Design and experimental verification of multiple delay feedback control for time-delay nonlinear oscillators

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Abstract This study aims to show that a multiple delay feedback control method can stabilize unstable fixed points of time-delay nonlinear oscillators. The boundary curves of stability in a control parameter space are derived using linear stability analysis. A simple procedure for designing a feedback gain is provided. The main advantage of this procedure is that the designed controller can stabilize a system even if the controller delay times are long. These analytical results are experimentally verified using electronic circuits.

Keywords delayed feedback control · time-delay nonlinear oscillators · electronic circuits · chaos

1 Introduction

Various methods for controlling chaos have been proposed and applied to real systems, such as electronic circuits, mechanical systems, and chemical reactions [1–5]. One such method, delayed feedback control (DFC), proposed by Pyragas [6], has created considerable interest in the field of nonlinear science [7] and the control theory [8]. The DFC method has been used to stabilize unstable periodic orbits (UPOs) and unstable fixed points (UFPs). Recently, the stabilization of UFPs has been investigated theoretically [9–13], and applied to inverted pendulums [13,14] and laser systems [15].

Multiple delay feedback control (MDFC), in which the controller has two or more time delays, was proposed by Ahlborn and Parlitz [16–18]. The UFPs are stabilized using the MDFC, with an appropriate combination of time delays. This method can achieve stabilization even for long delay times. Therefore, it is useful when controlling fast dynamic systems [16] or when either a computer with an AD/DA converter [19] or a bucket brigade delay (BBD) device [20] (i.e., a series of sample and hold circuits) are used to implement the time delays. The MDFC has been studied in detail from physical [16] and theoretical [17,21,22] viewpoints.

The dynamics of time-delay nonlinear oscillators have gained increasing attention both from the theoretical viewpoint [23,24] as well as for practical applications [25–33]. In particular, as time delays in engineering nonlinear systems such as the metal cutting process [32,33] and the contact rotating systems [34] can induce undesirable oscillations, it would be important to investigate the stabilization of the time-delay induced oscillations. In order to avoid oscillations, the system parameters must be chosen on the basis of stability analysis. If this avoidance requires a major system change, it might have to be abandoned due to practical restrictions. An alternative method is to suppress the oscillations using feedback control. This is a practical method, since it does not require a major system change.

In 1995, Namajūnas *et al.* showed that the DFC method can stabilize UFPs in time-delay chaotic oscillators both theoretically and experimentally [35]. However, it is not easy to provide a procedure for designing the controller parameters, the feedback gain and the controller delay, since the characteristic equation includes the two delay terms (i.e., the oscillator delay and the controller delay). Furthermore, the controller delay must be chosen within several narrow stability intervals. Since these intervals are almost smaller than the oscillator delay, the controller delay should be set to around the oscillator delay or less than it. These

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features make controller design difficult; for example, (a) numerical calculations for solving the characteristic equation including the two delay terms are needed to determine the controller parameters; (b) the controller delay should be set within the narrow stability intervals; (c) the controller delay cannot be longer than around the oscillator delay.

In recent years, these problems of the DFC method have been partially solved. For problem (a), Guan *et al.* provided a systematic procedure for designing the delayed feedback controller on the basis of the Lyapunov–Krasovskii functional approach [36]. However, this procedure cannot be used for long time-delay oscillators. For problem (b), Gjurchinovski and Urumov proposed a time-varying delay feedback control [37]. Although the main advantage of this proposal is that the stability regions in a controller parameter space increases when compared with those for the original delay feedback control, it is not easy to show a procedure for designing the controller and to employ the long controller delay. From problem (c), we notice that, for short time-delay oscillators, the controller delay of the DFC method has to be short. However, it is difficult to realize the short controller delay by the computer or the BBD device, since they have a finite speed operation.

The present study shows that the MDFC method provides answers to such unsolved problems for the stabilization of UFPs in time-delay nonlinear oscillators. The stability boundary curves in a control parameter space are derived using linear stability analysis. A simple procedure for designing the feedback gain and the controller delays, which is based on the observation of the root locus movement of the characteristic equation, is provided. The main advantages of this procedure are as follows: it is guaranteed that the UFPs can be stabilized by the designed controller for any oscillator delay if the oscillator parameters are within a large region in an oscillator parameter space; the controller delays, which retain a proportional relation with a certain bias, can be freely selected. These advantages are useful for the following practical situations: the UFPs in long time-delay oscillators can be stabilized by the designed controller; there is no need to numerically solve the characteristic equation in designing the controller; the controller delays can be arbitrarily chosen. This arbitrarily chosen indicates that the UFPs in short time-delay oscillators can be stabilized even by the slow computer or the slow BBD device. Furthermore, these analytical results are experimentally verified by electronic circuits.

2 Time-delay nonlinear oscillators

Consider a first-order delay differential equation [23],

$$\dot{x} = -\alpha x + f(x_\tau) + u, \tag{1}$$

where $x \in \boldsymbol{R}$ is the state variable and $x_\tau := x(t-\tau)$ represents the delayed one. $\alpha > 0$ is the system parameter. $f : \boldsymbol{R} \to \boldsymbol{R}$ is the nonlinear function. The fixed point is described by $x^* : 0 = -\alpha x^* + f(x^*)$. The multiple delayed feedback control signal $u \in \boldsymbol{R}$ is given by

$$u = k(2x - x_{T_1} - x_{T_2}), \tag{2}$$

where $x_{T_i} = x\left(t - T_i\right),\ i = 1, 2$ are the delayed state variables and $k \in \boldsymbol{R}$ is the feedback gain (see Fig. 1). Note that controller (2) with $T_1 = T_2$ is identical to the original (i.e., single) delayed feedback controller. Oscillator (1) with controller (2) also has the fixed point x^*.

The control system linearized at $x = x^*$ is described by

$$\dot{z} = -\alpha z + \beta z_\tau + k\left\{2z - z_{T_1} - z_{T_2}\right\}, \tag{3}$$

where $z \in \boldsymbol{R}$ is the variation of state x around $x = x^*$, that is, $z := x - x^*$. Here β is the slope of $f(x)$ at x^*, that is, $\beta = \left\{\mathrm{d}f(x)/\mathrm{d}x\right\}_{x=x^*}$. The characteristic equation of linear system (3),

$$g(\lambda) := \lambda + \alpha - \beta e^{-\lambda\tau} - k\left(2 - e^{-\lambda T_1} - e^{-\lambda T_2}\right) = 0, \tag{4}$$

can be used to evaluate the stability of $x = x^*$; the fixed point x^* is stable if and only if all of the roots of Eq. (4) lie in the open left-half of the complex plane. It should be noted that the stability analysis is valid only in the vicinity of x^*; this fact implies that our stability analysis cannot guarantee the global stability of x^*.

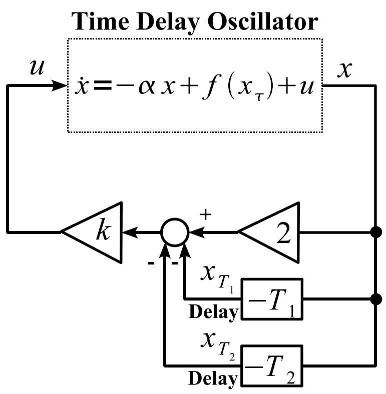
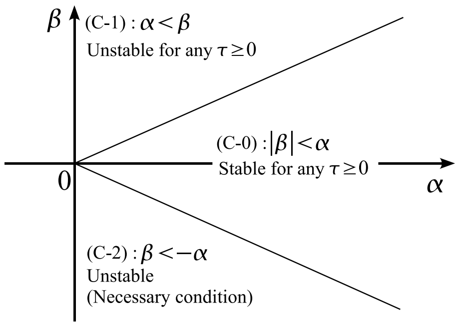
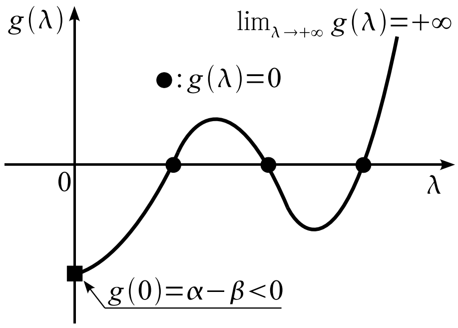


Fig. 1 Block diagram of multiple delayed feedback control of time-delay oscillators.



(a)



(b)

Fig. 2 Sketches: (a) stability domain of x^* without control (i.e., $T_1 = T_2 = 0$) in the α-β plane; (b) function $g(\lambda)$ with the odd number property.

3 Stability analysis

The present paper assumes that oscillator (1) without control (i.e., $T_1 = T_2 = 0$) behaves oscillatory; hence, the fixed point x^* is assumed to be unstable throughout this paper. The stability of x^* is governed by the characteristic equation, $\lambda + \alpha - \beta e^{-\lambda\tau} = 0$. According to the well-known results of the first-order delay differential equation [38], in the oscillator parameter plane as sketched in Fig. 2(a), we know that there are three conditions: (C-0) $|\beta| < \alpha$; (C-1) $\alpha < \beta$; (C-2) $\beta < -\alpha$. Since x^* is stable for any $\tau \geq 0$ under condition (C-0), there is no need to stabilize x^*. Hence, we remove this condition from consideration. In contrast, x^* is unstable for any $\tau \geq 0$ under condition (C-1). Further, the stability of x^* depends on τ under condition (C-2); thus, $\beta < -\alpha$ is a necessary condition for x^* to be unstable. From these arguments, we have to focus only on the two conditions, (C-1) and (C-2).

For oscillator (1) with control (i.e., $T_{1,2} > 0$), it is straightforward to derive a simple instability condition: if $\lim_{\lambda\to+\infty} g(\lambda) = +\infty$ and $g(0) = \alpha-\beta < 0$, then $g(\lambda)$ crosses the positive real axis $\lambda \in [0; +\infty)$ at least once as sketched in Fig. 2(b), that is, there exists at least one positive real root for $g(\lambda) = 0$. This fact yields that, if condition (C-1) is satisfied, the fixed point x^* in oscillator (1) cannot be stabilized by control signal (2) for any $k \in \boldsymbol{R}$ and $T_{1,2} > 0$. Therefore, throughout this paper, we focus only on the fixed points that satisfy condition (C-2). The instability condition (C-1) can be considered as the odd-number property for time-delay oscillators. It should be noted that the previous methods, such as the original DFC [35] and the time-varying DFC [37], never stabilize x^* under condition (C-1) due to their odd-number property. Section 6.3 mentions the property in detail.

Let us estimate the stability region in a control parameter space (T_1, T_2) on the basis of Eq. (4). The stability changes only when at least one root crosses the imaginary axis. To simplify the stability analysis, the roots on the axis are checked. Substituting $\lambda = i\lambda_I$ into Eq. (4), its real and imaginary parts are obtained:

$$\begin{aligned}&-2k + \alpha - \beta\cos\lambda_I\tau + k(\cos\lambda_I T_1 + \cos\lambda_I T_2) = 0,\\&\lambda_I + \beta\sin\lambda_I\tau - k(\sin\lambda_I T_1 + \sin\lambda_I T_2) = 0.\end{aligned} \tag{5}$$

The marginal stability curves are given by roots $T_{1,2}$ of Eqs. (5). The procedure for obtaining the curves is as follows: set a value of T_1; solve Eqs. (5) numerically for T_2 and λ_I; plot (T_1, T_2); change the value of T_1, and then return to the first step. To investigate the direction in which the roots cross the imaginary axis, the sign of the real part of $\mathrm{d}\lambda/\mathrm{d}T_2$,

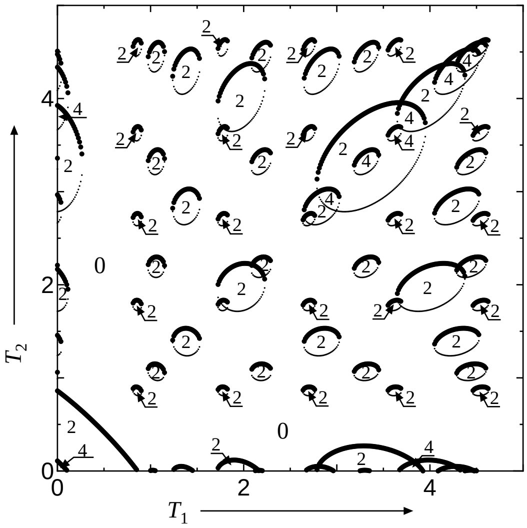
$$\begin{aligned}&\mathrm{Re}\left[\frac{\mathrm{d}\lambda}{\mathrm{d}T_2}\right]_{\lambda=i\lambda_I} =\\&\quad\mathrm{Re}\left[\frac{i\lambda_I k e^{-i\lambda_I T_2}}{1+\tau\beta e^{-i\lambda_I\tau} - k\left(T_1 e^{-i\lambda_I T_1} + T_2 e^{-i\lambda_I T_2}\right)}\right],\end{aligned} \tag{6}$$

is checked, where T_2 and λ_I are the values estimated in the above procedure. With increasing T_2, a positive (negative) value of Eq. (6) corresponds to a root crossing the axis from left to right (right to left).

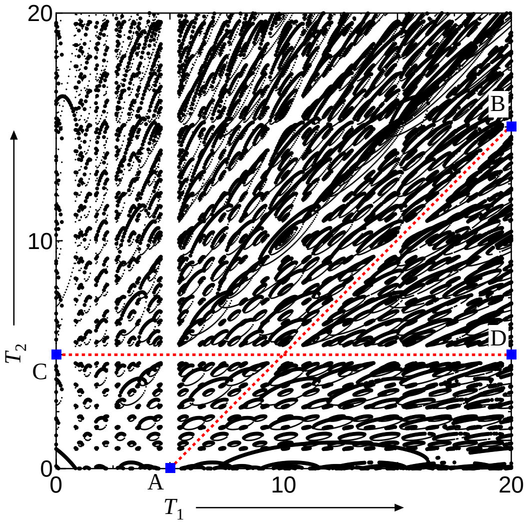
A numerical example illustrates the above procedure. The parameters are fixed at

$$\alpha = 1.0,\ \beta = -3.0,\ \tau = 5.0. \tag{7}$$

Let the feedback gain be fixed at $k \approx -6.4641$; the reason for setting this value will be explained in Sec. 5.2. Figure 3 (a) shows the marginal stability curves estimated by this procedure. The bold (thin) lines express the curves with negative (positive) term (6). These curves separate the parameter space into several regions. We know that when T_2 increases and crosses the bold (thin) line upward, we subtract (add) 2 from (to) the number



(a)



(b)

Fig. 3 Marginal stability curves of x^* ($\alpha = 1.0$, $\beta = -3.0$, $\tau = 5.0$, $k \approx -6.4641$): (a) $T_{1,2} \in [0, 5]$, (b) $T_{1,2} \in [0, 20]$
.

of unstable roots. Obviously, for $T_1 = T_2 = 0$, Eq. (4) is reduced to $g(\lambda) = \lambda + \alpha - \beta e^{-\lambda\tau}$. According to the stability analysis on scalar delayed systems [39], we notice that the number of unstable roots is 4. From these results, the numbers of unstable roots in the parameter space (T_1, T_2) are automatically obtained as shown in Fig. 3 (a). For example, in the regions labeled 2, there exist two unstable roots. Obviously, if (T_1, T_2) are within the region 0, the fixed point x^* is stable. Figure 3 (b) is a large area display of Fig. 3 (a). It must be emphasized that the single delay feedback control method (i.e., $T_1 = T_2$) can stabilize x^* only for a small range (e.g., $T_1 = T_2 \lesssim 5.4$ in Fig. 3(b)). Controller (2) with an appropriate combination of T_1 and T_2, however, can stabilize it over a wide parameter region (e.g., dotted line A-B).

4 Controller design

This section provides a simple procedure for designing controller (2), such that both delay times, T_1 and T_2, are as long as possible. From Fig. 3(b), it can be seen that there are two long stability strips, A-B and C-D[1]. On the strip A-B, T_1 and T_2 can be set to arbitrary long; however, on the strip C-D, T_2 has to be fixed at a finite time $T_2 \approx \tau = 5$. Thus, the strip C-D is not suitable for designing the controller. Figure 3(b) suggests that if T_1 and T_2 keep the relation with τ,

$$T_2 = T_1 - \tau, \ (T_1 > \tau), \tag{8}$$

illustrated by the dotted line A-B, then the fixed point x^* remains stable. Now we provide a design procedure analytically on the assumption that T_1 and T_2 keep relationship (8). In order to derive the procedure, the stability analysis is divided into the following two cases: (i) $T_1 = \tau$ and $T_2 = 0$ (i.e., point A in Fig. 3(b)); (ii) $T_2 = T_1 - \tau$ with $T_1 \geq \tau$ (i.e., dotted line A-B in Fig. 3(b)).

For case (i), substitution of $T_1 = \tau$ and $T_2 = 0$ into Eq. (4) leads $\lambda + \alpha - k + (k - \beta)e^{-\lambda\tau} = 0$. According to the well-known stability condition of the first-order delay differential equation [38], we obtain sufficient condition for $g(\lambda)$ to be stable: $|k - \beta| < \alpha - k$. This condition can be rewritten as (C-3) $\beta < \alpha$ and (C-4) $k < (\alpha + \beta)/2$. Since $\alpha > 0$ and (C-2) $\beta < -\alpha \leftrightarrow \alpha + \beta < 0$ are assumed to be satisfied in the preceding section, we notice that (C-3) and (C-4) $k < (\alpha + \beta)/2 < 0$ always hold. This fact implies that if the gain is chosen as (C-4), then $g(\lambda)$ in case (i) is stable.

For case (ii), T_1 and T_2 are assumed to keep relation (8). Substituting this relation and $\lambda = i\lambda_I$ into Eq. (4) yields

$$\begin{aligned} &k(1 + \cos\lambda_I\tau)\cos\lambda_I T_1 + k\sin\lambda_I\tau\sin\lambda_I T_1 \\ &\qquad\qquad = 2k - \alpha + \beta\cos\lambda_I\tau, \\ &k(1 + \cos\lambda_I\tau)\sin\lambda_I T_1 - k\sin\lambda_I\tau\cos\lambda_I T_1 \\ &\qquad\qquad = \lambda_I + \beta\sin\lambda_I\tau. \end{aligned} \tag{9}$$

[1] Although there are several narrow strips in Fig. 3(b), this paper focuses only on the two typical strips A-B and C-D: diagonal strip and parallel strip to an axis.

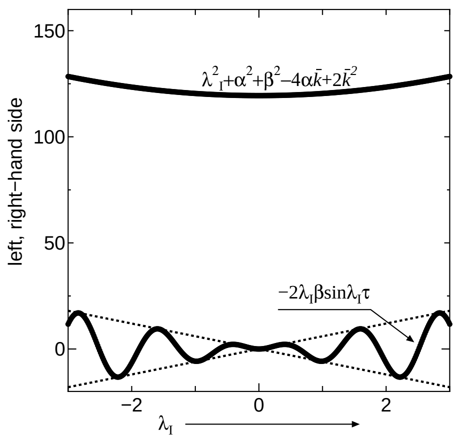


Fig. 4 Sketch of the left and right hand side of Eq. (12) ($\alpha = 1.0$, $\beta = -3.0$, $\tau = 5.0$, $k \approx -6.4641$).

We know that the following two statements are equivalent: the root of $g(\lambda) = 0$ with $T_2 = T_1 - \tau$, $T_1 \geq \tau$ never crosses the imaginary axis; at least one equation of (9) does not hold. Now, we shall employ the second statement in order to design the controller. Both sides of Eqs. (9) are squared and added,

$$\lambda_I^2 + \alpha^2 + \beta^2 - 4\alpha k + 2k^2 = 2h(k)\cos\lambda_I\tau - 2\lambda_I\beta\sin\lambda_I\tau, \tag{10}$$

where $h(k) := k^2 - 2\beta k + \alpha\beta$. Here, the feedback gain is fixed at $k = \bar{k}$: $h(\bar{k}) = 0$. As $\alpha\beta < 0$ holds due to (C-2) $\beta < -\alpha$ and $\alpha > 0$, the equation $h(k) = 0$ has positive and negative roots. However, $k < 0$ has to be held due to the stability condition of case (i), the feedback gain must be set to the negative root:

$$\bar{k} := \beta - \sqrt{\beta^2 - \alpha\beta} < 0. \tag{11}$$

The gain $\bar{k}$ simplifies Eq. (10) to

$$\lambda_I^2 + \alpha^2 + \beta^2 - 4\alpha\bar{k} + 2\bar{k}^2 = -2\lambda_I\beta\sin\lambda_I\tau. \tag{12}$$

If Eq. (12) does not hold, then at least one equation of (9) does not hold. Figure 4 sketches the left-hand and right-hand sides of Eq. (12): the parabolic bold curve is the left-hand side of the equation; the sine wave represents the right-hand side; the dotted lines, $\pm 2|\beta|$, are the upper and lower limits of the sine wave. If the parabolic curve and the dotted lines do not cross, Eq. (12) does not hold. Thus, it is easy to derive the sufficient condition under which the curve and the lines do not cross,

$$\alpha^2 + 2\bar{k}(\bar{k} - 2\alpha) > 0. \tag{13}$$

From inequality (11) (i.e., $\bar{k} < 0$), condition (13) always holds. In addition, $\bar{k}$ denoted by Eq. (11) must satisfy (C-4):

$$\beta - \sqrt{\beta^2 - \alpha\beta} < (\alpha+\beta)/2 \iff \beta - \alpha - 2\sqrt{\beta(\beta-\alpha)} < 0.$$

We notice that the above inequality holds under $\alpha > 0$ and (C-2).

The above arguments are summarized as follows: *For $\beta < -\alpha < 0$, if controller (2) uses $k = \bar{k} := \beta - \sqrt{\beta^2 - \alpha\beta}$ and $T_2 = T_1 - \tau$, then the fixed point x^* of oscillator (1) is stabilized for any long $T_1 \in [\tau, \infty)$.*

The above summary allows us to design controller (2) by the following steps: (step 1) α, β, and τ are given; (step 2) if α and β satisfy $\beta < -\alpha < 0$, then go to the next step, otherwise we have to abandon to design it; (step 3) k is set to $\bar{k} := \beta - \sqrt{\beta^2 - \alpha\beta}$; (step 4) T_1 and T_2 are maintained to satisfy the relation $T_2 = T_1 - \tau$. Even for any long $T_1 \in [\tau, \infty)$, controller (2) designed by this procedure stabilizes the fixed point x^* of oscillator (1).

It must be emphasized that the previous methods never stabilize x^* under condition (C-1) due to their odd-number property and there is no guarantee that they reliably stabilize it under condition (C-2) [35][37]. The MDFC method never stabilize it under condition (C-1); however, it is guaranteed that x^* is *stabilizable* by the designed controller for any oscillator delay $\tau > 0$ under condition (C-2).

The procedure requires the parameter values (i.e., α, β, and τ) to design k, T_1, and T_2; however, in practical situations, we may obtain the values with uncertainty such as the lower and upper limits of the values. Ishii *et al.* provided a procedure to design the single delayed feedback controller for one-dimensional discrete-time chaotic systems [40]. This procedure required only the lower and upper limits of the parameter values; the procedure is simple because the controlled systems do not include time delays and have discrete-time dynamics. On the other hand, it is not easy to provide a simple procedure for our problem, since the controlled system includes three time delays (i.e., τ, T_1, and T_2) and have continuous-time dynamics. The robust design which requires only the values with uncertainty for our problem still remains as an attractive future work.

5 Electronic circuit experiments

In this section, the theoretical results derived in previous sections are confirmed by electronic circuit experiments.

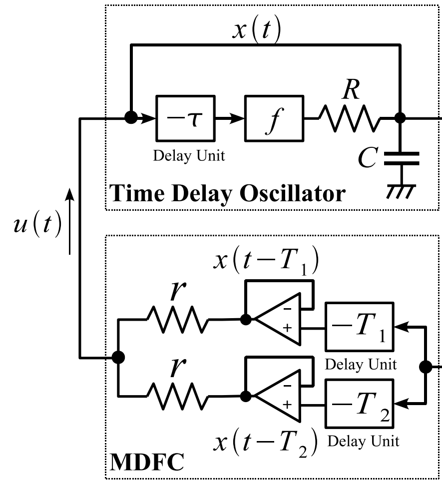


Fig. 5 Schematic drawing of the time-delay nonlinear oscillator with MDFC.

5.1 Time-delay electronic oscillators

The circuit diagram of the time-delay nonlinear oscillator with MDFC is illustrated in Fig. 5. Here, $x(t)$ denotes the voltage of oscillator. The boxes labeled $-\tau$, $-T_1$, $-T_2$ and f are the time-delay units and the nonlinear function unit. The time-delay units are almost the same as previous studies [25, 41]. The circuit diagrams of the delay units and the nonlinear function unit are described in Appendix A. This oscillator is governed by the circuit equation,

$$C\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \frac{1}{R}\left\{f\left(x(t-\tau)\right) - x(t)\right\} + u(t), \tag{14}$$

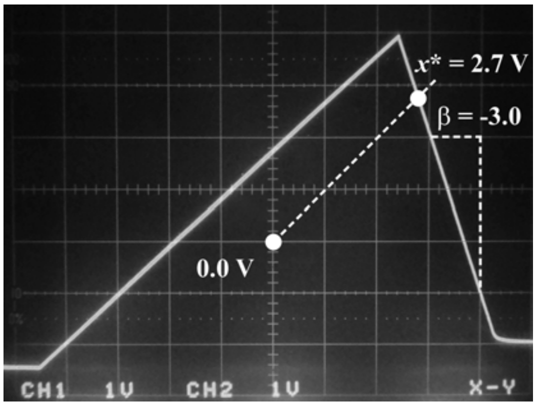
where R and C are a resistor and capacitor, respectively. The control signal $u(t)$ is the current from the MDFC circuit to the oscillator. The MDFC circuit exports the current $u(t)$,

$$u(t) = -\frac{1}{r}\left\{2x(t) - x(t-T_1) - x(t-T_2)\right\}. \tag{15}$$

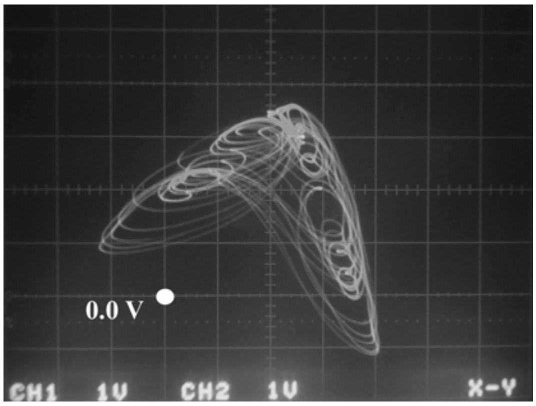
In order to analyze the above circuits, we treat them as the dimensionless oscillator (1) by the following relations,

$$\begin{aligned}&\tilde{t} := \frac{t}{RC},\ \tilde{\tau} := \frac{\tau}{RC},\ \tilde{T}_{1,2} := \frac{T_{1,2}}{RC},\ x := x(\tilde{t}),\\ &\dot{x} := \frac{\mathrm{d}x(\tilde{t})}{\mathrm{d}\tilde{t}},\ x_\tau := x(\tilde{t}-\tilde{\tau}),\ x_{T_{1,2}} := x(\tilde{t}-\tilde{T}_{1,2}),\\ &u := u(\tilde{t}),\ k := -R/r.\end{aligned} \tag{16}$$

These relations indicate that circuit equation (14) with MDFC is identical to oscillator (1) with controller (2) for $\alpha = 1$.



(a)



(b)

Fig. 6 Nonlinear function $f(x)$ and chaotic attractor: (a) x vs. $f(x)$ characteristic (horizontal axis: $x(t)$ (1V/div), vertical axis: $f(x(t))$ (1V/div)); (b) chaotic attractor for parameter set (17) (horizontal axis: $x(t-\tau)$ (1V/div), vertical axis: $x(t)$ (1V/div)).

5.2 Experimental results

The input-output characteristic of the nonlinear function unit, which is similar to that of the well-known Mackey-Glass system [23, 28, 35], is shown in Fig. 6(a). From this figure, the fixed point x^* and the slope $\beta(x^*)$ are estimated as $x^* = 2.7$ V and $\beta(2.7) = -3.0$, respectively. Throughout this paper, the circuit parameters for our experiments are as follows:

$$R = 1.0\ \mathrm{k}\Omega,\ C = 1.0\ \mu\mathrm{F},\ \tau = 5.0\ \mathrm{ms}, \tag{17}$$

where the oscillator exhibits chaotic behavior as shown in Fig. 6(b). The relations (16) indicate that circuit parameters (17) are identical to dimensionless parameters (7). Now let us design the controller according to the design procedure: (step 1) α, β, and τ are given; (step 2) $\alpha = 1$ and $\beta = -3$ satisfy $\beta < -\alpha < 0$, then go to

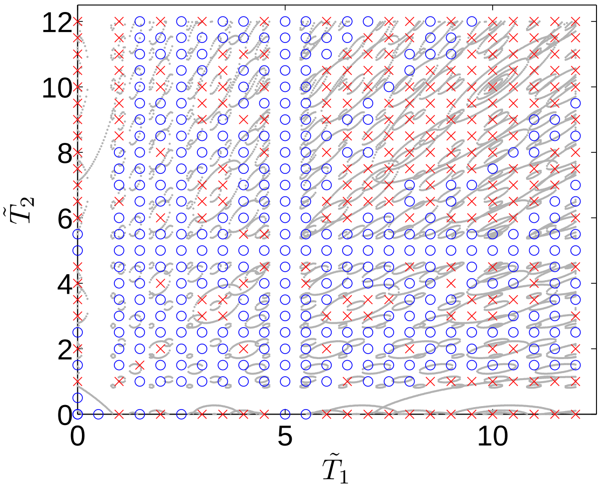


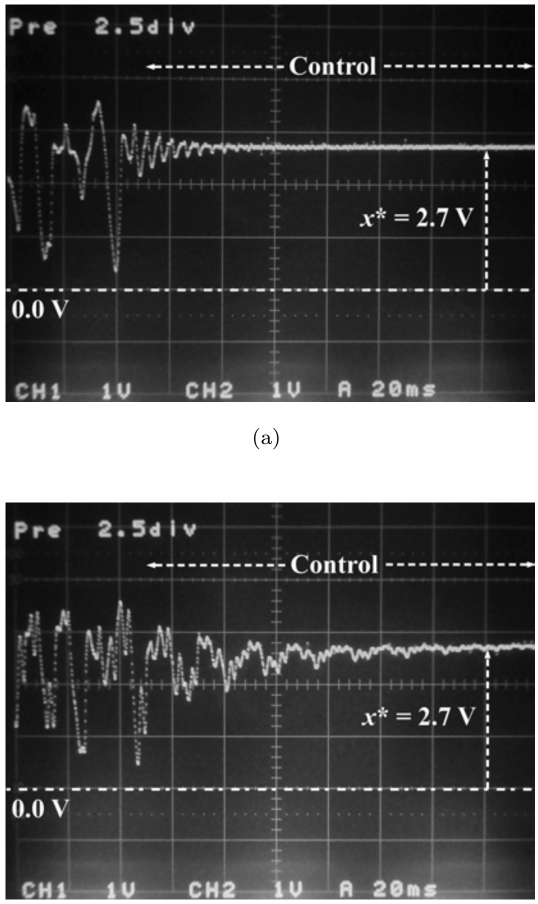
Fig. 7 $\tilde{T}_1 - \tilde{T}_2$ parameter space for comparison of theoretical and experimental results: the symbol ◯ ($\times$) denotes the occurrence (non occurrence) of stabilization experimentally; the gray lines are the stability curves estimated theoretically.

the next step; (step 3) k is set to $\bar{k} := \beta - \sqrt{\beta^2 - \alpha\beta} \approx -6.4641$; (step 4) T_1 and T_2 are maintained to satisfy the relation $T_2 = T_1 - \tau$. The designed gain k is approximately implemented by setting $r = 154\ \Omega$. It is guaranteed that if T_1, T_2, and τ follow relation (8), then the controller delays T_1 and T_2 can be as long as required.

Figures 3 (a) and 3 (b) correspond to the stability curves for parameter set (17) and the designed feedback resistor $r = 154\ \Omega$. The stability regions on the circuit experiments are shown in Fig. 7, where the symbol ◯ ($\times$) denotes the occurrence (non occurrence) of stabilization experimentally[2]. The occurrence or not is judged by the following steps: (i) oscillator (14) without control runs chaotically; (ii) control current (15) starts to flow into oscillator (14) at an arbitrary time; (iii) $x(t)$ converges on x^* within $x^* \pm 0.1$ V; (iv) steps (i)$\sim$(iii) are repeated several times; (v) if we observe the convergence of x^* (i.e., step (iii)) each time, then the symbol ◯ is filled in Fig. 7. Since the system dynamics, in principle, is not influenced by interchange of T_1 and T_2, we checked experimentally only the upper region of the diagonal line $T_2 = T_1$ in Fig. 7. The lower region is the copy of the upper one. From a comparison with the results, it can be stated that the stability region estimated by the numerical procedure roughly agrees with that obtained by our circuit experiments.

The above steps guarantee that the stabilization occurs for several initial states embedded within the chaotic attractor. Therefore, we may say that our stability analysis for the vicinity of x^* is valid for most initial states on the chaotic attractor.

[2] As the small delay, which is less than 1.0 ms, is difficult to realize experimentally, the data on $T_1 = 0.5$ ms and $T_2 = 0.5$ ms cannot be obtained.



(a)

(b)

Fig. 8 Time series data of the circuit voltage $x(t)$ [V] just before and just after the control: (a) parameter set (A) ($T_1 = 5.0$ ms and $T_2 = 0.0$ ms); (b) parameter set (B) ($T_1 = 20.0$ ms and $T_2 = 15.0$ ms). Horizontal axis: t (20 ms/div); vertical axis: x (1 V/div).

The time series data of the electronic oscillator controlled by MDFC is shown in Fig. 8(a). The control signal corresponding to point A in Fig. 3 (b) ($T_1 = 5.0$ ms and $T_2 = 0.0$ ms) is applied to the oscillator. The oscillator without control behave chaotically and then $x(t)$ converges on x^*. Figure 8(b) shows the time series data at point B in Fig. 3 (b) ($T_1 = 20.0$ ms and $T_2 = 15.0$ ms). This figure shows that stabilization occurs even with long delay times.

These experiments employ popular-priced circuit devices, which have an error of several percent. Thus, the designed controller inevitably has much more errors. However, as shown in Fig. 7, the controller works well on the circuit experiments. This fact experimentally verifies that the stabilization is robust to external noise and parameter uncertainty.

6 Discussion

6.1 Competition with other methods

Let us investigate the control performance of the MDFC method for stabilizing UFPs in time-delay oscillators. It is well known that a tracking filter and the original DFC [35] are the typical control methods for stabilizing time-delay oscillators. This subsection compares the MDFC method with these typical methods. The filter is described by

$$u = k(x - v), \ \frac{\mathrm{d}v}{\mathrm{d}t} = \omega_c(x - v),$$

where v is the additional variable, k is the feedback gain, and ω_c is the parameter. The original DFC is given by Eq. (2) with $T_1 = T_2$. Figure 9 (a) shows the largest real part of the roots $\mathrm{Re}[\lambda_{\max}]$ of the characteristic equation for the fixed point x^* controlled by the tracking filter. The parameter ω_c corresponds to the cutoff frequency. In order to stabilize x^*, the parameter should be within an interval $\omega_c \in (0, 1.235)$. The largest real part for the original DFC with $T_1 = T_2$ is shown in Fig. 9 (b). The controller delay time $T_1 = T_2$ should be chosen from several narrow intervals for the stabilization. Figure 9 (c) illustrates the largest real part for the MDFC on the dotted line A-B in Fig. 3(b). It can be seen that the largest real part never exceeds zero for any $T_2 > 0$. Thus, if the two controller delays retain the proportional relation, $T_2 = T_1 - \tau$, then the controller delays can be arbitrarily chosen.

The convergence speed of the controlled orbit in the vicinity of x^* depends on $\mathrm{Re}[\lambda_{\max}]$. From a practical point of view, it is desirable to reduce $\mathrm{Re}[\lambda_{\max}]$ to improve the convergent performance. It is useful to design the optimal control parameters which get the best convergent performance; this optimal control problem is considered as an important future work.

6.2 Previous studies related to our results

Although, to the authors' knowledge, there have been few efforts to investigate the stabilization of UFPs in time-delay nonlinear oscillators using the MDFC method, some previous methods for time-delay oscillators are related to our results. These related studies with the exception of the studies [35–37] mentioned in Section 1 are reviewed below.

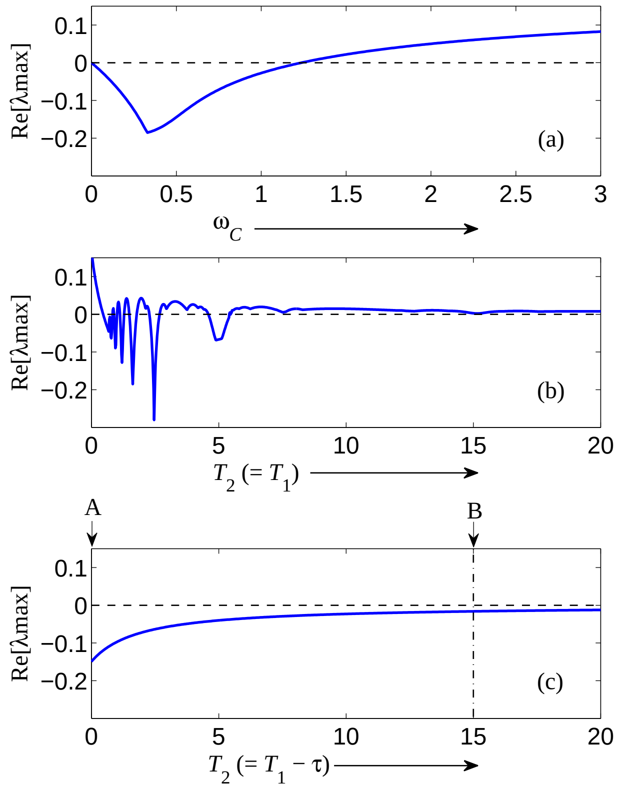


Fig. 9 Largest real part of the roots $\mathrm{Re}[\lambda_{\max}]$ of the characteristic equation for the fixed point x^*: (a) the tracking filter; (b) the original DFC with $T_1 = T_2$; and (c) the MDFC with $T_2 = T_1 - \tau$. The parameters are the same as the numerical and experimental results in previous sections ($\alpha = 1.0$, $\beta = -3.0$, $\tau = 5.0$, $k \approx -6.4641$).

Blyuss *et al.* analyzed the stability of UFPs in time-delayed pendulum-mass-spring-damper systems controlled by the DFC method [42]. Xu *et al.* provided a delay-dependent condition, which is described by a linear matrix inequality, for the stabilization of UFPs in time-delay nonlinear oscillators controlled by the DFC method [43]. Rezaie *et al.* applied a dynamic DFC method to the problem of Hopf bifurcation control for time-delay nonlinear oscillators [44]. Vasegh and Sedigh analyzed the stability of UPOs in time-delay oscillators controlled by the DFC method [45,46]. Our previous study showed that UFPs in simple two-dimensional oscillators without time delay can be stabilized using diffusive connections with two long time delays [47]. If the two delay times retain a proportional relation with a certain bias, the stabilization occurs independent of the delay times and the network topology. Our present study considers the specific case of a single oscillator; however, a time-delay oscillator is used instead of a two-dimensional one.

6.3 Odd number property

It was well known that the DFC method had a crucial disadvantage that it never stabilizes UPOs and UFPs, which have the odd number property [48–50]. This property for UPOs has been refuted recently [51–53]; whereas, that for UFPs is valid [12,13]. To overcome this property for UFPs, an observer-based controller [54] and a dynamic controller [55], both of which can be designed by a systematic procedure, have been proposed. Furthermore, an adaptive controller based on a conventional low-pass filter, which is the same as the tracking filter, also has the odd number property; however, an unstable filter was proposed to overcome this property [56,57]. The multiple DFC method [5,21] and the time-varying-delay method [58] also have this property [5,21]. As in Sec. 3, the MDFC for time-delay oscillators also has this property. Although this disadvantage could be overcome using the above techniques, we do not discuss it in detail because it will lead to digression from our main topic.

7 Conclusion

This paper demonstrated that the MDFC method can stabilize UFPs in time-delay nonlinear oscillators. The simple procedure for designing the feedback gain and the controller delays was provided. The main advantage of this procedure is that if T_1, T_2, and τ maintain the relation, the fixed point can be stabilized by long controller delays T_1 and T_2. The stability analysis and the design procedure were experimentally verified by electronic circuits.

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A Electronic circuits

The delay unit circuit shown in Fig. 10 imports the voltage $x(t)$ and exports the delayed voltage $x(t-\tau)$. This circuit consists of the four parts: delay part, input part, output part, and low-pass filter. The delay part employs the main device MN3011 (Panasonic) as a bucket brigade delay-line device. This device exports the delayed voltage. The delay time (1.0−20.0 ms) depends on the function generator frequency (10 − 240kHz). It should be noted that this device can delay the voltage only in a range 3.5 − 6.0V. However, the voltage of oscillator (14) is not within this range. To solve this problem, the input and output parts are added to the delay unit. The input part transforms the voltage $x(t)$ into the range. Since the output part has the opposing function, the output part exports the delayed voltage within the original range. As the delay device has a high frequency switch operation, its output includes the high-frequency noise. The low-pass filter removes the noise from the delayed voltage.

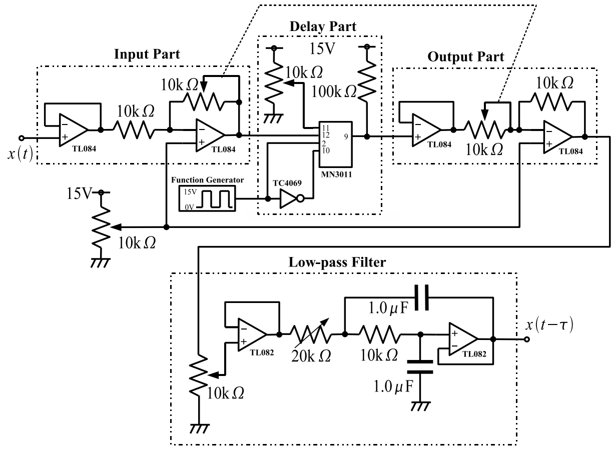

Fig. 10 Delay unit circuit.

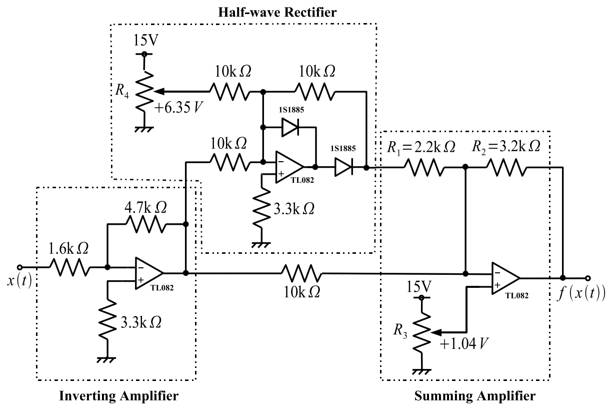

Fig. 11 Nonlinear function circuit.

The nonlinear unit circuit is shown in Fig. 11. This circuit consists of the inverting amplifier, the half-wave rectifier, and the summing amplifier. The inverting amplifier inverts and amplifies the input voltage $x(t)$. The half-wave rectifier works as the piecewise linear function which has a break point. The summing amplifier adds up the output voltages of the inverting amplifier and the half-wave rectifier. As a result, the relation between the input $x(t)$ and the output $f(x(t))$ is obtained as shown in Fig. 6(a). The break point at the peak can be adjusted by R_1 and R_2. The nonlinear function can be shifted up and down by changing R_3.

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