Optimal operation and supply chain coordination in a closed-loop supply chain with loss-averse attitude

メタデータ	言語: English 出版者: 公開日: 2020-10-29 キーワード (Ja): キーワード (En): 作成者: Kusukawa, Etsuko, Paku, Yuri メールアドレス: 所属:
URL	http://hdl.handle.net/10466/00017118

Optimal operation and supply chain coordination in a closed-loop supply chain with loss-averse attitude

Etsuko Kusukawa* and Yuri Paku

Course of Electrical and Electronic Systems,
College of Engineering,
Osaka Prefecture University,
Sakai, Osaka, 599-8531, Japan
Email: kusukawa@eis.osakafu-u.ac.jp
Email: sxb01153@edu.osakafu-u.ac.jp
*Corresponding author

Abstract: From social concerns about 3R activity worldwide, it is urgently needed to construct a closed-loop supply chain (CLSC). It is necessary to consider the negative range of profit due to the uncertain product demand for the optimal operation in the CLSC. This paper proposes the optimal operation for a CLSC with loss-averse attitude of the decision maker's profit. A retailer collects used products from customers by paying an incentive, a manufacturer produces recycled parts with acceptable quality and compensates a part of the collection cost to the retailer. This paper discusses some loss-averse attitudes (LAAs) of the decision maker's profit. Using loss-averse analysis, the optimal operations regarding the product order quantity, the unit collection incentive, and the lower limit of quality level under decentralised CLSC (DCLSC) and integrated CLSC (ICLSC) are determined. The analysis numerically investigates the effects of LAAs on the optimal operations and benefits of profit sharing approaches under ICLSC.

Keywords: supply chain management; closed-loop supply chain; uncertain demand; loss aversion; prospect theory; profit sharing.

Reference to this paper should be made as follows: Kusukawa, E. and Paku, Y. (2018) 'Optimal operation and supply chain coordination in a closed-loop supply chain with loss-averse attitude', *Asian J. Management Science and Applications*, Vol. 3, No. 3, pp.252–278.

Biographical notes: Etsuko Kusukawa is an Associate Professor at Course of Electrical and Electronic Systems, College of Engineering in Osaka Prefecture University, Japan. She earned her PhD in Industrial Engineering in 2001 from Osaka Prefecture University in Japan. Her research interests are supply chain management including closed-loop supply chain, green supply chain, dual supply chain and quality control. She has published research articles in *Industrial Engineering & Management System* and *International Journal of E-Business Development*.

Yuri Paku earned her Bachelor's degree in Electrical and Electronic Engineering in 2017 from Osaka Prefecture University, Japan. Her research interest is supply chain management including closed-loop supply chain.

1 Introduction

From social concerns about reduce-reuse-recycle (3R) activity for resource saving and environment problem worldwide, it is urgently-needed to construct a new supply chain management which incorporates reverse chains/logistics into traditional forward chains/logistics (Souza, 2013; Govindan et al., 2014; Schenkel et al., 2015; Govindan et al., 2017). The traditional forward chains consist of the flows from procurement of new materials through production of new products to sales of the products. The reverse chains are composed of the flows from collection of used products through recycling parts from the used products to reuse the recycled parts. Also, a supply chain which organises the forward chains and the reverse chains has been called a closed-loop supply chain (CLSC), a reverse supply chain, or a green supply chain (Govindan et al., 2013; Govindan et al., 2014; Gurtu et al., 2015; Watanabe et al., 2013; Watanabe and Kusukawa, 2014). This paper calls the supply chain with the forward chains and the reverse chains as a CLSC.

Remanufacturing is an approach used by many companies from different industries such as Dell, Hewlett-Packard (HP), IBM, Kodak and Xerox. An example of the different remanufactured products includes the following: photocopies, cellular telephones, single-use cameras, car's engines and transmissions and retreaded tires (Radhi and Zhang, 2016). It is necessary to take some measures and policies in order to promote 3R activities in the CLSC.

In order to conduct and promote 3R activities, in general, it is considerable for the system operation in a CLSC to face the uncertainty in demand of a single type of products and a variety of qualities of a single type of used products collected from customers/a market. For the above topics, many previous studies regarding a CLSC determined the optimal operation in the CLSC so as to maximise the expected profit or minimise the expected cost under above uncertainties. There are some overviews of above topics (Souza, 2013; Govindan et al., 2013; Stindt and Sahamie, 2014; Schenkel et al., 2015; Bazan et al., 2016; Sundari and Vijayalakshmi, 2016; Govinda et al., 2017). The effects of a collection incentive contract on promotion of the collection and recycling activities in a CLSC with a retailer and a manufacturer were discussed in Watanabe et al. (2013) and Watanabe and Kusukawa (2014). One is the incentive which is paid from a retailer to customers, the other is the incentive which is paid a manufacturer from a retailer. Two types of incentive enabled to promote the collection and the recycling of used products. The optimal operations under decentralised CLSC (DCLSC) and the integrated CLSC (ICLSC) were made for the product order quantity, the unit collection incentive of used products and the lower limit of quality level for recycling used products. Yamaguchi and Kusukawa (2018) discussed a CLSC consisting of a buyer (a retailer), a manufacturer, and a recycler. They focused on the effects of:

1 the uncertainty in product demand

2 the uncertainty in collection quantity of used products

3 a variety of quality of the parts extracted from used products on the optimal operation in a CLSC.

They incorporated a flexible ordering policy (FOP) that the buyer's order quantity is between the minimum order quantity and the maximum order quantity with the manufacturer into the optimal operations under DCLSC and ICLSC. Thus, previous papers mentioned above determined the optimal operation in a CLSC so as to maximise the expectation of the decision maker's profit. However, the decision making so as to maximise only the expectation of the decision maker's profit is insufficient because it ignores the loss aversion of the decision maker's profit due to the uncertain product demand.

Thus, for decision makers, the uncertain product demand is a risk in the operation of a CLSC which causes the negative profits of the decision makers. It is necessary to discuss how the loss aversion of the decision makers' profit loss due to the uncertain product demand affects the optimal operation in the CLSC. For example, some decision makers may be averse to the situation where their profits become negative when the product demand is less than the product order quantity (LAE: loss-averse attitude for excess quantity) (Wang and Webster, 2007), some decision makers may be averse to the situation where their profits become negative when the product demand is larger than the product order quantity (LAS: loss-averse attitude for shortage quantity) and the other ones may be averse to both situations mentioned above (LA) (Wang and Webster, 2009; Hu et al., 2016). Thus, the decision making so as to maximise only the expectation of the decision maker's profit is insufficient because it ignores the loss aversion of the decision maker's profit due to the uncertain product demand. Regarding topic, Wang and Webster (2009) discussed a loss-averse newsvendor problem considering the excess cost and shortage cost (LA), and determined the optimal order quantity as to the degrees of loss aversion regarding LA. Wang and Webster (2007) discussed a forward decentralised supply chain consisting of a loss-averse retailer (LAE) with the excess cost, but without shortage cost and the risk-neutral manufacturer. They showed that: the loss-averse newsvendor with LAE ordered less than a risk-neutral retailer and the optimal order quantity decreases the degree of loss aversion regarding LAE. Also, they discussed supply chain coordination using gain/loss-sharing-and-buyback contract between the retailer and the manufacturer. Hu et al. (2016) studied discussed a forward three-echelon supply chain consisting of a loss-averse retailer (LAE) with the excess cost and shortage cost (LA), a loss-neutral distributor, and a loss-neutral manufacturer. They derived the three players' optimal policies, and find that compared with a loss-neutral scenario, the loss-averse retailer gains fewer profits and a lower utility. They showed that the loss-averse retailer orders less when it faces a high overage cost and orders more when it faces a high shortage cost, compared with the loss-neutral scenario. They discussed supply chain coordination via revenue sharing contracts. However, previous papers mentioned above did not discuss the optimal operation and supply chain coordination under a CLSC.

Thus, little research has discussed the optimal operation in a CLSC simultaneously considering:

1 the optimal operation in a CLSC using risk analysis considering all loss aversion attitudes (LAE, LAS, LA) of the decision maker's profit

2 a variety of qualities of used products

3 collection effort of used products as incentive

4 compensation of collection cost of used products

5 profit sharing to shift profitably to that under ICLSC from the optimal operation under DCLSC as supply chain coordination.

Regarding 1, it is necessary to discuss how four types of loss-averse attitude, N, LAE, LAS, LA, of the decision maker's profit loss due to the uncertain product demand affect the optimal operation in a CLSC. This paper attempts to incorporate above issues 1–5 into optimal operation for a CLSC. Table 1 shows the comparison of the contributions of this paper with those of previous papers.

Table 1 The comparison of the contributions of this paper with those of previous papers

Assumptions	*Model*								
			Closed-loop Supply chain						
Authors	*Newsvendor problem*	*Forward supply chain*	*Collection effort of used products*	*Compensation of collection of used products*	*Quality of parts*	*Risk analysis loss aversion*	*Stackelberg model*	*Supply chain coordination*	*Number of supply chain members*
Watanabe and Kusukawa (2014)			✓	✓	✓		✓	✓	2
Yamaguchi and Kusukawa (2018)			✓	✓	✓		✓	✓	3
Wang and Webster (2007)	✓	✓				✓ LAE	✓	✓	2
Wang and Webster (2009)	✓					✓ LA			1
Hu et al. (2016)	✓	✓				✓ LA	✓	✓	3
This paper			✓	✓	✓	✓ LAE LAS LA	✓	✓	2

This paper clarifies the optimal operation in a CLSC considering the loss-averse attitude of the decision maker. This paper uses the loss-averse analysis based on prospect theory (Wang and Webster, 2009) regarding the uncertain product demand. The CLSC consists of a retailer and a manufacturer. A contract for cooperation regarding collection of used products is concluded between both members. The retailer collects used products from consumers by paying an incentive, and then delivers the collected used products to a manufacturer. The retailer orders a single type of products from the manufacturer and sells them in a market. The manufacturer produces the same quantity of products ordered by the retailer, using recyclable parts with acceptable quality in used products and compensates a part of the collection cost of used products to the retailer as to the quantity of the recycled parts. This paper discusses four types of loss-averse attitude of the decision maker's profit loss, N, LAE, LAS, LA, caused by the uncertain product demand. Considering the negative region of the decision maker's profit under DCLSC and ICLSC as to the decision maker's loss-averse attitude, the theoretically optimal decisions for the product order quantity, the unit collection incentive, and the lower limit of quality level, under DCLSC and ICLSC are made. DCLSC maximises the expected utility functions of the decision makers, based on the Stackelberg game (Nagarajan and Sosic, 2008; Watanabe et al., 2013; Watanabe and Kusukawa, 2014). ICLSC maximises the expected utility function of the whole system. In general, the optimal operation under ICLSC is more desirable than that under DCLSC from the aspect of the total optimisation. Therefore, profit sharing as supply chain coordination is introduced into the optimal operation under ICLSC to encourage all members to shift to the optimal operation under ICLSC as to loss-averse attitude of the decision maker from those under DCLSC, by guaranteeing the increments of all members' expected profits under the optimal operation of ICLSC. Concretely, two profit sharing approaches as supply chain coordination are discussed:

1 profit ratio of each member under ICLSC (Watanabe et al., 2013)

2 profit investment ratio of each member under ICLSC (Watanabe and Kusukawa, 2014).

The numerical analyses in this paper investigates how:

1 four types of loss-averse attitude of the decision maker regarding the uncertain product demand

2 quality of recyclable parts

affect the optimal operations under DCLSC and ICLSC. Also, the optimal operation under DCLSC is compared with that under ICLSC as to each loss-averse attitude of the decision maker. Finally, it is shown how two profit sharing approaches as supply chain coordination:

1 profit ratio of each member

2 profit investment ratio of each member

can bring benefit to all members under ICLSC when the optimal operation shifts from DCLSC to ICLSC. The rest of this paper is organised as follows. Section 2 provides notation used in mathematical expressions of this paper. Section 3 provides model descriptions. Section 4 formulates the profits and the expected profits in a CLSC. Section 5 discusses the loss-averse analysis based on prospect theory in a CLSC. Sections 6 and 7 describe the decision procedures for the optimal operations under DCLSC and ICLSC. Section 8 discusses profit sharing as supply chain coordination. Section 9 conducts numerical analyses, shows the results of the optimal operation, and describes managerial insights. Section 10 summarises conclusions and future researches for this paper.

2 Notation

- N: risk neutral attitude.
- LAE: loss-averse attitude for excess quantity.
- LAS: Loss-averse attitude for shortage quantity.
- LA: loss-averse attitude for excess quantity and shortage quantity.

- $i \in \{\text{N, LAE, LAS, LA}\}$: a set of loss-averse attitude of the decision maker.
- λ^E: the degree of LAE ($\lambda^E > 1$).
- λ^S: the degree of LAS ($\lambda^S > 1$).
- $j \in \{\text{R, M, S}\}$: a set of members in the CLSC (R: a retailer, M: a manufacturer, S: the whole system).
- $k \in \{\text{D, I}\}$: a set of methods of decision making for a CLSC(D: the DCLSC, I: the ICLSC).
- Q: product order quantity.
- t: the unit collection incentive (collection cost) of used products.
- u: lower limit of quality level to remanufacture recyclable parts after disassembly of used products, referred to simply as lower limit of quality level ($0 \leq u \leq 1$).
- $A(t)$: collection quantity of used products for t.
- $R(t)$: compensation per the recycled part which a manufacturer pays to a retailer who paid the unit collection incentive t to collect the quantity of used products $A(t)$ [this indicates that a manufacture compensates a part of collection cost of used products paid by a retailer as to the quantity of recycled parts in collection quantity of used products $A(t)$].
- α: the degree of compensation for t.
- t_U: upper limit of t.
- c_t: delivery cost per unit of used products collected from customers, to a manufacturer.
- c_a: disassembly and inspection cost per unit of used products.
- ℓ: quality level of recyclable parts ($0 \leq \ell \leq 1$) after disassembly of used products.
- $g(\ell)$: probability density function (pdf) of quality level ℓ.
- $c_r(\ell)$: the unit remanufacturing cost of recyclable parts with quality level ℓ.
- c_d: the unit disposal cost of un-recycled parts.
- c_n: the unit procurement cost of new parts.
- c_m: the unit production cost of products.
- m_a: the unit margin obtained from wholesales of products.
- w: the unit wholesale price of products, referred to simply as the unit wholesale price.
- p: sales price per unit of products.
- s: shortage penalty cost per unit of products whose demand is unsatisfied.
- h_r: the unit inventory holding cost of unsold products.

- x: demand of product in a market.
- $f(x)$: probability density function of demand x.
- $F(x)$: cumulative distribution function of demand x.
- $\pi_j(Q, t, u)$: profit of member $j \in \{R, M, S\}$.
- $E[\pi_j(Q, t, u)]$: the expected profit of member $j \in \{R, M, S\}$.
- $EU_j^i(Q, t, u)$: the expected utility function of a member $j \in \{R, M, S\}$ with loss-averse attitude $i \in \{N, LAE, LAS, LA\}$.
- Q_k^i: the optimal order quantity under a method of decision-making $k \in \{D, I\}$ with loss-averse attitude $i \in \{N, LAE, LAS, LA\}$.
- t_k: the optimal unit collection incentive under a method of decision-making $k \in \{D, I\}$.
- u_k: the optimal lower limit of quality level under a method of decision-making $k \in \{D, I\}$.

3 Model descriptions

A CLSC with a retailer and a manufacturer is considered. The CLSC has an operational flow from collection of a single type of used products, through remanufacturing a single type of recycled parts from the used products, to sales of a single type of products from the recycled parts and new parts in a single period. A single type of products such as consumer electronics (mobile phone, personal computer), semiconductor, and electronic component is sold in a market.

3.1 Operational flows of a CLSC

Figure 1 shows the operational flow of the CLSC addressed in this paper.

1. The retailer pays the unit collection incentive t of used products (collection incentive) to collect them from customers and delivers the collected quantity $A(t)$ of used products for t to the manufacturer at the unit cost c_t [(i)–(iii) in Figure 1].
2. The manufacturer disassembles the used products to a single type of recyclable parts and inspects all the recyclable parts at the unit cost c_a. After the disassembly, the manufacturer classifies the recyclable parts into the quality level ℓ ($0 \le \ell \le 1$) [(iv) in Figure 1]. The manufacturer remanufactures recycled parts from all the recyclable parts with quality level ℓ more than the lower limit of quality level u ($0 \le u \le 1$) at the unit remanufacturing cost $c_r(\ell)$ [(v) in Figure 1]. The manufacturer disposes all the recyclable parts with lower quality level than u at the unit cost c_d [(vi) in Figure 1].
3. The manufacturer craves to increase the quantity of recycled parts by remanufacturing more recyclable parts with higher quality level than lower limit of quality level u as parts from the viewpoints his profit and environment. So, he pays

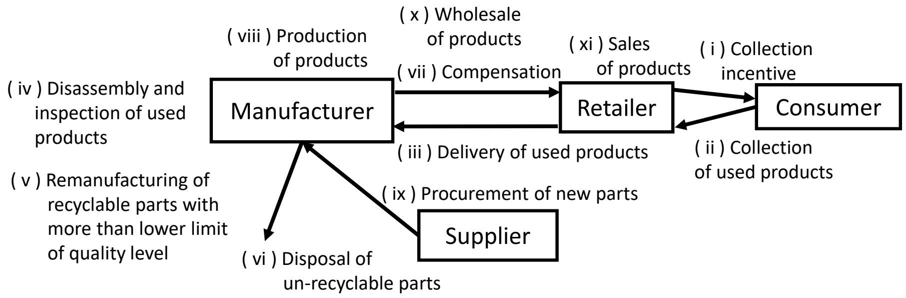
the compensation to the retailer for the cooperation to collect more quantity of the used products [(vii) in Figure 1]. Concretely, the manufacture compensates a part of collection cost of used products to the retailer who paid the unit collection incentive t to collect the quantity $A(t)$ of the used products as to the quantity of recycled parts, which used recyclable parts with higher quality level than u, in collection quantity of used products $A(t)$. That is, the manufacture pays the compensation $R(t)$ per recycled part to the retailer who paid the unit collection incentive t to collect the quantity $A(t)$ of the used products.

4 The retailer places an order of the products Q with the manufacturer. The manufacturer produces the products at the unit cost c_m to satisfy the order quantity Q from the retailer [(viii) in Figure 1]. If the required quantity of parts to produce Q is unsatisfied with the quantity of the recycled parts, the manufacturer procures the required quantity of new parts at the unit cost c_n from external supplier [(ix) in Figure 1].

5 The manufacturer sells the quantity Q of the products to the retailer at the unit wholesale price w [(x) in Figure 1].

6 The retailer sells the products in a market at the unit sales price p during a single period [(xi) in Figure 1]. The unit inventory holding cost h_r is charged only based on the ending inventory of the period, while the unit shortage penalty cost s for the unsatisfied product demand is incurred.

Figure 1 The operational flow of CLSC addressed in this paper



3.2 Model assumptions

1 The collection quantity of used products, $A(t)$ varies according to the unit collection incentive t. In general, the higher t is, the more a retailer can collect used products from customers. From the aspect of a retailer's profit, the feasible range of t is $0 \le t \le t_U < p$. This paper assumes that the collection quantity $A(t)$ is not enough to satisfy the order quantity Q of the products even if the retailer pays the upper limit t_U of the unit collection incentive t. Here, u is the lower limit of quality level to remanufacture recyclable parts after disassembly of used products ($0 \le u \le 1$). Under this assumption, the shortage of parts to produce Q of the products occurs. In this case, this paper assumes that the manufacturer procures the required quantity of new parts at the unit cost c_n from external supplier.

2 A single type of recyclable parts is extracted from the unit of used products. The manufacturer remanufactures a single type of parts, using the single type of recyclable parts with higher quality level than u.

3 The variability of quality level ℓ of the recyclable parts is modelled as a probability distribution with the probability density function $g(\ell)$.

4 The unit remanufacturing cost $c_r(\ell)$ to produce a recycled part from a recyclable part with ℓ varies as to the quality level ℓ $(0 \le \ell \le 1)$ of recyclable part. The lower quality level of recyclable part is, the higher the unit remanufacturing cost $c_r(\ell)$ is. Here, $\ell = 0$ indicates the worst quality level of the recyclable part, while $\ell = 1$ indicated the best quality level of recyclable part. Therefore, $c_r(\ell)$ is a monotone decreasing function with respect to quality level ℓ of a recyclable part.

5 The quality of each recycled part produced from recyclable parts is as good as that of products new parts.

6 The unit wholesale price w is calculated from the unit procurement cost c_n of new parts, the unit production cost c_m of products, and the unit margin m_a obtained from wholesales of products.

7 The variability of the product demand x is modelled as a probability distribution with the probability density function $f(x)$.

4 The profits and the expected profits of a CLSC

From Section 2, the retailer's profit consists of the collection incentive of used products from customers, the delivery cost of used products to a manufacturer, the procurement cost of products, the compensation revenue for the retailer's cooperation of the collection of used products from a manufacturer, the product sales, the inventory holding cost of unsold products, and the shortage penalty cost for unsatisfied product demand in a market. The retailer's profit for the product order quantity Q, the unit collection incentive t, and the lower limit of quality level u is formulated as

$$\begin{aligned}\pi_{\mathrm{R}}\left(Q,t,u\right) &= -tA(t) - c_t A(t) - wQ + R(t)\int_u^1 A(t)g(\ell)d\ell \\ &\quad + \begin{cases} px - h_r(Q-x) & (0 \le x \le Q) \\ pQ - s(x-Q) & (Q \le x) \end{cases}\end{aligned} \tag{1}$$

Taking expectation of the product demand x in equation (1), the retailer's expected profit for Q, t and u is derived as

$$\begin{aligned}E\left[\pi_{\mathrm{R}}(Q,t,u)\right] &= -tA(t) - c_t A(t) + R(t)\int_u^1 A(t)g(\ell)d\ell \\ &\quad + \int_0^Q \left\{px - h_r(Q-x)\right\} f(x)dx + \int_Q^\infty \left\{pQ - s(x-Q)\right\} f(x)dx - wQ\end{aligned} \tag{2}$$

From Section 2, the manufacturer's profit consists of the product wholesales, the disassembly and the inspection costs of the used products, the remanufacturing cost of recyclable parts, the compensation cost to the retailer, the disposal cost of the un-recycled parts, the procurement cost of new parts, and the production cost of products. Thus, the manufacturer's profit is unaffected by the product demand x. The manufacturer's expected profit for Q, t and u is equal to the manufacturer's profit for Q, t and u, that is

$$\begin{aligned} E\left[\pi_{\mathrm{M}}(Q,t,u)\right] &= \pi_{\mathrm{M}}(Q,t,u) \\ &= wQ - R(t)\int_u^1 A(t)g(\ell)d\ell - c_a A(t) - \int_u^1 A(t)c_r(\ell)g(\ell)d\ell \\ &\quad - c_d\int_0^u A(t)g(\ell)d\ell - c_n\left\{Q - \int_u^1 A(t)g(\ell)d\ell\right\} - c_m Q \end{aligned} \tag{3}$$

Under model assumptions 3.2 (1), the sixth term in equation (3) $-c_n\{Q - A(t)\int_u^1 g(\ell)d\ell\}$ (the procurement cost of new parts) is considered in the expected manufacturer's profit.

The whole system's profit is obtained as the sum of the retailer's profit and the manufacturer's profit. The whole system's profit for Q, t and u is obtained as

$$\begin{aligned} \pi_{\mathrm{S}}(Q,t,u) &= \pi_{\mathrm{R}}(Q,t,u) + \pi_{\mathrm{M}}(Q,t,u) \\ &= -tA(t) - c_t A(t) - c_a A(t) - A(t)\int_u^1 c_r(\ell)g(\ell)d\ell \\ &\quad - c_d A(t)\int_0^u g(\ell)d\ell - c_n\left\{Q - A(t)\int_u^1 g(\ell)d\ell\right\} \\ &\quad - c_m Q + \begin{cases} px - (Q-x)h_r & (0 \le x \le Q) \\ pQ - (x-Q)s & (Q \le x). \end{cases} \end{aligned} \tag{4}$$

The whole system's expected profit is obtained as the sum of the retailer's expected profit and the manufacturer's expected profit. The whole system's expected profit for Q, t and u is obtained as

$$\begin{aligned} E\left[\pi_{\mathrm{S}}(Q,t,u)\right] &= E\left[\pi_{\mathrm{R}}(Q,t,u)\right] + E\left[\pi_{\mathrm{M}}(Q,t,u)\right] \\ &= -tA(t) - c_t A(t) - c_a A(t) - A(t)\int_u^1 c_r(\ell)g(\ell)d\ell \\ &\quad - c_d A(t)\int_0^u g(\ell)d\ell - c_n\left\{Q - A(t)\int_u^1 g(\ell)d\ell\right\} - c_m Q \\ &\quad + \int_0^Q \left\{px - h_r(Q-x)\right\} f(x)dx + \int_Q^\infty \{pQ - s(x-Q)\} f(x)dx \end{aligned} \tag{5}$$

From equations (4) and (5), the terms on the wholesales of products and the compensation for the collection incentive accruing between the retailer and the manufacturer are cancelled out.

5 Loss-averse analysis of profits in a CLSC for uncertain product demand

First, the prospect theory (Wang and Webster, 2009) to use loss-averse analysis is discussed. Let W_0 denote the reference level (the initial wealth) of the decision maker at the beginning of the selling season. A simple piecewise-linear form of loss aversion utility function is considered as

$$U(W)=\begin{cases} W-W_0 & W\geq W_0 \\ \lambda(W-W_0) & W<W_0 \end{cases} \tag{6}$$

where $\lambda \geq 1$ is defined as the loss aversion coefficient of the decision maker. Therefore, there exists a kink at the reference level W_0 if $\lambda > 1$, and higher values of λ imply higher levels of loss aversion of the decision maker. Without loss of generality, the reference level of the decision maker is normalised to $W_0 = 0$.

Next, loss-averse analysis for the uncertain product demand is discussed for the profits of a retailer and the whole system who are affected by the product demand x and decision makers in the CLSC. Loss-averse analysis is conducted for four types of loss-averse attitude of the retailer and the whole system regarding the uncertain product demand: N (risk-neutral attitude without loss aversion), LAE (loss-averse attitude for excess quantity), LAS (loss-averse attitude for shortage quantity) and LA (loss-averse attitude for excess quantity and shortage quantity). N makes a decision without loss-averse attitudes so as to maximise the decision maker's expected profit. LAE hates the decision maker's profit loss due to excess quantity when $x \leq Q$. LAS hates the decision maker's profit loss due to shortage quantity when $x > Q$. LA hates the decision maker's profit loss due to excess quantity and shortage quantity when $x \leq Q$ and $x > Q$.

By mapping the retailer's profit in equation (1) into equation (6), the retailer's expected utility function in LA is obtained as

$$\begin{aligned} EU_{\mathrm{R}}^{\mathrm{LA}}(Q,t,u) &= E\left[\pi_{\mathrm{R}}(Q,t,u)\right]+\left(\lambda^{\mathrm{E}}-1\right)\int_0^{q_1^{\mathrm{D}}}\{px-h_r(Q-x)-wQ\}f(x)dx \\ &\quad +\left(\lambda^{\mathrm{S}}-1\right)\int_{q_2^{\mathrm{D}}}^{\infty}\{pQ-s(x-Q)-wQ\}f(x)dx \end{aligned} \tag{7}$$

λ^{E} (>1) denotes the degree of loss aversion regarding LAE when $x \leq Q$. λ^{S} (>1) denotes the degree of loss aversion regarding LAS when $x > Q$. q_1^{D} is x satisfying $px - (Q - x)h_r - wQ = 0$ when $x \leq Q$. Therefore, $q_1^{\mathrm{D}} = (w+h_r)Q/(p+h_r)$. q_2^{D} is x satisfying $pQ - (x - Q)s - wQ = 0$ when $x > Q$. Therefore, $q_2^{\mathrm{D}} = (p+s-w)Q/s$. The case that $\lambda^{\mathrm{S}} = 1$ in equation (7) indicates the retailer's expected utility function in LAE. The case that $\lambda^{\mathrm{E}} = 1$ in equation (7) indicates that in LAS. The case that $\lambda^{\mathrm{E}} = \lambda^{\mathrm{S}} = 1$ indicates N. The retailer's the expected utility function for Q, t and u is equal to the retailer's expected profit for Q, t, and u in equation (2).

By mapping the whole system's profit into equation (4) into equation (6), the whole system's expected utility function in LA is obtained as

$$\begin{aligned} EU_{\mathrm{S}}^{\mathrm{LA}}(Q,t,u) &= E\left[\pi_{\mathrm{S}}(Q,t,u)\right] \\ &\quad +\left(\lambda^{\mathrm{E}}-1\right)\int_0^{q_1^{\mathrm{I}}}\left\{px-(Q-x)h_r-(c_m+c_n)Q\right\}f(x)dx \\ &\quad +\left(\lambda^{\mathrm{S}}-1\right)\int_{q_2^{\mathrm{I}}}^{\infty}\left\{pQ-(x-Q)s-(c_m+c_n)Q\right\}f(x)dx \end{aligned} \tag{8}$$

From the whole system's expected profit in equation (4), the whole system's expected utility function in equation (8) uses $-(c_m + c_n)Q$ as the term of loss averse analysis. This is because the whole system's profit cancels out the term of wQ, but incurs the term of $(c_m + c_n)Q$.

q_1^{I} is x satisfying $px - (Q - x)h_r - (c_m + c_n)Q = 0$ when $x \le Q$. Therefore, $q_1^{\mathrm{I}} = (c_m + c_n + h_r)Q/(p + h_r)$. q_2^{I} is x satisfying $pQ - (x - Q)s - (c_m + c_n)Q = 0$ when $x > Q$. Therefore, $q_2^{\mathrm{I}} = \{p + s - (c_m + c_n)\}Q/s$. The case that $\lambda^S = 1$ in equation (8) indicates the whole system's expected utility function in LAE. The case that $\lambda^E = 1$ in equation (8) indicates that in LAS. The case that $\lambda^E = \lambda^S = 1$ indicates N. The whole system's the expected utility function for Q, t and u is equal to the whole system's expected profit for Q, t, and u in equation (2).

The manufacturer's profit is unaffected by the product demand x. The manufacturer's the expected utility function for Q, t and u as to attitude $i \in$ {N, LAE, LAS, LA} is equal to the manufacturer's expected profit for Q, t and u in equation (3).

6 Optimal operation under DCLSC

The optimal operations under DCLSC with four types of loss-averse attitude of the decision maker are discussed. In DCLSC, the optimal decision approach for the Stackelberg game is adopted. This paper regards a retailer and a manufacturer as the leader and the follower of the decision making under DCLSC. Then, the retailer determines the optimal product order quantity Q_{D}^{i} ($i \in$ {N, LAE, LAS, LA}) in attitude i and the optimal unit collection incentive t_{D} so as to maximise the retailer's expected profit with attitude N and maximise the retailer's expected utility function with attitudes LAE, LAS and LA under degrees of loss aversion, λ^{E} and λ^{S} of the retailer. The manufacturer determines the optimal lower limit of quality level u_{D} so as to maximise the manufacturer's expected profit under Q_{D}^{i} and t_{D}.

The optimal product order quantities $Q_{\mathrm{D}}^{\mathrm{N}}$, $Q_{\mathrm{D}}^{\mathrm{LAE}}$, $Q_{\mathrm{D}}^{\mathrm{LAS}}$ and $Q_{\mathrm{D}}^{\mathrm{LA}}$ under DCLSC with attitudes N, LAE, LAS and LA for degrees of loss aversion, λ^{E} and λ^{S} of the retailer, are determined so as to maximise the retailer's expected utility function with LAE, LAS, LA, and maximise the retailer's expected profit with N.

In case of attitude LA, the first and second partial derivatives of the retailer's expected utility function in equation (7) in terms of Q are obtained as

$$\begin{aligned}\frac{\partial U_{\mathrm{R}}^{\mathrm{LA}}(Q,t,u)}{\partial Q} &= \lambda^{\mathrm{S}}(p+s-w) - (p+h_r+s)F(Q) \\ &\quad - (\lambda^{\mathrm{E}}-1)(w+h_r)F\left(q_1^{\mathrm{D}}\right) - (\lambda^{\mathrm{S}}-1)(p+s-w)F\left(q_2^{\mathrm{D}}\right)\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial^2 U_{R}^{\mathrm{LA}}(Q,t,u)}{\partial^2 Q} &= -(p+h_r+s)\,f(Q) \\ &\quad - (\lambda^{\mathrm{E}}-1)\frac{(w+h_r)^2}{(p+h_r)}\,f\left(q_1^{\mathrm{D}}\right) - (\lambda^{\mathrm{S}}-1)\frac{(p+s-w)^2}{s}\,f\left(q_2^{\mathrm{D}}\right) < 0\end{aligned} \tag{10}$$

Here, $q_1^{\mathrm{D}} = (w+h_r)Q/(p+h_r)$ and $q_2^{\mathrm{D}} = (p+s-w)Q/s$. The elicitation processes in equations (9) and (10) are derived in Appendix A. Equation (10) is negative since p, h_r, s, w, $f(Q)$, $f(q_1^{\mathrm{D}}) > 0$, $f(q_2^{\mathrm{D}}) > 0$, $p > w$, $\lambda^{\mathrm{E}} > 1$, $\lambda^{\mathrm{S}} > 1$. Therefore, equation (7) is concave in terms of Q. Therefore, $Q_{\mathrm{D}}^{\mathrm{LA}}$ in attitude LA is obtained so as to satisfy the following first-order condition (FOC) of equation (9), using numerical search.

$$\begin{aligned}\frac{\partial U_{\mathrm{R}}^{\mathrm{LA}}(Q,t,u)}{\partial Q} &= -w+p+s-\left(p+h_r+s\right)F(Q)-\left(\lambda^{\mathrm{E}}-1\right)\left(w+h_r\right)F\left(q_1^{\mathrm{D}}\right)\\ &\quad +\left(\lambda^{\mathrm{S}}-1\right)(p+s-w)-\left(\lambda^{\mathrm{S}}-1\right)(p+s-w)F\left(q_2^{\mathrm{D}}\right)=0\end{aligned} \tag{11}$$

Similar way to LA, $Q_{\mathrm{D}}^{\mathrm{LAE}}$ in attitude LAE is obtained so as to satisfy the first-order condition (FOC) of equation (11) when $\lambda^S = 1$, using numerical search, $Q_{\mathrm{D}}^{\mathrm{LAS}}$ in attitude LAS is obtained so as to satisfy FOC of equation (11) when $\lambda^E = 1$, using numerical search. When $\lambda^E = 1$ and $\lambda^S = 1$ in FOC of equation (11), Q_D^N in attitude N is obtained as

$$Q_{\mathrm{D}}^{\mathrm{N}} = F^{-1}\left(\frac{-w+p+s}{p+h_r+s}\right) \tag{12}$$

Q_{D}^{i} as to attitude i under DCLSC is unaffected by t and u.

Under Q_{D}^{i} in attitude i of the retailer, the unit collection incentive t and the lower limit of quality level u are optimised. The first-order partial derivative of the manufacturer's the expected profit in equation (3) in terms of u under Q_{D}^{i} and t is obtained as

$$\frac{\partial U_M^i\left(Q_{\mathrm{D}}^i,t,u\right)}{\partial u} = \frac{\partial E\left[\pi_M\left(Q_{\mathrm{D}}^i,t,u\right)\right]}{\partial u} = A(t)g(u)\{c_r(u)+R(t)-c_d-c_n\} \tag{13}$$

It can be seen that equation (13) is unaffected by Q_{D}^{i}, but affected by t under DCLSC. Here, equation (13) is zero if the following condition:

$$c_r(u)+R(t)-c_d-c_n=0 \tag{14}$$

is satisfied. From (4) in Section 2.2 model assumptions, the unit remanufacturing cost $c_r(u)$ is a monotone decreasing function for u. Therefore, the provisional lower limit of quality level under t, $u_{\mathrm{D}}(t)$, is obtained as u satisfying equation (14). $u_{\mathrm{D}}(t)$ is unaffected by Q_{D}^{i}.

By varying t within $0 \le t \le t_U$, determine the optimal combination of Q_{D}^{i}, t and $u_{\mathrm{D}}(t)$ under DCLSC in attitude i ($\in$ N, LAE, LAS, LA) of the retailer so as to maximise the retailer's expected profit in attitude N and the retailer's expected utility function in attitude LAE, LAS and LA. Therefore, the optimal operation under DCLSC can be obtained as

$$\left(Q_{\mathrm{D}}^i,t,u_{\mathrm{D}}(t)\right)=\left(Q_{\mathrm{D}}^i,t_{\mathrm{D}},u_{\mathrm{D}}\right)\left(\in\{\mathrm{N, LAE, LAS, LA}\}\right)$$

Thus, t_{D} and u_{D} are determined mutually between a retailer and a manufacturer under DCLSC. Here, terms regarding loss aversion with the degrees of loss aversion λ^E and λ^S in the retailer's expected utility function for attitude i is unaffected by t and u. Therefore, the optimal decisions for t and u under DCLSC are unaffected by attitude i.

7 Optimal operation under ICLSC

The optimal operation under ICLSC is discussed. Under ICLSC, the optimal product order quantity Q_{I}^{i} ($i \in$ {N, LAE, LAS, LA}) in attitude *i* of the decision maker in the whole system, the optimal unit collection incentive t_I, and the lower limit of quality level u_I in attitude *i* are determined to maximise the whole system's expected profit in attitude *N* and maximise the whole system's expected utility function with attitudes LAE, LAS and LA under degrees of loss aversion, λ^E and λ^S of the decision maker in the whole system. The optimal product order quantities $Q_{\mathrm{I}}^{\mathrm{LAE}}$, $Q_{\mathrm{I}}^{\mathrm{LAS}}$ and $Q_{\mathrm{I}}^{\mathrm{LA}}$ under ICLSC with attitude attitudes LAE, LAS and LA for degrees of loss aversion, λ^{E} and λ^{S} of the decision maker in the whole system, are determined so as to maximise the whole system's expected utility function.

In case of attitude LA, the first and second partial derivatives of the whole system's expected utility function in equation (8) in terms of *Q* are obtained as

$$\begin{aligned}\frac{\partial U_{\mathrm{S}}^{\mathrm{LA}}(Q,t,u)}{\partial Q} &= \lambda^{\mathrm{S}}\left(p+s-c_m-c_n\right)-\left(p+h_r+s\right)F(Q)\\ &\quad -\left(\lambda^{\mathrm{E}}-1\right)\left(c_m+c_n+h_r\right)F\left(q_1^{\mathrm{I}}\right)\\ &\quad -\left(\lambda^{\mathrm{S}}-1\right)\left(p+s-c_m-c_n\right)F\left(q_2^{\mathrm{I}}\right)\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial^2 U_R^{LA}(Q,t,u)}{\partial^2 Q} &= -\left(p+h_r+s\right)f\left(Q\right)-\left(\lambda^{\mathrm{E}}-1\right)\frac{\left\{p+s-(c_m+c_n)\right\}^2}{\left(p+h_r\right)}f\left(q_1^{\mathrm{I}}\right)\\ &\quad -\left(\lambda^{\mathrm{S}}-1\right)\frac{\left\{p+s-(c_m+c_n)\right\}^2}{s}f\left(q_2^{\mathrm{I}}\right)<0.\end{aligned} \tag{16}$$

Here, $q_1^{\mathrm{I}}=(c_m+c_n+h_r)Q/(p+h_r)$ and $q_2^{\mathrm{I}}=\{p+s-(c_m+c_n)\}Q/s$. The elicitation processes in equations (15) and (16) are derived in Appendix B. Equation (16) is negative since p, h_r, s, c_m, c_n, $f(Q)$, $f(q_1^{\mathrm{I}})>0$, $f(q_2^{\mathrm{I}})>0$, $f(q_2^{\mathrm{D}})>0$, $p>(c_m+c_n)$, $\lambda^{\mathrm{E}}>1$, $\lambda^{\mathrm{S}}>1$. Therefore, equation (8) is concave in terms of *Q*. Therefore, $Q_{\mathrm{I}}^{\mathrm{LA}}$ in attitude LA is obtained so as to satisfy the following first-order condition (FOC) of equation (15), using numerical search.

$$\begin{aligned}\frac{\partial U_{\mathrm{S}}^{\mathrm{LA}}(Q,t,u)}{\partial Q} &= -c_m-c_n+p+s-\left(p+h_r+s\right)F(Q)\\ &\quad -\left(\lambda^{\mathrm{E}}-1\right)\left(c_m+c_n+h_r\right)F\left(q_1^{\mathrm{I}}\right)+(\lambda^{\mathrm{S}}-1)\left(p+s-c_m-c_n\right)\\ &\quad -\left(\lambda^{\mathrm{S}}-1\right)\left(p+s-c_m-c_n\right)F\left(q_2^{\mathrm{I}}\right)=0\end{aligned} \tag{17}$$

Similar way to LA, $Q_{\mathrm{I}}^{\mathrm{LAE}}$ in attitude LAE is obtained so as to satisfy the first-order condition (FOC) of equation (17) when $\lambda^S = 1$, using numerical search, $Q_{\mathrm{I}}^{\mathrm{LAS}}$ in attitude LAS is obtained so as to satisfy FOC of equation (17) when $\lambda^E = 1$, using numerical search. When $\lambda^E = 1$ and $\lambda^S = 1$ in FOC of equation (17), $Q_{\mathrm{I}}^{\mathrm{N}}$ in attitude N is obtained as

$$Q_{\mathrm{I}}^{\mathrm{N}} = F^{-1}\left(\frac{-c_m - c_n + p + s}{p + h_r + s}\right) \tag{18}$$

Q_{I}^{i} as to attitude i under ICLSC is unaffected by t and u.

Under Q_{I}^{i} in attitude i of the decision maker in the whole system, the unit collection incentive t and the lower limit of quality level u are optimised.

The first-order partial derivatives of the whole system's expected profit under attitude N and the whole system's expected utility functions under attitude i in terms of u under Q_{I}^{i} and t are obtained as

$$\frac{\partial U_S^i\left(Q_{\mathrm{I}}^i, t, u\right)}{\partial u} = \frac{\partial E\left[\pi_S\left(Q_{\mathrm{I}}^i, t, u\right)\right]}{\partial u} = A(t)g(t)\left\{c_r(u) - c_d - c_n\right\} \tag{19}$$

It can be seen that equation (19) is unaffected by Q_{I}^{i} and t. Similar way to ICSL, the optimal lower limit of quality level u_{I} under ICLSC to satisfy

$$c_r(u) - c_d - c_n = 0. \tag{20}$$

From equation (20), u_{I} is unaffected by Q_{I}^{i} and t. Therefore, the value of u_{I} is unaffected by attitude i.

The whole system's expected profit under attitude N and the whole system's expected utility function under attitude $i \in$ {LAE, LAS, LA} have no term depending on both of t and Q. Also, the optimal decision for t under ICLSC is unaffected by Q. Therefore, the value of the optimal collection incentive t_{I} of attitude $i \in$ {N, LAE, LAS, LA} of the decision maker in the whole system is same.

By varying t within $0 \leq t \leq t_U$, determine the optimal combination of Q_{I}^{i}, t and u_{I} under ICLSC in attitude i so as to maximise the whole system's expected profit in attitude N and the whole system's expected utility function in attitudes LAE, LAS, LA. Therefore, the optimal operation under ICLSC can be obtained as

$$\left(Q_{\mathrm{I}}^i, t, u_{\mathrm{I}}\right) = \left(Q_{\mathrm{I}}^i, t_{\mathrm{I}}, u_{\mathrm{I}}\right)(i \in \{\mathrm{N, LAE, LAS, LA}\})$$

From the same reason as DCLSC, the optimal decisions for t and u under ICLSC are unaffected by attitude i.

8 Profit sharing as supply chain coordination

It is necessary to guarantee the increase of profits of all members under ICLSC when the optimal operation shifts from DCLSC to ICLSC and the whole system's expected profit in ICLSC is higher than that under DCLSC. This paper discusses supply chain coordination to promote the shift to the optimal operation under ICLSC from that under DCLSC. As supply chain coordination, it is discussed two profit sharing approaches based on:

1 profit ratio of each firm under ICLSC (PS I)

2 profit investment ratio of each firm under ICLSC (PS II) between a retailer and a manufacturer under ICLSC.

It is verified how two profit sharing approaches can bring the profitability to a retailer and a manufacturer under ICLSC.

8.1 PS I using profit ratio of each firm

Using the expected profits of a retailer, a manufacturer and the whole system under the optimal operation of ICLSC with attitude $i \in$ {N, LAE, LAS, LA}, the profit ratios, ρ_{R}^{i}, ρ_{M}^{i}, of the retailer and the manufacture under attitude *i* are calculated as

$$\rho_{\mathrm{R}}^{i} = \frac{E\left[\pi_{\mathrm{R}}\left(Q_{\mathrm{I}}^{i}, t_{\mathrm{I}}, u_{\mathrm{I}}\right)\right]}{E\left[\pi_{\mathrm{S}}\left(Q_{\mathrm{I}}^{i}, t_{\mathrm{I}}, u_{\mathrm{I}}\right)\right]} \tag{21}$$

$$\rho_{M}^{i} = 1 - \rho_{R}^{i} \tag{22}$$

Here, $\Delta\pi_{\mathrm{S}}^{i}$ ($i \in$ {N, LAE, LAS, LA}) denotes the increment of the whole system's profit from the optimal operation under DCLSC with attitude *i* to that under ICLSC with attitude *i*. $\Delta\pi_{\mathrm{S}}^{i}$ is calculated as

$$\Delta\pi_{\mathrm{S}}^{i} = E\left[\pi_{\mathrm{S}}\left(Q_{\mathrm{I}}^{i}, t_{\mathrm{I}}, u_{\mathrm{I}}\right)\right] - E\left[\pi_{\mathrm{S}}\left(Q_{\mathrm{D}}^{i}, t_{\mathrm{D}}, u_{\mathrm{D}}\right)\right] \tag{23}$$

Using $\Delta\pi_{\mathrm{S}}^{i}$, ρ_{R}^{i} and ρ_{M}^{i}, the allocated amount to each member after profit sharing based on profit ratios under attitude *i*, ρ_{R}^{i}, ρ_{M}^{i}, are calculated as

$$\varphi_{R}^{\mathrm{PSI}(i)} = \Delta\pi_{S}^{i} \times \rho_{R}^{i} \tag{24}$$

$$\varphi_{M}^{\mathrm{PSI}(i)} = \Delta\pi_{S}^{i} \times \rho_{M}^{i} \tag{25}$$

Using $\varphi_{R}^{I(i)}$ and $\varphi_{M}^{I(i)}$, the expected profit of each member with the allocated amount with attitude *i* can be obtained as

$$\tilde{\pi}_{R}^{\mathrm{PSI}(i)}\left(Q_{I}^{i}, t_{I}, u_{I}\right) = E\left[\pi_{R}\left(Q_{D}^{i}, t_{D}, u_{D}\right)\right] + \varphi_{R}^{\mathrm{PSI}(i)} \tag{26}$$

$$\tilde{\pi}_{M}^{\mathrm{PSI}(i)}\left(Q_{I}^{i}, t_{I}, u_{I}\right) = E\left[\pi_{M}\left(Q_{D}^{i}, t_{D}, u_{D}\right)\right] + \varphi_{M}^{\mathrm{PSI}(i)} \tag{27}$$

8.2 PS II using profit investment ratio of each firm

Using the expected profits and the total costs of a retailer and a manufacturer under the optimal operation of ICLSC with attitude $i \in$ {N, LAE, LAS, LA}, the profit investment ratios, R_{R}^{i}, R_{M}^{i}, of the retailer and the manufacturer under attitude *i* are calculated as

$$R_{\mathrm{R}}^{i} = \frac{E\left[\pi_{\mathrm{R}}\left(Q_{I}^{i}, t_{I}, u_{I}\right)\right]}{ETC\left[\pi_{\mathrm{R}}\left(Q_{I}^{i}, t_{I}, u_{I}\right)\right]} \tag{28}$$

$$R_{\mathrm{M}}^{i}=\frac{E\left[\pi_{\mathrm{M}}\left(Q_I^i,t_I,u_I\right)\right]}{ETC\left[\pi_{\mathrm{M}}\left(Q_I^i,t_I,u_I\right)\right]} \tag{29}$$

The profit investment ratios, R_{R}^{i}, R_{M}^{i}, under attitude *i* are normalised as

$$R_{\mathrm{R}}^{adj(i)}=R_{\mathrm{R}}^{i}\Big/\left(R_{\mathrm{R}}^{i}+R_{\mathrm{M}}^{i}\right) \tag{30}$$

$$R_{\mathrm{M}}^{adj(i)}=R_{\mathrm{M}}^{i}\Big/\left(R_{\mathrm{R}}^{i}+R_{\mathrm{M}}^{i}\right) \tag{31}$$

Using $\Delta\pi_{\mathrm{S}}^{i}$, R_{R}^{i} and R_{M}^{i}, the allocated amount to each member after profit sharing based on profit investment ratios, $\varphi_{\mathrm{R}}^{\mathrm{PSII}(i)}$, $\varphi_{\mathrm{M}}^{\mathrm{PSII}(i)}$, under attitude *i* are calculated as

$$\varphi_{\mathrm{R}}^{\mathrm{PSII}(i)}=\Delta\pi_{\mathrm{S}}\times R_{\mathrm{R}}^{adj(i)} \tag{32}$$

$$\varphi_{\mathrm{M}}^{\mathrm{PSII}(i)}=\Delta\pi_{\mathrm{S}}\times R_{\mathrm{M}}^{adj(i)} \tag{33}$$

Using $\varphi_{\mathrm{R}}^{\mathrm{PSII}(i)}$ and $\varphi_{\mathrm{M}}^{\mathrm{PSII}(i)}$, the profit of each member with the allocated amount under attitude *i* can be obtained as

$$\tilde{\pi}_{\mathrm{R}}^{\mathrm{PSII}(i)}\left(Q_I^i,t_I,u_I\right)=\pi_{\mathrm{R}}\left(Q_D^i,t_D,u_D\right)+\varphi_{\mathrm{R}}^{\mathrm{PSII}(i)} \tag{34}$$

$$\tilde{\pi}_{\mathrm{M}}^{\mathrm{PSII}(i)}\left(Q_I^i,t_I,u_I\right)=\pi_{\mathrm{M}}\left(Q_D^i,t_D,u_D\right)+\varphi_{\mathrm{M}}^{\mathrm{PSII}(i)} \tag{35}$$

9 Numerical experiments

This section numerically investigates how:

1 four types of loss-averse attitude of the decision maker regarding the uncertain product demand

2 quality of recyclable parts

affect the optimal operations under DCLSC and ICLSC. Concretely, the optimal operation regarding the product order quantity, the unit collection incentive, the lower limit of quality level and the expected profits under DCLSC are compared with those under ICLSC by changing the degrees of loss aversion of LAE and LAS, λ^{E} and λ^{S} of the decision maker, and the quality distribution of recyclable parts. Moreover, it shows that profit sharing approaches as supply chain coordination (SCC), based on:

1 profit ratio

2 profit investment ratio

can bring the more expected profits to a retailer and a manufacturer under ICLSC and enable to shift to the optimal operation under ICLSC from that under DCLSC.

The following numerical examples are used as $p = 150$, $s = 175$, $h_r = 15$, $c_a = 1$, $c_d = 1$, $c_t = 1$, $c_n = 35$, $c_m = 2$, $m_a = 15$. $A(t)$, $c_r(\ell)$ and w are respectively set as $A(t) = 100 + 50t$, $c_r(\ell) = 40(1 - 0.9\ell)$ and $w = c_n + c_m + m_a$, satisfying the properties of functions in

Subsection 2.2 (1), (4), and (6). The degree of compensation α for the retailer's unit collection incentive *t* is set as $\alpha = 0.7$.

The product demand *x* follows the normal distribution with the mean $\mu = 1{,}000$ and the variance $\sigma^2 = 300^2$.

In this paper, the variation in the quality level ℓ ($0 \leq \ell \leq 1$) of recyclable parts in used products is modelled by using the beta distribution. This is because the beta distribution is widely used to measure relative parameters like level ℓ ($0 \leq \ell \leq 1$), or anything that is between 0–1. Also, the beta distribution can express various shapes of distribution of recyclable parts in used products by using probability density function $g(\ell)$ of the beta distribution with parameters (*a*, *b*):

$$g(\ell) = \frac{\Gamma(a+b)}{\Gamma(a)+\Gamma(b)} \ell^{a-1}(1-\ell)^{b-1} \tag{35}$$

where $\Gamma(\cdot)$denotes the gamma function.

This paper provides the following four cases of quality distribution *B*(*a*, *b*) of recyclable parts:

Case 1 *B*(1, 1): a situation where quality of each recyclable parts is distributed uniformly.

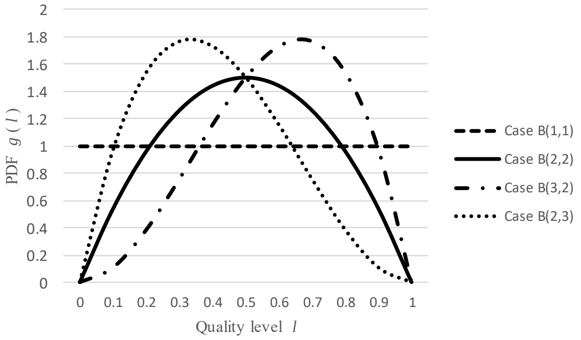
Case 2 *B*(2, 2): a situation where there are many recyclable parts with middle quality level.

Case 3 *B*(3, 2): a situation where there are many recyclable parts with the relatively high quality level.

Case 4 *B*(2, 3): a situation where there are many recyclable parts with the relatively low quality level.

Figure 2 shows quality distribution of recyclable parts in used products modelled as the beta distribution *B*(*a*, *b*) in cases 1–4.

Figure 2 Quality distribution of recyclable parts in used products modelled as the beta distribution *B*(*a*, *b*) in cases 1–4



9.1 Effect of loss-averse attitude of the decision maker on the optimal operation in a CLSC

From Sections 5 and 6, the loss-averse attitude of the decision maker affects the optimal product order quantities under DCLSC and ICLSC. Table 1 shows the effect of degrees of loss aversion λ^{E} and λ^{S} on the optimal product order quantity Q_k^i ($i \in$ {N, LAE, LAS, LA}), $k \in$ {D, I}) under DCLSC and ICLSC. Note that Q_k^i is unaffected by the quality distribution of recyclable parts. From Table 1, the following results can be seen: The magnitude relation for the optimal order quantity is $Q_k^{\mathrm{LAS}} > Q_k^{\mathrm{N}} > Q_k^{\mathrm{LA}} > Q_k^{\mathrm{LAE}}$. The higher λ^{E} is, the smaller $Q_{\mathrm{D}}^{\mathrm{LAE}}$ and $Q_{\mathrm{I}}^{\mathrm{LAE}}$ are. The higher λ^{S} is, the larger $Q_{\mathrm{D}}^{\mathrm{LAS}}$ and $Q_{\mathrm{I}}^{\mathrm{LAS}}$ are.

The higher λ^{E} and λ^{S} are, the smaller $Q_{\mathrm{D}}^{\mathrm{LA}}$ and $Q_{\mathrm{I}}^{\mathrm{LA}}$ are. The changes of $Q_{\mathrm{D}}^{\mathrm{LAS}}$ and $Q_{\mathrm{I}}^{\mathrm{LAS}}$ for λ^{S} are similar to those of $Q_{\mathrm{D}}^{\mathrm{N}}$ and $Q_{\mathrm{I}}^{\mathrm{N}}$. The changes of $Q_{\mathrm{D}}^{\mathrm{LA}}$ and $Q_{\mathrm{I}}^{\mathrm{LA}}$ for λ^{E} and λ^{S} are similar to those of $Q_{\mathrm{D}}^{\mathrm{LAE}}$ and $Q_{\mathrm{I}}^{\mathrm{LAE}}$ for λ^{E}. Therefore, the optimal product order quantity with LA is affected by that with LAE. The optimal decisions under LAE and LA tend to reduce the optimal product order quantity so as to reduce the inventory holding cost of unsold products. The decision making under LAS tends to increase the optimal product order quantity so as to reduce the shortage penalty cost of unsatisfied product.

9.2 Effect of quality distribution on optimal operations in a CLSC

From Sections 5 and 6, the quality distribution of recyclable parts does not affect the optimal product order quantities under DCLSC and ICLSC.

Table 2 Effect of degrees of loss aversion λ^{E} and λ^{S} on the optimal order quantities under DCLSC and ICLSC

Loss-averse attitude	*Optimal decisions*	*Degrees of loss aversion λ^E and λ^S*						
		2	*5*	*10*	*15*	*20*	*25*	*30*
N	$Q_{\mathrm{D}}^{\mathrm{N}}$	1,256	1,256	1,256	1,256	1,256	1,256	1,256
	$Q_{\mathrm{I}}^{\mathrm{N}}$	1,307	1,307	1,307	1,307	1,307	1,307	1,307
LAE (λ^{E})	$Q_{\mathrm{D}}^{\mathrm{LAE}}$	1,245	1,218	1,183	1,155	1,131	1,111	1,092
	$Q_{\mathrm{I}}^{\mathrm{LAE}}$	1,302	1,289	1,270	1,253	1,238	1,224	1,212
LAS (λ^{S})	$Q_{\mathrm{D}}^{\mathrm{LAS}}$	1,256	1,258	1,261	1,263	1,265	1,268	1,270
	$Q_{\mathrm{I}}^{\mathrm{LAS}}$	1,307	1,307	1,308	1,308	1,308	1,309	1,309
LA (λ^{E}, λ^{S})	$Q_{\mathrm{D}}^{\mathrm{LA}}$	1,246	1,222	1,194	1,176	1,163	1,154	1,147
	$Q_{\mathrm{I}}^{\mathrm{LA}}$	1,302	1,289	1,271	1,255	1,241	1,230	1,219

Table 2 shows results of the optimal unit collection incentive t_k ($k \in$ {D, I}) and the optimal lower limit of quality level u_k ($k \in$ {D, I}) in the four cases of quality distribution of recyclable parts. Note that t_k and u_k are unaffected by the optimal product order quantity Q_k^i ($i \in$ {N, LAE, LAS, LA}, $k \in$ {D, I}) from equations (13) and (19).

From Table 2, the following results can be seen:

- The better the quality of recyclable parts is as case 3, t_D and t_I become the highest among other cases in order to increase the number of collected used products. In this case, more the recycled parts can be remanufactured and the more the retailer can receive compensation from the manufacturer.
- The worse the quality of recyclable parts is as case 4, t_D and t_I become the lowest among other cases in order to reduce the quantity of collected used products.
- Under DCLSC, u_D and t_D change as to cases 1–4 of the quality distribution of the recyclable parts based on equation (35). This is because the manufacturer tends to reduce the profit loss due to the compensation to the retailer as to the quality distribution.
- Under ICLSC, only t_I is affected by the quality distribution of the recyclable parts.

9.3 Comparison of results under DCLSC and ICLSC

Table 3 shows results of the expected profits and profit sharing approaches under ICLSC with attitude LA. The quality distribution of recyclable parts is case 4.

Table 3 The optimal decisions of the unit collection inventive and lower limit of quality level under DCLSC and ICLSC

	Quality distribution of recyclable parts			
Optimal decision (DCSL)	*Case 1 B(1, 1)*	*Case 2 B(2, 2)*	*Case 3 B(3, 2)*	*Case 4 B(2, 3)*
t_D	2.94	3.94	6.13	2.27
u_D	0.25	0.3	0.4	0.22
Optimal decision (ICSL)	*Case 1 B(1, 1)*	*Case 2 B(2, 2)*	*Case 3 B(3, 2)*	*Case 4 B(2, 3)*
t_I	4.61	4.52	6.3	2.74
u_I	0.11	0.11	0.11	0.11

First, the expected profits for the optimal operation with LA under DCLSC are compared with those under ICLSC. From Table 3, the following results can be seen: The expected profits of the manufacturer and the whole system with LA under ICLSC are higher than those under DCLSC. Thus, the optimal operation with attitude *i* under ICLSC can bring the profitability to the manufacturer and the whole system. However, only the expected profit of the retailer with LA under ICLSC is lower than that under DCLSC. The retailer's expected profit accounts for most of the whole system's expected profit under ICLSC and the retailer is the leader of the decision making with attitude *i* under DCLSC. It is necessary to guarantee the profitability to all members under ICLSC when the optimal operation shifts from DCLSC to ICLSC.

Next, the profitability of profit sharing under ICLSC with LA is discussed. The number in the parenthesis in Table 3 indicates decrease or increase in the expected profit of each member under ICLSC without and with profit sharing, comparing that under DCLSC. From Table 3, the following results can be seen: The all members' expected profits under ICLSC with PS I and PS II are higher than those under DCLSC. Thus, the optimal operation under ICLSC with PS I and PS II can guarantee the increases of all members' expected profits.

Next, the benefits of PS I and PS II are compared. The expected profit of the manufacturer under ICLSC of this paper is lower than that of the retailer. Therefore, the profitability of the manufacturer in PS I based on each member's profit ratio is less than that of the retailer. The manufacturer has a disadvantage in PS I. In PS II, the earning (cost) performance of each member is reflected because the allocated amount of each member is calculated based on profit investment ratio. Thus, PS II can be recommended for all members under ICLSC. Thus, the shift to the optimal operation under ICLSC with profit sharing can encourage to promote not only the collection activity of used products, but also the recycling activity, improving the expected profits of the whole system and all members under ICSLC as the total optimisation.

Table 4 Results of the expected profits and profit sharing approaches under ICLSC with attitude LA (quality distribution of recyclable parts: case 4)

Loss-averse attitude	*Expected profits*	*DCLSC*	*ICLSC*	*ICLSC*	
				PS I	*PS II*
LA ($\lambda^E = \lambda^S = 2$)	Retailer	69,572	69,428 (–144)	70,024 (+452)	69,966 (+394)
	Manufacturer	19,785	20,515 (+730)	19,919 (+134)	19,977 (+192)
	Whole system	89,357	89,943 (+586)	89,943 (+586)	89,943 (+586)

The above results in other attitudes and other cases of the quality distribution of recyclable parts are same.

10 Conclusions

This paper incorporated loss-averse analysis based on prospect theory regarding the uncertain product demand into the optimal operation in a CLSC with a retailer and a manufacturer. Concretely, the four types of loss-averse attitude regarding the uncertain product demand were discussed: N (risk-neutral attitude without loss aversion), LAE (loss-averse attitude for excess quantity), LAS (loss-averse attitude for shortage quantity) and LA (loss-averse attitude for excess quantity and shortage quantity). Considering the negative region of the decision maker's profit under DCLSC and ICLSC as to the decision maker's loss-averse attitude, the optimal operation for the product order quantity, the unit collection incentive, and the lower limit of quality level under DCLSC and ICLSC were made by mathematical analyses and numerical searches. Under DCLSC, the retailer and the manufacturer were regarded as the leader and the follower of the decision making. The optimal operation under DCLSC based on the Stackelberg game was made so as to maximise the expected profit of the retailer in N, and maximise the expected utility function of the retailer with LAE LAS and LA. The manufacturer determined the optimal operation so as to maximised the own expected profit under the retailer's optimal decision. Under ICLSC, the optimal operation was made so as to maximise the expected profit of the whole system in N, and maximise the expected utility function of the whole system with LAE LAS and LA.

Results of theoretical analysis and numerical analysis in this paper gave the following managerial insights:

1 The optimal decision for the order quantity under DCLSC and ICLSC depends on the loss-averse attitude of the decision maker, while the optimal decisions for the unit collection incentive and the lower limit of quality level are unaffected by the loss-averse attitude.

2 The higher degrees of loss aversion regarding LAE, LAS and LA are, the smaller the optimal product quantities with LAE and LA under DCLSC and ICLSC are, but the larger those with LAS under DCLSC and ICLSC are. The change of the optimal product order quantity with LA is affected by that with LAE for degrees of loss aversion regarding LAE.

3 The optimal lower limit of quality level and the optimal unit collection incentive under DCLSC are affected by the compensation for the collection incentive of used products and the quality distribution of recyclable parts.

4 The shift to the optimal operation under ICLSC with profit sharing can encourage to promote not only the collection activity of used products, but also the recycling activity, improving the expected profits of the whole system and all members under ICSLC as the total optimisation.

As future researches, it will be necessary to incorporate the following topics into a CLSC model in this paper:

1 the optimal operation and supply chain coordination for a CLSC with loss-averse attitude of the decision maker, considering the uncertainties in not only the product demand but also the collectable quantity of used products from customers

2 incorporation of carbon emission and cap-and-trade policy into the optimal operation in a CLSC.

References

Bazan, E., Jaber, M.Y. and Zanoni, S. (2016) 'A review of mathematical inventory models for reverse logistics and the future of its modeling: an environmental perspective', *Applied Mathematical Modelling*, Vol. 40, Nos. 5–6, pp.4151–4178.

Govindan, K., Popiuc, M.N. and Diabat, A. (2013) 'Overview of coordination contracts within forward and reverse supply chains', *Journal of Cleaner Production*, Vol. 47, pp.319–334.

Govindan, K., Soleimani, H. and Kannan, D. (2017) 'A review of reverse logistics and closed-loop supply chains: a *Journal of Cleaner Production* focus', *Journal of Cleaner Production*, Vol. 142, Part 1, pp.371–384.

Govindan, K., Soleimani, H. and Kannan, D. (2014) 'Reverse logistics and closed-loop supply chain: a comprehensive review to explore the future', *European Journal of Operational Research*, Vol. 240, No. 3, pp.603–626.

Gurtu, A., Searcy, C. and Jaber, M.Y. (2015) 'An analysis of keywords used in the literature on green supply chain management', *Management Research Review*, Vol. 38, No. 2, pp.166–194.

Hu, B., Meng, C., Xu, D. and Son, Y-J. (2016) 'Three-echelon supply chain coordination with a loss-averse retailer and revenue sharing contracts', *International Journal of Production Economics*, Vol. 179, pp.192–202.

Nagarajan, M. and Sosic, G. (2008) ‘Game-theoretic analysis of cooperation among supply chain agents: review and extensions’, *European Journal of Operational Research*, Vol. 187, No. 3, pp.719–745.

Radhi, M. and Zhang, G. (2016) ‘Optimal configuration of remanufacturing supply network with return quality decision’, *International Journal of Production Research*, Vol. 54, No. 5, pp.1487–1502.

Schenkel, M., Caniëls, M.C.J., Krikke, H. and van der Laan, E. (2015) ‘Understanding value creation in closed loop supply chains – past findings and future directions’, *Journal of Manufacturing Systems*, Vol. 37, pp.729–745.

Souza, G.C. (2013) ‘Closed-loop supply chains: a critical review, and future research’, *Decision Sciences*, Vol. 44, No. 1, pp.7–38.

Stindt, D. and Sahamie, R. (2014) ‘Review of research on closed loop supply chain management in the process industry’, *Flexible Services and Manufacturing Journal*, Vol. 26, Nos. 1–2, pp.268–293.

Sundari, P.R.T. and Vijayalakshmi, C. (2016) ‘A comprehensive review of closed loop supply chain’, *Pure and Applied Mathematics*, Vol. 12, No. 4, pp.2785–2792.

Wang, C.X. and Webster, S. (2007) ‘Channel coordination for a supply chain with a risk-neutral manufacturer and a loss-averse retailer’, *Decision Science*, Vol. 38, No. 3, p.361.

Wang, C.X. and Webster, S. (2009) ‘The loss-averse newsvendor problem’, *Omega*, Vol. 37, No. 1, pp.93–105.

Watanabe, T. and Kusukawa, E. (2014) ‘Optimal operation for green supply chain considering demand information, collection incentive and quality of recycling parts’, *Industrial Engineering and Management Systems*, Vol. 13, No. 2, pp.129–147.

Watanabe, T., Kusuawa, E. and Arizono, I. (2013) ‘Optimal operation for green supply chain in consideration of collection incentive and quality for recycling of used products’, *Industrial Engineering & Management Systems*, Vol. 12, No. 4, pp.317–329.

Yamaguchi, T. and Kusukawa, E. (2018) ‘Optimal operation with flexible ordering policy for closed-loop supply chain considering the uncertainties in product demand and collection quantity of used products’, *Industrial Engineering and Management Systems*, Vol. 17, No. 3, in press.

Appendix A

The retailer’s expected utility function with LA in equation (7) can be rewritten as

$$\begin{aligned}
EU_{\mathrm{R}}^{\mathrm{LA}}(Q,t,u) &= -tA(t)-c_tA(t)+R(t)\int_u^1 A(t)g(\ell)d\ell \\
&\quad -(p+h_r+s)\int_0^Q F(x)+(p+s-w)Q-sE[x] \\
&\quad +\left(\lambda^{\mathrm{E}}-1\right)\left(p+h_r\right)q_1^{\mathrm{D}}F\left(q_1^{\mathrm{D}}\right)-\left(\lambda^{\mathrm{E}}-1\right)\left(p+h_r\right)\int_0^{q_1^{\mathrm{D}}} F(x)dx \\
&\quad -\left(\lambda^{\mathrm{E}}-1\right)\left(w+h_r\right)QF\left(q_1^{\mathrm{D}}\right) \\
&\quad +\left(\lambda^{\mathrm{S}}-1\right)(p+s-w)Q-\left(\lambda^{\mathrm{S}}-1\right)(p+s-w)QF\left(q_2^{\mathrm{D}}\right) \\
&\quad -\left(\lambda^{\mathrm{S}}-1\right)sE[x]+\left(\lambda^{\mathrm{S}}-1\right)sq_2^{\mathrm{D}}F\left(q_2^{\mathrm{D}}\right)-\left(\lambda^{\mathrm{S}}-1\right)s\int_0^{q_2^{\mathrm{D}}} F(x)dx
\end{aligned} \tag{A-1}$$

In case of attitude LA, the first and second partial derivatives of the retailer's expected utility function in equation (7) in terms of Q are obtained as

$$
\begin{aligned}
\frac{\partial EU_{\mathrm{R}}^{\mathrm{LA}}(Q,t,u)}{\partial Q} &= (p+s-w)-(p+h_r+s)F(Q) \\
&\quad +(\lambda^{\mathrm{E}}-1)(p+h_r)\frac{dq_1^{\mathrm{D}}}{dQ}F(q_1^{\mathrm{D}}) \\
&\quad +(\lambda^{\mathrm{E}}-1)(p+h_r)q_1^{\mathrm{D}}f(q_1^{\mathrm{D}})\frac{dq_1^{\mathrm{D}}}{dQ} \\
&\quad -(\lambda^{\mathrm{E}}-1)(p+h_r)F(q_1^{\mathrm{D}})\frac{dq_1^{\mathrm{D}}}{dQ} \\
&\quad -(\lambda^{\mathrm{E}}-1)(w+h_r)F(q_1^{\mathrm{D}})-(\lambda^{\mathrm{E}}-1)(w+h_r)Q\,f(q_1^{\mathrm{D}})\frac{dq_1^{\mathrm{D}}}{dQ} \\
&\quad +(\lambda^{\mathrm{S}}-1)(p+s-w)-(\lambda^{\mathrm{S}}-1)(p+s-w)F(q_2^{\mathrm{D}}) \\
&\quad -(\lambda^{\mathrm{S}}-1)(p+s-w)Q\,f(q_2^{\mathrm{D}})\frac{dq_2^{\mathrm{D}}}{dQ}+(\lambda^{\mathrm{S}}-1)s\frac{dq_2^{\mathrm{D}}}{dQ}F(q_2^{\mathrm{D}}) \\
&\quad +(\lambda^{\mathrm{S}}-1)sq_2^{\mathrm{D}}\,f(q_2^{\mathrm{D}})\frac{dq_2^{\mathrm{D}}}{dQ}-(\lambda^{\mathrm{S}}-1)s\,F(q_2^{\mathrm{D}})\frac{dq_2^{\mathrm{D}}}{dQ} \\
&= (p+s-w)-(p+h_r+s)F(Q)-(\lambda^{\mathrm{E}}-1)(w+h_r)F(q_1^{\mathrm{D}}) \\
&\quad +(\lambda^{\mathrm{E}}-1)(p+h_r)\frac{(w+h_r)Q}{(p+h_r)}f(q_1^{\mathrm{D}})\frac{(w+h_r)}{(p+h_r)} \\
&\quad -(\lambda^{\mathrm{E}}-1)(w+h_r)Q\,f(q_1^{\mathrm{D}})\frac{(w+h_r)}{(p+h_r)} \\
&\quad +(\lambda^{\mathrm{S}}-1)(p+s-w)-(\lambda^{\mathrm{S}}-1)(p+s-w)F(q_2^{\mathrm{D}}) \\
&\quad -(\lambda^{\mathrm{S}}-1)(p+s-w)Q\,f(q_2^{\mathrm{D}})\frac{dq_2^{\mathrm{D}}}{dQ} \\
&\quad +(\lambda^{\mathrm{S}}-1)s\frac{(p+s-w)Q}{s}f(q_2^{\mathrm{D}})\frac{dq_2^{\mathrm{D}}}{dQ} \\
&= (p+s-w)-(p+h_r+s)F(Q)-(\lambda^{\mathrm{E}}-1)(w+h_r)F(q_1^{\mathrm{D}}) \\
&\quad +(\lambda^{\mathrm{S}}-1)(p+s-w)-(\lambda^{\mathrm{S}}-1)(p+s-w)F(q_2^{\mathrm{D}}) \\
&= \lambda^{\mathrm{S}}(p+s-w)-(p+h_r+s)F(Q) \\
&\quad -(\lambda^{\mathrm{E}}-1)(w+h_r)F(q_1^{\mathrm{D}})-(\lambda^{\mathrm{S}}-1)(p+s-w)F(q_2^{\mathrm{D}})
\end{aligned}
\tag{A-2}
$$

$$\begin{aligned}\frac{\partial^2 EU_{\mathrm{R}}^{\mathrm{LA}}(Q,t,u)}{\partial^2 Q} &= -(p+h_r+s)\,f(Q)-(\lambda^{\mathrm{E}}-1)(w+h_r)\,f(q_1^{\mathrm{D}})\frac{dq_1^{\mathrm{D}}}{dQ}\\ &\quad -(\lambda^{\mathrm{S}}-1)(p+s-w)f(q_2^{\mathrm{D}})\frac{dq_2^{\mathrm{D}}}{dQ}\\ &= -(p+h_r+s)\,f(Q)-(\lambda^{\mathrm{E}}-1)(w+h_r)\,f(q_1^{\mathrm{D}})\frac{(w+h_r)}{(p+h_r)}\\ &\quad -(\lambda^{\mathrm{S}}-1)(p+s-w)f(q_2^{\mathrm{D}})\frac{(p+s-w)}{s}\\ &= -(p+h_r+s)\,f(Q)-(\lambda^{\mathrm{E}}-1)\frac{(w+h_r)^2}{(p+h_r)}\,f(q_1^{\mathrm{D}})\\ &\quad -(\lambda^{\mathrm{S}}-1)\frac{(p+s-w)^2}{s}\,f(q_2^{\mathrm{D}})\end{aligned} \tag{A-3}$$

The elicitation processes in equations (9) and (10) can be derived as equations (A-2) and (A-3).

Appendix B

The whole system's expected utility function with LA in equation (8) can be rewritten as

$$\begin{aligned}EU_{\mathrm{S}}^{\mathrm{LA}}(Q,t,u) &= -tA(t)-c_tA(t)-c_aA(t)-\int_u^1 A(t)c_r(\ell)g(\ell)d\ell\\ &\quad -c_d\int_0^u A(t)g(\ell)d\ell+c_n\int_u^1 A(t)g(\ell)d\ell\\ &\quad -(p+h_r+s)\int_0^Q F(x)+\{p+s-(c_n+c_m)\}Q\\ &\quad -sE[x]+(\lambda^{\mathrm{E}}-1)(p+h_r)\,q_1^{\mathrm{I}}F(q_1^{\mathrm{I}})\\ &\quad -(\lambda^{\mathrm{E}}-1)(p+h_r)\int_0^{q_1^{\mathrm{I}}} F(x)dx\\ &\quad -(\lambda^{\mathrm{E}}-1)\{(c_m+c_n)+h_r\}QF(q_1^{\mathrm{I}})\\ &\quad +(\lambda^{\mathrm{S}}-1)\{p+s-(c_m+c_n)\}Q\\ &\quad -(\lambda^{\mathrm{S}}-1)\{p+s-(c_m+c_n)\}QF(q_2^{\mathrm{I}})-(\lambda^{\mathrm{S}}-1)sE[x]\\ &\quad +(\lambda^{\mathrm{S}}-1)sq_2^{\mathrm{I}}F(q_2^{\mathrm{I}})-(\lambda^{\mathrm{S}}-1)s\int_0^{q_2^{\mathrm{I}}} F(x)dx\end{aligned} \tag{B-1}$$

In case of attitude LA, the first and second partial derivatives of the whole system's expected utility function in equation (8) in terms of Q are obtained as

$$
\begin{aligned}
\frac{\partial EU_{\mathrm{S}}^{\mathrm{LA}}(Q,t,u)}{\partial Q} =& \left\{ p+s-(c_n+c_m) \right\} - \left(p+h_r+s \right) F(Q) \\
& +(\lambda^{\mathrm{E}}-1)\left(p+h_r \right) \frac{dq_1^{\mathrm{I}}}{dQ} F\left(q_1^{\mathrm{I}} \right) \\
& +(\lambda^{\mathrm{E}}-1)\left(p+h_r \right) q_1^{\mathrm{I}} f\left(q_1^{\mathrm{I}} \right) \frac{dq_1^{\mathrm{I}}}{dQ} \\
& -(\lambda^{\mathrm{E}}-1)\left(p+h_r \right) F\left(q_1^{\mathrm{I}} \right) \frac{dq_1^{\mathrm{I}}}{dQ} \\
& -(\lambda^{\mathrm{E}}-1)\left\{ (c_m+c_n)+h_r \right\} F\left(q_1^{\mathrm{I}} \right) \\
& -(\lambda^{\mathrm{E}}-1)\left\{ (c_m+c_n)+h_r \right\} Q\, f\left(q_1^{\mathrm{I}} \right) \frac{dq_1^{\mathrm{I}}}{dQ} \\
& +(\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} \\
& -(\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} F\left(q_2^{\mathrm{I}} \right) \\
& -(\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} Q\, f\left(q_2^{\mathrm{I}} \right) \frac{dq_2^{\mathrm{I}}}{dQ} \\
& +(\lambda^{\mathrm{S}}-1)\, s \frac{dq_2^{\mathrm{I}}}{dQ} F\left(q_2^{\mathrm{I}} \right) + (\lambda^{\mathrm{S}}-1)\, s\, q_2^{\mathrm{I}}\ f\left(q_2^{\mathrm{I}} \right) \frac{dq_2^{\mathrm{I}}}{dQ} \\
& -(\lambda^{\mathrm{S}}-1)\, s\, F\left(q_2^{\mathrm{I}} \right) \frac{dq_2^{\mathrm{I}}}{dQ}
\end{aligned}
\tag{B-2}
$$

$$
\begin{aligned}
=& \left\{ p+s-(c_n+c_m) \right\} - \left(p+h_r+s \right) F(Q) \\
& -(\lambda^{\mathrm{E}}-1)\left\{ (c_m+c_n)+h_r \right\} F\left(q_1^{\mathrm{I}} \right) \\
& +(\lambda^{\mathrm{E}}-1)\left(p+h_r \right) \frac{\left\{ (c_m+c_n)+h_r \right\} Q}{\left(p+h_r \right)} f\left(q_1^{\mathrm{I}} \right) \frac{\left\{ (c_m+c_n)+h_r \right\}}{\left(p+h_r \right)} \\
& -(\lambda^{\mathrm{E}}-1)\left\{ (c_m+c_n)+h_r \right\} Q\, f\left(q_1^{\mathrm{I}} \right) \frac{\left\{ (c_m+c_n)+h_r \right\}}{\left(p+h_r \right)} \\
& +(\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} \\
& -(\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} F\left(q_2^{\mathrm{I}} \right) \\
& -(\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} Q\, f\left(q_2^{\mathrm{I}} \right) \frac{dq_2^{\mathrm{I}}}{dQ} \\
& +(\lambda^{\mathrm{S}}-1)\, s \frac{\left\{ p+s-(c_m+c_n) \right\} Q}{s}\ f\left(q_2^{\mathrm{I}} \right) \frac{dq_2^{\mathrm{I}}}{dQ} \\
=& \left\{ p+s-(c_n+c_m) \right\} - \left(p+h_r+s \right) F(Q) \\
& -(\lambda^{\mathrm{E}}-1)\left\{ (c_m+c_n)+h_r \right\} F\left(q_1^{\mathrm{I}} \right) + (\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} \\
& -(\lambda^{\mathrm{S}}-1)\left\{ p+s-(c_m+c_n) \right\} F\left(q_2^{\mathrm{I}} \right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 EU_{\mathrm{S}}^{\mathrm{LA}}(Q,t,u)}{\partial^2 Q} &= -\left(p+h_r+s\right) f(Q) - (\lambda^{\mathrm{E}}-1)\left\{(c_m+c_n)+h_r\right\} f\left(q_1^{\mathrm{I}}\right)\frac{dq_1^{\mathrm{I}}}{dQ} \\
&\quad -(\lambda^{\mathrm{S}}-1)\left\{p+s-(c_m+c_n)\right\} f\left(q_2^{\mathrm{I}}\right)\frac{dq_2^{\mathrm{I}}}{dQ} \\
&= -\left(p+h_r+s\right) f(Q) - (\lambda^{\mathrm{E}}-1)\frac{\left\{(c_m+c_n)+h_r\right\}^2}{\left(p+h_r\right)} f\left(q_1^{\mathrm{I}}\right) \\
&\quad -(\lambda^{\mathrm{S}}-1)\frac{\left\{p+s-(c_m+c_n)\right\}^2}{s} f\left(q_2^{\mathrm{I}}\right)
\end{aligned}
\tag{B-3}
$$

The elicitation processes in equations (15) and (16) can be derived as equations (B-2) and (B-3).