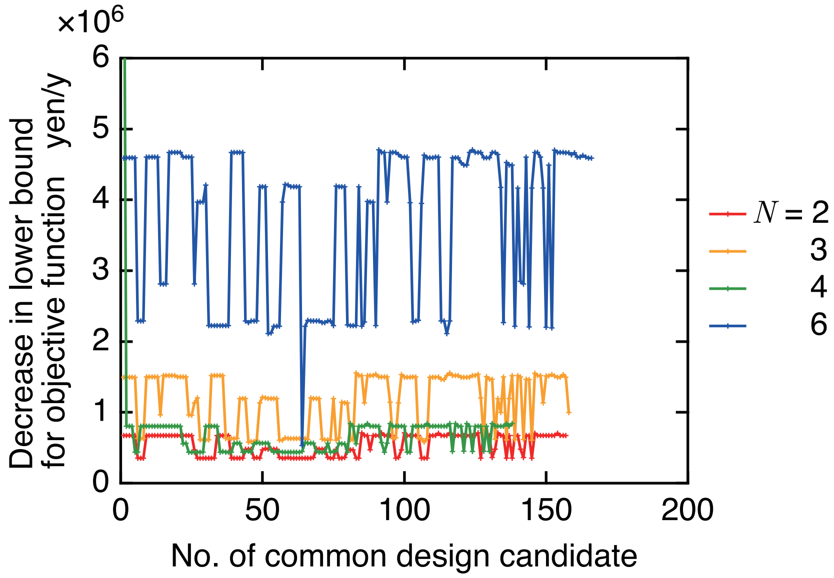
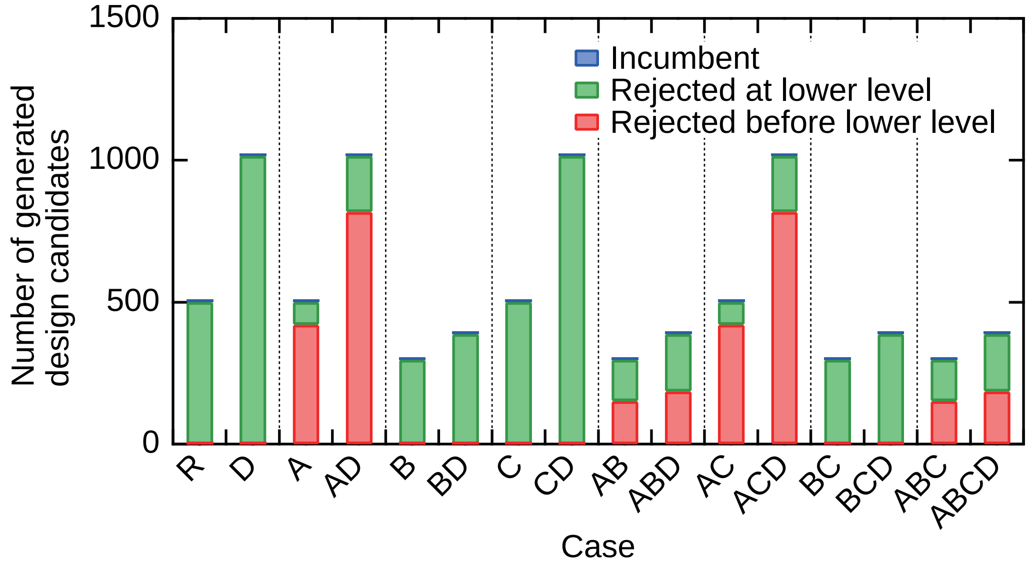
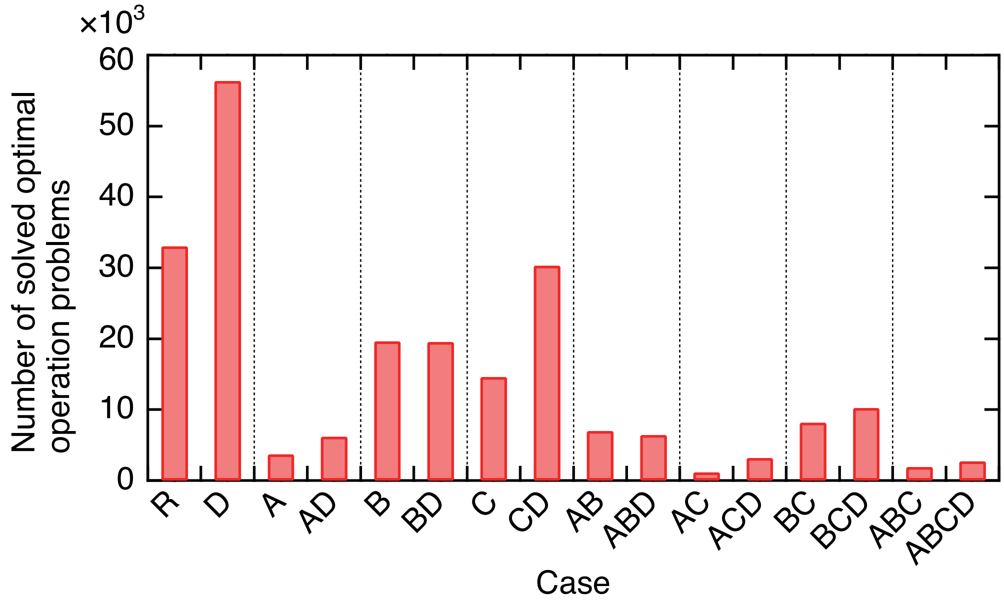


Fig. 9 Decrease in lower bound for value of objective function (Case ABCD)

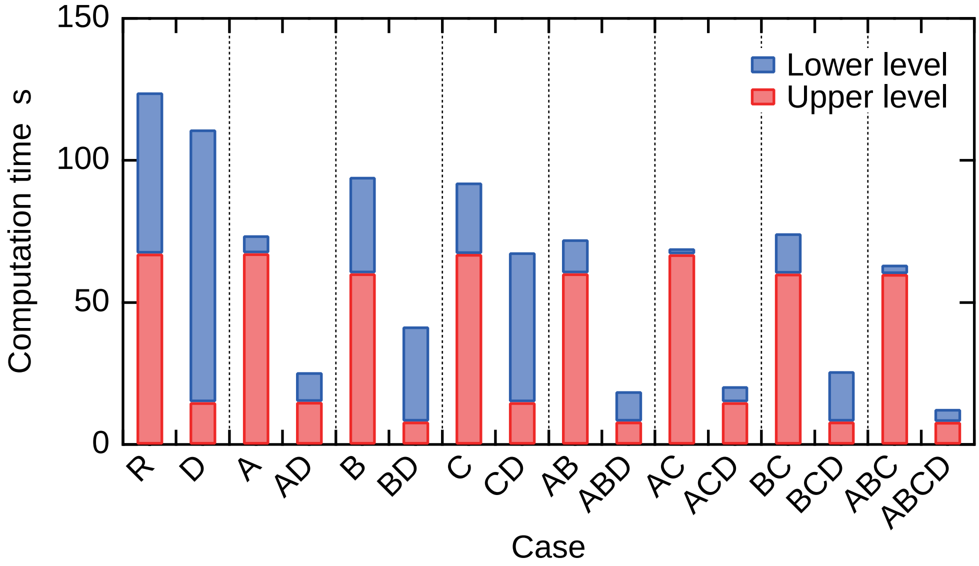


(a) Number of generated design candidates



(b) Number of solved optimal operation problems

Fig. 10 Effect of strategies for enhancing computation efficiency on evaluation criteria ($N = 4$)



(c) Computation time

Fig. 10 Effect of strategies for enhancing computation efficiency on evaluation criteria

($N = 4$)