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	作成者: Masamura, Shinnosuke, Iwamoto, Tetsu,
	Sugitani, Yoshiki, Konishi, Keiji, Hara, Naoyuki
	メールアドレス:
	所属:
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Experimental investigation of amplitude death in delay-coupled double-scroll circuits with randomly time-varying network topology

Shinnosuke Masamura · Tetsu Iwamoto · Yoshiki Sugitani* · Keiji Konishi · Naoyuki Hara

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Abstract The present study experimentally investigates amplitude death in delay-coupled double-scroll circuits with a time-varying network topology that randomly changes at a regular interval. Circuit experiments show that amplitude death can occur in the time-varying network. Furthermore, the experimental results well agree with the analytical results obtained based on a time-averaged adjacency matrix when the interval is much shorter than the natural period of the double-scroll circuits.

Keywords Amplitude death · Delayed coupling · Timevarying network topology

1 Introduction

A variety of nonlinear phenomena in coupled oscillators are induced by mutual interactions among oscillators [2]. These interactions are roughly classified into weak and strong types. The former mainly influences the phase of each oscillator. Phase synchronization can thus occur in coupled oscillators [3,4]. The latter influences not only the phase but also the amplitude of each oscillator. Strong interaction can induce oscillation quenching in coupled oscillators [5–7]. Amplitude death, a quenching phenomenon, is the stabilization

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Department of Electrical and Information Systems,

Osaka Prefecture University

1-1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531 Japan

Y. Sugitani*

4-12-1 Nakanarusawa, Hitachi, Ibaraki 316-8511 Japan E-mail: yoshiki.sugitani.0301@vc.ibaraki.ac.jp of the steady state in coupled oscillators. This phenomenon occurs in both coupled limit-cycle oscillators and coupled chaotic oscillators [8]. Recently, amplitude death has been found in various types of coupled oscillator, such as quantum self-oscillators [9], fractional-order oscillators [10, 11], and reaction diffusion systems [12, 13]. In addition, partial amplitude death, where an oscillator either converges onto its own equilibrium point or maintains oscillation, has been reported [14].

Amplitude death has great potential application for the suppression of undesired oscillations in various engineering systems [15–20]. It is of considerable importance for stable operation of coupled permanent-magnet synchronous motors [15]. An unnecessary oscillation of the dc bus voltage is quenched by coupling a pair of dc microgrids [16, 17]. Amplitude death can be used to suppress thermoacoustic oscillations, which can harm gas turbine engines [18–20].

Early research on amplitude death concluded that a frequency mismatch between oscillators is required to induce amplitude death [21,22]. However, some coupling schemes that can cause amplitude death in identical coupled oscillators have been proposed, such as delayed connections [23, 24], dynamic connections [25], conjugate connections [26], mean-field connections [27], and direct and indirect connections [28]. Delayed connections have attracted much attention [6,29] because time delay is inherent in interactions in real systems (e.g., autapse connections in neural networks [30–32]). Amplitude death induced by delayed connections has been studied for ring networks [33], all-to-all networks [11], small world networks [34], random networks [34], and scale-free networks [35].

Most studies on amplitude death have considered only time-invariant networks. However, it is well known that a lot of networks in the real world have time-varying topologies, such as ad-hoc networks [36], physiological networks [37], neural networks [38], epidemic systems [39], and gene reg-

S. Masamura \cdot T. Iwamoto \cdot K. Konishi \cdot N. Hara

Department of Electrical and Electronic Systems Engineering, Ibaraki University



Fig. 1 Illustration of a randomly time-varying network with 5 oscillators. (a) Time variation of the network with regular interval Δt ; (b) constraint matrix H, which limits the network topology; (c) elements of the adjacency matrix E(t).

ulatory networks [40]. We are thus interested in amplitude death in time-varying networks.

Stability analysis for such networks is quite difficult because the local stability of amplitude death is equivalent to that of linear time-varying systems with delay. For this difficult task, we previously numerically investigated amplitude death in a simple time-varying network [41] whose topology randomly changed at a regular interval and whose nodes were the two-dimensional Stuart-Landau oscillators. However, our previous study did not address two significant problems related to the practical usage of amplitude death. First, amplitude death has not been experimentally confirmed in time-varying networks; experimental investigation of the robustness against noise and parameter mismatch, which are unavoidable in real systems, is quite important from an engineering point of view. Second, our previous study focused only on the Stuart-Landau oscillator, which is a two-dimensional periodic normal form of Hopf bifurcation. Thus, we need to confirm whether amplitude death occurs in more general oscillators, since the dynamics of many real oscillators is not only periodic but also usually

more complex (e.g., chaotic) due to their high dimensionality of three or higher.

The present study addresses these problems by performing real circuit experiments on amplitude death in a timevarying network. In our experiments, a three-dimensional chaotic circuit [42] is employed as the oscillator. The timevarying topology, implemented using analog switches that are opened and closed by binary signals, randomly changes at a regular interval. It is shown that amplitude death occurs in the time-varying network¹. Furthermore, it is observed that if the regular interval of the time-varying topology is sufficiently shorter than the natural period of the chaotic circuit, the results of the approximate stability analysis based on a time-averaged adjacency matrix are in good agreement with the behavior of the circuit.

¹ Various known collective behaviors, such as complete synchronization, partial synchronization, and chimera state, that can occur in delayed coupled oscillators [43] are not covered in the present study.



Fig. 2 Circuit diagram of coupled oscillator circuits for N = 2. Each OSC consists of a double-scroll circuit and a delay unit; SW consists of two switching devices and coupling resisters.

2 Delay-coupled oscillators with randomly time-varying network

Let us briefly review randomly time-varying networks [41]. Figure 1(a) shows delay-coupled oscillators whose network topology changes at regular interval $\Delta t > 0$. Two oscillators are connected with probability $p \in [0, 1]$ and are not connected with probability 1 - p according to constraint matrix $H = \{h_{j,l}\}$, as shown in Fig. 1(b). The element $h_{j,l} \in \{0, 1\}$ governs the connection between oscillators as follows: if $h_{j,l} = h_{l,j} = 1$, the *j*-th and *l*-th oscillators are allowed to be connected; if $h_{j,l} = h_{l,j} = 0$, they are not allowed to be connected; self-feedback is not allowed (i.e., $h_{j,j} \equiv 0$). For instance, as illustrated in Figs. 1(a) and (b), oscillator 1 (OSC 1) is allowed to be connected to oscillator 5 (OSC 5) because $h_{1,5} = h_{5,1} = 1$, but not to oscillator 4 (OSC 4) because $h_{1,4} = h_{4,1} = 0$.

The time-varying network topology at time *t* is described by an adjacency matrix $\{E(t)\}_{j,l} = h_{j,l}\varepsilon_{j,l}(t)$, where $\varepsilon_{j,l}(t) = \varepsilon_{l,j}(t) \in \{0, 1\}$ is a random binary signal with interval Δt . We have $\varepsilon_{j,l}(t) = \varepsilon_{l,j}(t) = 1$ with a probability of *p* and $\varepsilon_{j,l}(t) = \varepsilon_{l,j}(t) = 0$ with a probability of 1 - p. Based on this random signal, the *j*-th and *l*-th oscillators are coupled with probability *p* if $h_{j,l} = h_{l,j} = 1$. The random signals $\varepsilon_{j,l}(t) = \varepsilon_{l,j}(t)$ for $j, l \in \{1, ..., N\}$ are independent of each other. Figure 1(c) shows an example of three elements of adjacency matrix E(t). Since we have $h_{1,2}\varepsilon_{1,2}(t) = 1$ for $t \in [(n-1)\Delta t, n\Delta t)$ and $h_{1,2}\varepsilon_{1,2}(t) = 0$ for $t \in [n\Delta t, (n+2)\Delta t)$, OSC 1 and OSC 2 are connected only for $t \in [(n-1)\Delta t, n\Delta t)$ in Fig. 1(a). Additionally, since we have $h_{1,4}\varepsilon_{1,4}(t) \equiv 0$ because $h_{1,4} = 0$, OSC 1 and OSC 4 are never connected.

Let *N* be the number of oscillators. The dynamics of the *j*-th oscillator is expressed as

$$\begin{cases} \dot{\mathbf{x}}^{(j)}(t) = \mathbf{F}\left(\mathbf{x}^{(j)}(t)\right) + \mathbf{b}u^{(j)}(t) \\ y^{(j)}(t) = \mathbf{c}\mathbf{x}^{(j)}(t) \end{cases} \quad (j = 1, \dots, N), \tag{1}$$

where $\mathbf{x}^{(j)}(t) \in \mathbb{R}^m$ and $y^{(j)}(t) \in \mathbb{R}$ are the state variable and the output signal of the *j*-th oscillator, respectively. $\mathbf{F}(\mathbf{x})$: $\mathbb{R}^m \to \mathbb{R}^m$ is a nonlinear function, which is assumed to have at least one equilibrium point \mathbf{x}^* satisfying $\mathbf{F}(\mathbf{x}^*) = \mathbf{0}$. Vector $\mathbf{b} \in \mathbb{R}^m$ denotes the input coefficient vector and vector $\mathbf{c} \in \mathbb{R}^{1 \times m}$ denotes the output coefficient vector. The input signal $u^{(j)}(t) \in \mathbb{R}$ is given by

$$u^{(j)}(t) = k \sum_{l=1}^{N} h_{j,l} \varepsilon_{j,l}(t) \left\{ y^{(l)}(t-\tau) - y^{(j)}(t) \right\},$$
(2)

where $k \in \mathbb{R}$ is the coupling strength. $y^{(l)}(t - \tau) \in \mathbb{R}$ denotes the delayed output signal with connection delay $\tau \ge 0$ from the *l*-th oscillator. Coupled oscillators (1) and (2) have the homogeneous steady state

$$\left[\boldsymbol{x}^{(1)^{T}}(t) \cdots \boldsymbol{x}^{(N)^{T}}(t)\right]^{T} = \left[\boldsymbol{x}^{*T} \cdots \boldsymbol{x}^{*T}\right]^{T}.$$
(3)

We now focus on the local stability of this steady state. Linearizing coupled oscillators (1) and (2) at steady state (3)



Fig. 3 Time-series data of voltages $v_b^{(1),(2)}$ at $(k,\tau) = (4.85, 0.50)$ for a pair of oscillator circuits with random on-off switching (p = 0.5) for (a) $\Delta t = 0.080$ ms and (b) $\Delta t = 2.000$ ms. The errors δ for (a) and (b) correspond to points A and B in Fig. 4, respectively.

yields the following time-varying linear system with delay:

$$\Delta \dot{\boldsymbol{X}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{A}) \Delta \boldsymbol{X}(t) + k \bigg[(\boldsymbol{E}(t) \otimes \boldsymbol{b}\boldsymbol{c}) \Delta \boldsymbol{X}(t-\tau) \\ - \bigg\{ \operatorname{diag} \left(d_1(t), \dots, d_N(t) \right) \otimes \boldsymbol{b}\boldsymbol{c} \bigg\} \Delta \boldsymbol{X}(t) \bigg], \quad (4)$$

where $\Delta \mathbf{X}(t) := \left[\Delta \mathbf{x}^{(1)^T}(t) \cdots \Delta \mathbf{x}^{(N)^T}(t) \right]^T$, $\Delta \mathbf{x}^{(j)} := \mathbf{x}^{(j)} - \mathbf{x}^*$ (*j* = 1, ..., *N*), and $\mathbf{A} := \{\partial \mathbf{F}(\mathbf{x}) / \partial \mathbf{x}\}_{\mathbf{x}=\mathbf{x}^*}$. The symbol \otimes denotes the Kronecker product. The time-varying degree of the *j*-th oscillator is defined by $d_j(t) := \sum_{l=1}^N \{\mathbf{E}(t)\}_{j,l}$. It should be emphasized that the stability analysis of linear system (4) is a difficult task because of its time-varying matrices [i.e., $\mathbf{E}(t)$ and diag $(d_1(t), \ldots, d_N(t))$] and time delay. In Sec. 4, we approximately analyze the stability of linear system (4) based on a time-averaged adjacency matrix.

3 Circuit experiments on amplitude death

We now experimentally investigate amplitude death in the delay-coupled double-scroll circuits with a randomly time-varying network topology. This section considers two cases: a pair of oscillator circuits (N = 2) and a network consisting of five oscillator circuits (N = 5).

3.1 Pair of oscillator circuits (N = 2)

Consider the pair of double-scroll circuits [42] coupled by a delayed connection with random on-off switching shown in Fig. 2. Each oscillator circuit, which has capacitors C_a and C_b , linear resistor R_d , nonlinear resistor NR, and inductor L,

is governed by the circuit equation

$$\begin{cases} C_{a} \frac{dv_{a}^{(j)}}{dt} = \frac{1}{R_{d}} \left(v_{b}^{(j)} - v_{a}^{(j)} \right) - h \left(v_{a}^{(j)} \right) \\ C_{b} \frac{dv_{b}^{(j)}}{dt} = \frac{1}{R_{d}} \left(v_{a}^{(j)} - v_{b}^{(j)} \right) + i_{L}^{(j)} + i_{u}^{(j)} , \quad (j = 1, 2). \tag{5}$$

$$L \frac{di_{L}^{(j)}}{dt} = -v_{b}^{(j)}$$

The current through NR with parameters $m_{0,1}$ and B_p is given by $h(v) := m_0 v + (m_1 - m_0)\{|v + B_p| - |v - B_p|\}/2$. The voltages, $v_b^{(1)}$ and $v_b^{(2)}$, across C_b are applied to the delay unit², and then delayed voltages $v_b^{(1)}(t - T)$ and $v_b^{(2)}(t - T)$ with delay time *T* are applied to coupling resistor *r* through the switching devices³. The coupling currents flowing to each oscillator are given by

$$i_{u}^{(1)} = \frac{1}{r} h_{1,2} \varepsilon_{1,2}(t) \left\{ v_{b}^{(2)}(t-T) - v_{b}^{(1)}(t) \right\},$$

$$i_{u}^{(2)} = \frac{1}{r} h_{2,1} \varepsilon_{2,1}(t) \left\{ v_{b}^{(1)}(t-T) - v_{b}^{(2)}(t) \right\}.$$
(6)

A switching device passes (does not pass) the delayed voltage directly to the coupling resistor when the binary voltage signal applied to the device is at a high (low) level (see Appendix A); that is, the two oscillator circuits are coupled (i.e., $h_{1,2}\varepsilon_{1,2}(t) = h_{2,1}\varepsilon_{2,1}(t) = 1$) if the signal is high, and are isolated (i.e., $h_{1,2}\varepsilon_{1,2}(t) = h_{2,1}\varepsilon_{2,1}(t) = 0$) if the signal is low.

² The delay unit is mainly implemented by a peripheral interface controller (PIC) [44]. The voltage $v_{\rm b}^{(j)}$ is stored in the memory of the PIC through its built-in 8-bit analog-to-digital (A/D) converter with sampling period 25 μ s. The delayed digital signal is output from the PIC and converted to delayed voltage $v_{\rm b}^{(j)}(t-T)$ by a digital-to-analog (D/A) converter.

³ Resistor r includes the on-resistance of the switching device.

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Fig. 4 Stability region in coupling parameter (k, τ) space for a pair of oscillators (N = 2) for $(a) \Delta t = 0.080 \text{ ms}$, $(b) \Delta t = 0.800 \text{ ms}$, $(c) \Delta t = 2.000 \text{ ms}$, and (d) a time-invariant network. Circles denote coupling parameter sets (k, τ) for the experiment, and their color represents error δ . The curves and regions respectively denote the marginal stability curves and the stability regions analytically derived based on the time-averaged adjacency matrix.

The nominal parameters of the oscillator circuits are fixed at

$$C_{\rm a} = 0.1 \,\mu\text{F}, \ C_{\rm b} = 1.0 \,\mu\text{F}, \ L = 180 \,\text{mH}, \ R_{\rm d} = 1.8 \,\text{k}\Omega,$$

 $m_0 = -0.4 \,\text{mS}, \ m_1 = -0.8 \,\text{mS}, \ B_{\rm p} = 1 \,\text{V},$ (7)

throughout the present study, where each isolated oscillator circuit shows a double-scroll attractor [45]. Note that oscillators (5) with coupling currents (6) can be written as coupled oscillators (1) and (2) with

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{bmatrix} \eta \{ x_2 - x_1 - \hat{g}(x_1) \} \\ x_1 - x_2 + x_3 \\ -\gamma x_2 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \quad (8)$$

where

$$x_{1} := \frac{v_{a}}{B_{p}}, \quad x_{2} := \frac{v_{b}}{B_{p}}, \quad x_{3} := \frac{i_{L}R_{d}}{B_{p}}, \quad \eta := \frac{C_{b}}{C_{a}}, \quad \gamma := \frac{R_{d}^{2}C_{b}}{L},$$
$$a := m_{1}R_{d}, \quad b := m_{0}R_{d}, \quad k := \frac{R_{d}}{r}, \quad \tau := \frac{T}{R_{d}C_{b}}, \quad (9)$$

$$\hat{g}(x) := bx + \frac{1}{2}(b-a)\{|x-1| - |x+1|\}.$$

The dimensionless time $t/R_{\rm d}C_{\rm b}$ is used instead of the real time *t*. Coupled oscillators (1) and (2) with Eq. (8) have three equilibrium points, namely $\mathbf{x}_{\pm}^* = [\pm p^* \ 0 \ \mp p^*]^T$ and $\mathbf{x}_0^T =$

 $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, where $p^* := (b - a)/(b + 1)$. Here, the natural period⁴ of the oscillator circuit around the equilibrium point x^*_+ is $T_f \approx 3$ ms.

Let us experimentally examine amplitude death in a pair of delay-coupled double-scroll circuits. The connection probability is fixed at p = 0.5 throughout the present study. In other words, there is a 50% chance that the binary voltage signal will be high. The constraint matrix is set to

$$\boldsymbol{H} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\tag{10}$$

(i.e., $h_{1,2} = h_{2,1} = 1$ in Eq. (6)); that is, the two oscillator circuits are allowed to be connected. The coupling strength and the connection delay are set to $(k, \tau) = (4.85, 0.50)$. Figure 3 shows the time-series data of $v_b^{(1),(2)}$ for the switching intervals $\Delta t = 0.080$ ms and 2.000 ms in the coupled circuits shown in Fig. 2. These switching intervals correspond to the fast and slow switching of the network topology, respectively (the natural period of the oscillator circuit $T_f \approx 3$ ms). The two isolated oscillator circuits start to be coupled by coupling current (6) at t = 0.50 s. With the short interval

⁴ The period $T_{\rm f} \approx 3 \, {\rm ms}$ is estimated using the eigenvalues of Jacobian matrix *A* at equilibrium point x_{\pm}^* . Operational amplifiers with a slew rate of 13 V/ μ s, which is sufficiently high compared with the period $T_{\rm f}$ and the voltage range of the oscillator circuit, are employed to make nonlinear resistor NR.



Fig. 5 Circuit diagram of a network with five oscillators (N = 5). OSC and SW are the same as those in Fig. 2.

 $\Delta t = 0.080$ ms, these voltages converge onto the equilibrium point (i.e., amplitude death), as shown in Fig. 3(a), whereas with the long interval $\Delta t = 2.000$ ms, they do not, as shown in Fig. 3(b). To quantitatively evaluate the stability of the equilibrium point in the circuit experiments, we define the mean absolute error δ [V] from the equilibrium point:

$$\delta := \left\langle \frac{1}{N} \sum_{j=1}^{N} \left| v_{\mathbf{b}}^{(j)}(t) - v_{\mathbf{b}}^* \right| \right\rangle.$$
(11)

Here, $\langle \cdot \rangle$ denotes the average over time interval $t \in [t_0 +$ 0.5, $t_0 + 1.0$], where t_0 is the start time of coupling and $v_b^* \equiv 0$ is steady-state voltage $v_{\rm b}^{(j)}(t)$ on the equilibrium point⁵. In Fig. 3(a), the voltages converge onto the equilibrium point with a small error ($\delta = 0.0065$ V). In contrast, in Fig. 3(b), the error is large ($\delta = 0.0489$ V). Figures 4(a), (b), and (c) show error δ in coupling parameter (k, τ) space for intervals $\Delta t = 0.080 \,\mathrm{ms}, 0.800 \,\mathrm{ms}, \mathrm{and} 2.000 \,\mathrm{ms}, \mathrm{respectively}.$ The circles denote coupling parameter sets (k, τ) for the circuit experiments. Their color represents the value of δ . Black circles denote amplitude death. It can be seen that the number of stable parameter sets decreases as interval Δt increases. The curves in Fig. 4 denote the marginal stability curves, which are discussed in Sec. 4. Figure 4(d) shows the experimental results for the time-invariant network (i.e., $\varepsilon_{1,2}(t) =$ $\varepsilon_{2,1}(t) \equiv 1$) with adjacency matrix (10). At $\tau \approx 1.5$, there is no stable parameter set for k greater than 3. For the fast time-varying network (see Fig. 4(a)), stable parameter sets exist even for $k \approx 6$.

3.2 Oscillator network (N = 5)

We now consider the network consisting of five oscillator circuits (N = 5) shown in Fig. 5, where OSC and SW are

the same as those in Fig. 2. The coupling currents flowing to each oscillator are given by

$$i_{u}^{(j)}(t) = \frac{1}{r} \sum_{l=1}^{5} h_{j,l} \varepsilon_{j,l}(t) \left\{ v_{b}^{(l)}(t-\tau) - v_{b}^{(j)}(t) \right\}, \quad (j = 1, \dots, 5).$$
(12)

Currents (12) can be written in non-dimensional form (2) via variable transformation (9).

This subsection considers the following two constraint matrices:

$$\boldsymbol{H}_{\text{ring}} := \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{H}_{\text{all}} := \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}.$$
(13)

Note that H_{ring} and H_{all} represent the adjacency matrices of a ring network (see Fig. 1(b)) and an all-to-all network, respectively. Let us demonstrate the implementation of the time-varying topology based on constraint matrices $H_{\rm ring}$ and H_{all} . Figure 6 shows the time-series data of the five oscillator circuits and the binary voltage signals applied to SWs with interval $\Delta t = 0.800 \text{ ms}$ for H_{ring} and H_{all} . Here, (j-l) denotes the binary voltage signal applied to the SW between the *i*-th and *l*-th oscillator circuits, as shown in Fig. 5. The connection parameters are fixed at $(k, \tau) = (1.78, 0.50)$. Until t = 0.01 s, all the binary voltage signals are low; that is, all the oscillator circuits are isolated (i.e., $h_{i,l}\varepsilon_{i,l}(t) \equiv 0$, $\forall j, l \text{ in Eq. (12)}$). After t = 0.01 s, the signals randomly and independently take high or low levels with interval $\Delta t =$ 0.800 ms based on the constraint matrix. For H_{ring} , as shown in Fig. 6(a), voltages $v_{\rm b}^{(1)}, \ldots, v_{\rm b}^{(5)}$ converge onto the equilibrium point even though signals (2-5), (1-3), (2-4), (3-5), and (1 - 4) are always low due to constraint matrix H_{ring} . For H_{all} (see Fig. 6(b)), all the voltages of the oscillator circuits also converge onto the equilibrium point; in contrast to $H_{\rm ring}$, there are no binary voltage signals that always take a low level.

Figures 7(a), (b), and (c) show the experimental results for constraint matrix H_{ring} in (k, τ) space with intervals $\Delta t =$ 0.080 ms, 0.800 ms, and 2.000 ms, respectively. Figures 8(a), (b), and (c) show the results for H_{all} . It can be seen that for H_{ring} , the number of stable parameter sets (k, τ) decreases as interval Δt increases, whereas for H_{all} , this number depends little on Δt . These experimental results are consistent with our numerical results [41]. Note that with H_{ring} and H_{all} , each oscillator circuit is allowed to be connected with two and four neighboring oscillator circuits, respectively. This difference in the number of neighbors may have affected the results; however, the influence of the number of neighbors and interval Δt on the stability of amplitude death in a randomly time-varying network has not been sufficiently inves-

⁵ For calculating Eq. (11), we measured voltages $v_b^{(j)}$, $\forall j \in \{1, ..., N\}$ in parallel using an A/D board (see Appendix A).



Fig. 6 Time-series data for five oscillator circuits with randomly time-varying network topology (top) and binary voltage signals applied to SWs (bottom). (a) H_{ring} and (b) H_{all} . The symbol (j - l) represents the binary voltage signal applied to the SW between the *j*-th and *l*-th oscillator circuits in Fig. 5: the *j*-th and *l*-th oscillator circuits are connected if the signal is high, and are isolated if it is low. For (a), the binary voltage signals (2 - 5), (1 - 3), (2 - 4), (3 - 5), and (1 - 4) are always low due to constraint matrix H_{ring} .

tigated. A detailed investigation of this relation is left for future work.

Figures 7(d) and 8(d) show the experimental results for the time-invariant network with adjacency matrices H_{ring} and H_{all} , respectively. For $\tau \approx 3$ in Fig. 7(d), amplitude death does not occur for *k* greater than 3, whereas in Fig. 7(a), it occurs for *k* greater than 3. Similar results can also be observed in Figs. 8(d) and 8(a).

4 Stability analysis based on time-averaged adjacency matrix

The previous section experimentally verified amplitude death in a randomly time-varying network. This section analytically investigates the stability of amplitude death. As stated in Sec. 2, it is difficult to analyze the local stability of amplitude death in a time-varying network. Thus, this section approximately analyzes the stability based on an analysis method proposed in previous studies [46–48] and compares the results with the experimental results. 4.1 Stability analysis based on time-averaged adjacency matrix

For sufficiently long time $L \gg \Delta t$, the random binary signal in Eq. (2) can be averaged as

$$\frac{1}{L} \int_{t}^{t+L} \varepsilon_{j,l}(r) \mathrm{d}r = p.$$
(14)

If this averaging is approximately valid for time-variant system (4) with delay, then we can reduce it to a time-invariant system *with* delay as

$$\begin{aligned} d\dot{X}(t) &= (\boldsymbol{I}_N \otimes \boldsymbol{A}) \, \varDelta \boldsymbol{X}(t) + k \bigg[(\boldsymbol{p} \boldsymbol{H} \otimes \boldsymbol{b} \boldsymbol{c}) \varDelta \boldsymbol{X}(t - \tau) \\ &- \bigg\{ \boldsymbol{p} \text{diag}(d_1, \dots, d_N) \otimes \boldsymbol{b} \boldsymbol{c} \bigg\} \varDelta \boldsymbol{X}(t) \bigg], \end{aligned} \tag{15}$$

where pH is the time-averaged adjacency matrix, and $d_j := \sum_{l=1}^{N} h_{j,l}$ (j = 1, ..., N). Note that previous studies [46–48] mathematically proved the validity of the averaging shown above for time-variant systems *without* delay: the local stability of synchronization in a fast time-varying network, whi-



Fig. 7 Stability regions for H_{ring} for (a) $\Delta t = 0.080$ ms, (b) $\Delta t = 0.800$ ms, (c) $\Delta t = 2.000$ ms, and (d) a time-invariant network. Circles, curves, and shaded areas are the same as those in Fig. 4.

ch is governed by a linear time-variant system without delay, can be approximately reduced to the stability of a linear time-invariant system without delay. The present study focuses on time-variant system (4) with delay, for which the validity has not been proven. Thus, it is not presently guaranteed that the stability of system (4) with delay can be approximately reduced to that of system (15) with delay. Guaranteeing the above is a difficult task. Therefore, in the following, we analyze the stability of amplitude death under the assumption that the averaging in Eq. (14) is valid for time-variant system (4) with delay.

Suppose that the row sum of H is equivalent to D (i.e., $d_j = D, \forall j \in \{1, ..., N\}$). Then, diagonal matrix diag $(d_1, ..., d_N)$ can be rewritten as DI_N . Thus, linear system (15) with the time-averaged adjacency matrix is simplified to

$$\Delta \dot{X}(t) = \left\{ \boldsymbol{I}_N \otimes (\boldsymbol{A} - pk\boldsymbol{D}\boldsymbol{b}\boldsymbol{c}) \right\} \Delta \boldsymbol{X}(t) + kp(\boldsymbol{H} \otimes \boldsymbol{b}\boldsymbol{c}) \Delta \boldsymbol{X}(t-\tau).$$
(16)

The stability of linear system (16) is governed by the roots of the following characteristic equation:

$$G(s) = \det\left[sI_{Nm} - \{I_N \otimes (A - kpDbc)\} - kp(H \otimes bc)e^{-s\tau}\right].$$
(17)

Real symmetric matrix H can be diagonalized as $T^{-1}HT$ = diag (ρ_1, \ldots, ρ_N) using orthogonal matrix T, where ρ_q $(q = 1, \ldots, N) \in \mathbb{R}$ are the eigenvalues of H. This diagonalization

yields N characteristic equations

$$G(s) = \prod_{q=1}^{N} g(s, \rho_q),$$

$$g(s, \rho) := \det \left[s \boldsymbol{I}_m - \boldsymbol{A} + k p D \boldsymbol{b} \boldsymbol{c} - k p \rho \boldsymbol{b} \boldsymbol{c} e^{-s\tau} \right].$$
(18)

We see that linear system (16) with the time-averaged adjacency matrix is stable if and only if $g(s, \rho_q)$ is stable for all $q \in \{1, ..., N\}$. Equation (18) allows us to analytically obtain the marginal stability curves and the stability regions in coupling parameter (k, τ) space (see Appendix B) [49].

4.2 Comparison with experimental results

Let us derive the marginal stability curves with the Jacobian matrix of oscillator circuit (8) at equilibrium point x_{\pm}^* ,

$$A = \begin{bmatrix} -\eta(b+1) & \eta & 0\\ 1 & -1 & 1\\ 0 & \gamma & 0 \end{bmatrix},$$
 (19)

where the parameters

$$\eta = 10, \ \gamma = 18, \ b = -0.72, \ a = -1.44,$$

are calculated from circuit parameters (7). The marginal stability curves obtained using the procedure in Appendix B are shown in Figs. 4(a)-(c), 7(a)-(c), and 8(a)-(c). The bold



Fig. 8 Stability regions for H_{all} for (a) $\Delta t = 0.080$ ms, (b) $\Delta t = 0.800$ ms, (c) $\Delta t = 2.000$ ms, and (d) a time-invariant network. Circles, curves, and shaded areas are the same as those in Fig. 4.

(thin) curves represent the crossing direction of the characteristic roots of $g(s, \rho) = 0$: the roots cross the imaginary axis towards stability (instability) when parameter set (k, τ) crosses the curves with an increment k. The shaded areas denote the stability regions where all the roots of G(s) = 0 lie in the left-half complex plane; that is, linear system (16) is stable for sets (k, τ) within these regions.

For $\Delta t = 0.080$ ms (i.e., a fast time-varying network), shown in Figs. 4(a), 7(a), and 8(a), the stable parameter sets for our experiment (i.e., black circles) well agree with the stability regions obtained based on the time-averaged adjacency matrix (i.e., shaded areas) despite that our circuits include small parameter mismatch⁶. However, for $\Delta t = 2.000$ ms (i.e., a slow time-varying network), shown in Figs. 4(c) and 7(c), our analytical results do not well agree with the experimental results. Of note, in Fig. 8(c), our analytical results are in good agreement with the experimental results. This suggests that the upper limit of switching interval Δt , for which the experimental results agree with the analysis based on the time-averaged matrix, depends on the constraint matrix.

For the time-invariant network (i.e., Figs 4(d), 7(d), and 8(d)), the marginal stability curves and the stability regions can be derived by substituting p = 1 into Eq. (18). Compared with the regions for the time-varying network, those for the time-invariant network shrink in the direction of cou-

pling strength k. This is because coupling strength k is multiplied by probability p in Eq. (18). This implies that amplitude death occurs for larger coupling strength k as coupling probability p decreases.

5 Conclusion

The present study investigated amplitude death experimentally in delay-coupled double-scroll circuits with a randomly time-varying network topology. Our circuit experiments showed that amplitude death can occur in such a time-varying network. Furthermore, for the fast time-varying network, the approximate stability analysis based on the time-averaged adjacency matrix well agreed with the experimental results.

The present study considered only the typical constraint matrix of a time-varying network topology, which has the same sum for each row. Constraint matrices with different row sums (e.g., the adjacency matrix of a small-world network) will be investigated in future work. Furthermore, amplitude death induced in a time-varying network topology with different types of oscillator, such as fractional-order systems [10,11,50] and oscillators with hidden chaotic attractors [51], will also be the subject of future work.

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⁶ The resistors in our circuits have a tolerance of 1%. We selected capacitors that deviated by less than 0.6% from their nominal values.



Fig. 9 Experimental setup for N = 2. OSC and SW are the same as those in Fig. 2.

A Implementation of time-varying network and measurement circuit

This appendix shows how to implement a time-varying network and measurement circuit. For simplicity, we explain a pair of oscillator circuits (i.e., N = 2) even though we experimentally implemented a network with five oscillator circuits (i.e., N = 5), as shown in Fig. 5. Figure 9 illustrates the experimental setup, where OSC and SW are the same as those in Fig. 2. The digital output board (PEX-H293022, Interface Corp.) attached to Computer 1 outputs the binary voltage signal with interval Δt . This signal is generated as follows: (1) Computer 1 generates random variable $\varepsilon_{1,2}(t) = \varepsilon_{2,1}(t) \in \{0, 1\}$; this variable is multiplied by $h_{1,2} = h_{2,1}$; (2) the digital output board outputs a high (low) level signal if $h_{1,2}\varepsilon_{2,1}(t) = 1$ ($h_{1,2}\varepsilon_{2,1}(t) = 0$); (3) steps (1) and (2) are repeated at regular interval Δt . The binary voltage signal is applied to the two analog switching devices (ADG452) in SW. OSCs 1 and 2 are connected only when a high level signal is applied to the switching devices. Furthermore, since the digital output board can output 12 signals in parallel, the board can operate not only for one SW (i.e., N = 2), but for 10 SWs, as shown in Fig. 5. In the case of 10 SWs (i.e., N = 5), all 10 binary voltage signals are simultaneously generated at interval Δt , as shown in Fig. 6.

Voltages $v_b^{(1)}$ and $v_b^{(2)}$ of the oscillator circuits are fed into Computer 2 through a 12-bit A/D input board (PCI-3153, Interface Corp., sampling rate: 30 kHz, input voltage range: ± 5 V). The A/D input board can measure up to 16 voltages in parallel. The measured voltages are used for calculating error δ in Eq. (11).

In a previous study [52], a pair of double-scroll circuits (i.e., N = 2) with a time-varying network was implemented, in which the circuits were coupled by a static (i.e., non-delayed) connection. This previous study investigated synchronization numerically and experimentally in time-varying networks. Neither amplitude death nor a delayed connection was considered. In addition, three or more oscillators were not experimentally implemented.

B Procedure for drawing marginal stability curves

Here, we derive the marginal stability curves (see Chapter 5 in [49] for more details) of Eq. (18) in coupling parameter (k, τ) space. By substituting $s = i\lambda, \lambda \in \mathbb{R}$, the Jacobian matrix (19), and the input and output coefficient vectors (8) into $g(s, \rho) = 0$, we can obtain its real and imaginary parts,

$$\begin{cases} -\bar{\eta}\lambda kp\rho\sin\lambda\tau + \lambda^2 kp\rho\cos\lambda\tau + \theta_{\rm R}(\lambda) - \lambda^2 kpD = 0\\ -\lambda^2 kp\rho\sin\lambda\tau - \bar{\eta}\lambda kp\rho\cos\lambda\tau + \theta_{\rm I}(\lambda) + \bar{\eta}\lambda kpD = 0 \end{cases},$$
(20)

where $\bar{\eta} := \eta(b+1)$, $\theta_{R}(\lambda) := -\lambda^{2}(\bar{\eta}+1) + \bar{\eta}\gamma$, and $\theta_{I}(\lambda) := \lambda(\gamma+\eta b - \lambda^{2})$. Because $\sin^{2} x + \cos^{2} x = 1$, we get

$$\lambda^{2} p^{2} \left(\lambda^{2} + \bar{\eta}^{2}\right) \left(D^{2} - \rho^{2}\right) k^{2} + 2\lambda p D \left\{-\lambda \theta_{\mathrm{R}}(\lambda) + \bar{\eta} \theta_{\mathrm{I}}(\lambda)\right\} k + \theta_{\mathrm{R}}^{2}(\lambda) + \theta_{\mathrm{I}}^{2}(\lambda) = 0.$$
(21)

Equation (21) can be solved with respect to k,

$$k(\lambda) = \frac{-D\left\{-\lambda\theta_{\rm R}(\lambda) + \bar{\eta}\theta_{\rm I}(\lambda)\right\} \pm \sqrt{D_{\rm d}(\lambda)}}{\lambda p \left(\lambda^2 + \bar{\eta}^2\right) \left(D^2 - \rho^2\right)},\tag{22}$$

where $D_{\rm d}(\lambda)$ is given by

$$D_{\rm d}(\lambda) := -D^2 \Big\{ \lambda \theta_{\rm I}(\lambda) + \bar{\eta} \theta_{\rm R}(\lambda) \Big\}^2 + \rho^2 \left(\lambda^2 + \bar{\eta}^2 \right) \Big\{ \theta_{\rm I}(\lambda)^2 + \theta_{\rm R}(\lambda)^2 \Big\}.$$

Furthermore, from Eq. (20), we have

 $\tau(\lambda, n) =$

$$\frac{1}{\lambda} \left\{ \operatorname{Tan}^{-1} \frac{\bar{\eta}\theta_{\mathrm{R}}(\lambda) + \lambda\theta_{\mathrm{I}}(\lambda)}{\bar{\eta}\theta_{\mathrm{I}}(\lambda) - \lambda\theta_{\mathrm{R}}(\lambda) + \lambda k(\lambda)pD\left(\lambda^{2} + \bar{\eta}^{2}\right)} + n\pi \right\},$$
(23)

where n = 1, 2, ... Thus, the marginal stability curves can be theoretically drawn in (k, τ) space from Eqs. (22) and (23) using λ in the range of $D_d(\lambda) > 0$.

Furthermore, to estimate the stability region, the direction of the roots of $g(i\lambda, \rho) = 0$ crossing the imaginary axis is required. The direction is obtained based on the sign of the following equation:

$$\operatorname{Re}\left[\frac{\mathrm{d}s}{\mathrm{d}k}\right]_{s=i\lambda} = \operatorname{Re}\left[\frac{-ip\lambda(i\lambda+\bar{\eta})(D-\rho e^{-i\lambda\tau})}{-\lambda^2+\gamma-\eta+i\lambda\left\{1+kp\left(D-\rho e^{-i\lambda\tau}\right)\right\}+(i\lambda+\bar{\eta})\left[i2\lambda+1+kp\left\{D-(1-i\lambda\tau)\rho e^{-i\lambda\tau}\right\}\right]}\right].$$
(24)

A positive (negative) sign for Eq. (24) denotes that the roots cross the imaginary axis from left to right (right to left) as coupling strength *k* increases.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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