大阪公立大学 Osaka Metropolitan University 学術情報リポジトリ

Optimal Linear Codes Over the Field of Order 7

メタデータ	言語: eng 出版者: 公開日: 2021-04-19 キーワード (Ja): キーワード (En): 作成者: Nomura, Keita, Maruta, Tatsuya メールアドレス: 所属:
URL	http://hdl.handle.net/10466/00017339

Optimal Linear Codes Over the Field of Order 7

1st Keita Nomura
Department of Mathematical Sciences
Osaka Prefecture University
Sakai, Osaka 599-8531, Japan
sab01109@edu.osakafu-u.ac.jp

2nd Tatsuya Maruta
Department of Mathematical Sciences
Osaka Prefecture University
Sakai, Osaka 599-8531, Japan
maruta@mi.s.osakafu-u.ac.jp

***Abstract*—We construct some new linear codes over the field of order 7 to determine the exact value of the minimum length for which a linear code of dimension four with given minimum weight exists for some open cases. Most of the new codes are constructed as projective duals of some 7-divisible codes from some orbits of a projectivity in the projective space.**

***Index Terms*—linear code, divisible code, projective dual**

I. Introduction

Let $\mathbb{F}_q$ denote the field of order q and let $\mathbb{F}_q^n$ be the vector space of n-tuples over $\mathbb{F}_q$. For a vector $a \in \mathbb{F}_q^n$, the *weight* of a is the number of non-zero entries in a. An $[n, k, d]_q$ code $\mathcal{C}$ is a k-dimensional subspace of $\mathbb{F}_q^n$ with minimum weight d. Let A_i be the number of codewords of $\mathcal{C}$ with weight i. The list of non-zero A_i's is called the weight distribution of $\mathcal{C}$. The weight distribution with $(A_0, A_d, ...) = (1, w_d, ...)$ is expressed as $0^1 d^{w_d} \cdots$ in this paper. A fundamental and classical problem in coding theory is to determine the exact values of $n_q(k, d)$, the minimum length n for which an $[n, k, d]_q$ code exists [5]. The Griesmer bound gives a lower bound on the length n:

$$n \geq g_q(k, d) := \sum_{i=0}^{k-1} \left\lceil d/q^i \right\rceil ,$$

where $\lceil x \rceil$ stands for the minimum integer greater than or equal to x, see [5]. The values of $n_q(k, d)$ have been determined for all d only for some small order q with small dimensions k [9]. For linear codes over $\mathbb{F}_7$, the value of $n_7(k, d)$ is known for all d for $k \leq 3$ but not determined yet for many integers d for $k = 4$. The following two theorems are already known, see [1], [4], [6]–[8].

Theorem 1. *$n_7(3, d) = g_7(3, d) + 1$ for $d = 7, 13, 14,$ 19-21, 25-28, 31-35 and $n_7(4, d) = g_7(4, d)$ for any other d.*

Theorem 2. (i) *$n_7(4, d) = g_7(4, d)$ for $d \in \{$1-5, 8-10, 15, 36-42, $50, 246$-252, 288-385, 442-462, 491-504$\}$, and for all $d \geq 540$.*
(ii) *$n_7(4, d) = g_7(4, d) + 1$ for $d \in \{6, 7,$ 11-14, 18-21, $25, 26,$ 31-35, 43-48, $56, 69, 70,$ 75-77, 80-98, 190-196, 211-214, 239-245, 257-259, 264-287, 386-391, 428-441, 470-490, 519-539$\}$.*
(iii) *$n_7(4, d) \geq g_7(4, d) + 1$ for $d \in \{27, 28,$ 124-147, 169-189, 215-238, 392$\}$.*

This research is partially supported by JSPS KAKENHI Grant Number 20K03722.

The aim of this paper is to determine $n_7(4, d)$ for some values of d by constructing new codes. Our results are summarized as follows.

Theorem 3. (i) *There exists a $[g_7(4, d), 4, d]_7$ code for $d = 399$.*
(ii) *There exists a $[g_7(4, d) + 1, 4, d]_7$ code for $d = 175,$ 182, 189, 217, 224, 231, 238, 392.*
(iii) *There exists a $[g_7(4, d) + 2, 4, d]_7$ code for $d = 143$.*

Corollary 4. (i) *$n_7(4, d) = g_7(4, d)$ for $393 \leq d \leq 399$.*
(ii) *$n_7(4, d) = g_7(4, d) + 1$ for $d \in \{$169-189, 215-238, 392$\}$.*
(iii) *$n_7(4, d) = g_7(4, d) + 1$ or $g_7(4, d) + 2$ for $141 \leq d \leq 143$.*

We give the updated table for $n_7(4, d)$ as Table I. We give the values and bounds for $g = g_7(4, d)$ and $n = n_7(4, d)$ for all d except for $d > 567$ which are the cases satisfying $n_7(4, d) = g_7(4, d)$ by Theorem 2. In the table, "*a*-*b*" stands for $g_7(4, d) + a \leq n_7(4, d) \leq g_7(4, d) + b$.

II. Some methods for constructing new codes

In this section, we give some geometric methods to construct new codes. As usual, PG(r, q) denotes the projective geometry of dimension r over $\mathbb{F}_q$. The j-dimensional projective subspaces of PG(r, q) are called *j-spaces*. The 0-spaces, 1-spaces, 2-spaces and $(r-1)$-spaces are called *points, lines, planes* and *hyperplanes*, respectively.

Let $\mathcal{C}$ be an $[n, k, d]_q$ code with generator matrix G having no all-zero column. Then, the columns of G can be considered as a multiset of n points in $\Sigma = $ PG$(k-1, q)$ denoted by $\mathcal{G}$. We denote by $\mathcal{S}_j$ the set of j-spaces of Σ and by θ_j the number of points in a j-space, which can be calculated as $\theta_j = (q^{j+1} - 1)/(q - 1)$. An *$i$-point* is a point of Σ which appears exactly i times as columns of G. We denote by γ_0 the maximum multiplicity of a point of Σ in $\mathcal{G}$. Let Λ_i be the set of i-points in Σ and let $\lambda_i = |\Lambda_i|$, $0 \leq i \leq \gamma_0$, where $|T|$ denotes the number of elements in a set T. For any set S in Σ, the multiplicity of S, denoted by $m_{\mathcal{G}}(S)$, is naturally defined as $m_{\mathcal{G}}(S) = \sum_{i=1}^{\gamma_0} i \cdot |S \cap \Lambda_i|$. A line ℓ is called a *t-line* if $t = m_{\mathcal{G}}(\ell)$. A *$t$-plane* and so on are similarly defined. Then we obtain the partition $\bigcup_{i=0}^{\gamma_0} \Lambda_i$ of Σ such that $n = m_{\mathcal{G}}(\Sigma)$ and that the maximum multiplicity of a hyperplane of Σ is equal to $n-d$ [5]. Such a partition of Σ is called an $(n, n-d)$-*arc* of Σ. Conversely an $(n, n-d)$-arc of Σ gives $[n, k, d]_q$ codes all of which are equivalent. Denote by a_i the number of i-hyperplanes in Σ. The list of the non-zero values a_i is called the *spectrum* of $\mathcal{C}$. The spectrum can be obtained from the weight distribution as $a_i = A_{n-i}/(q-1)$ for $0 \le i \le n-d$.

For a non-zero element α of $\mathbb{F}_q$, let $R = \mathbb{F}_q[x]/(x^N - \alpha)$ be the ring of polynomials over $\mathbb{F}_q$ modulo $x^N - \alpha$. We associate the vector $(a_0, a_1, ..., a_{N-1}) \in \mathbb{F}_q^N$ with polynomial $a(x) = \sum_{i=0}^{N-1} a_i x^i \in R$. For $\mathbf{g} = (g_1(x), \cdots, g_s(x)) \in R^s$,

$$C_{\mathbf{g}} = \{(r(x)g_1(x), \cdots, r(x)g_s(x)) \mid r(x) \in R\}$$

is called the 1-*generator quasi-twisted code* with generator $\mathbf{g}$. $C_{\mathbf{g}}$ is called *quasi-cyclic* if $\alpha = 1$. When $s = 1$, $C_{\mathbf{g}}$ is called *constacyclic* or *pseudo-cyclic* and is simply called cyclic if $\alpha = 1$. Take a monic polynomial $g(x) = x^k - \sum_{i=0}^{k-1} a_i x^i$ in $\mathbb{F}_q[x]$ dividing $x^N - \alpha$ and let T be the companion matrix of $g(x)$. Let τ be the projectivity of $\mathrm{PG}(k-1, q)$ defined by T. We denote by $[g^n]$ or by $[a_0 a_1 \cdots a_{k-1}^n]$ the $k \times n$ matrix $[P, TP, T^2P, ..., T^{n-1}P]$, where P is the column vector $(1, 0, 0, \cdots, 0)^{\mathrm{T}}$. Then $[g^N]$ generates an α^{-1}-cyclic code. Thus, cyclic or pseudo-cyclic codes can be constructed from an orbit of a given projectivity. The matrix

$$[P, TP, T^2P, ..., T^{n_1-1}P; P_2, TP_2, ..., T^{n_2-1}P_2; \\ \cdots; P_s, TP_s, ..., T^{n_s-1}P_s]$$

is denoted by $[g^{n_1}] + P_2^{n_2} + \cdots + P_s^{n_s}$. Then, the $k \times sN$ matrix $[g^N] + P_2^N + \cdots + P_s^N$ defined from s orbits of τ of length N generates a quasi-twisted (or quasi-cyclic) code [11]. There are many good codes constructed from some orbits of projectivities, see [11].

An $[n, k, d]_q$ code $\mathcal{C}$ is called m-*divisible* (or m-div for short) if the weight of every codeword of $\mathcal{C}$ is divisible by an integer $m > 1$. It could happen that some quasi-twisted codes are divisible or can be extended to divisible codes, see the next section.

Lemma 5 ([12]). *Let $\mathcal{C}$ be an m-div $[n, k, d]_q$ code with $q = p^h$, p prime. Assume $\lambda_0 > 0$ and that $\mathcal{C}$ has spectrum*

$$(a_{n-d-(w-1)m}, a_{n-d-(w-2)m}, \cdots, a_{n-d-m}, a_{n-d}) \\ = (\alpha_{w-1}, \alpha_{w-2}, \cdots, \alpha_1, \alpha_0),$$

where $m = p^r$ for some $1 \le r < h(k-2)$ satisfying

$$\bigcap_{H \in \mathcal{S}_{k-2},\ m_{\mathcal{G}}(H) < n-d} H = \emptyset.$$

Then, there exists a t-div $[n^*, k, d^*]_q$ code $\mathcal{C}^*$ such that

$$t = \frac{q^{k-2}}{m},\ n^* = ntq - \frac{d}{m}\theta_{k-1},\ d^* = ((n-d)q - n)t$$

with spectrum

$$(a_{n^*-d^*-\gamma_0 t}, a_{n^*-d^*-(\gamma_0-1)t}, \cdots, a_{n^*-d^*-t}, a_{n^*-d^*}) \\ = (\lambda_{\gamma_0}, \lambda_{\gamma_0-1}, \cdots, \lambda_1, \lambda_0).$$

A generator matrix of $\mathcal{C}^*$ can be obtained by considering the $(n-d-jm)$-hyperplanes as the j-points in the dual space Σ^* of Σ for $0 \le j \le w-1$ [12]. $\mathcal{C}^*$ is called a *projective dual* of $\mathcal{C}$, see also [2] and [3].

Lemma 6 ([10]). *Let $\mathcal{C}$ be an $[n, k, d]_q$ code with generator matrix G giving the multiset $\mathcal{G}$ of n points in $\Sigma = \mathrm{PG}(k-1, q)$. If $\mathcal{G}$ contains a u-space U and if $d > q^u$, then an $[n-\theta_u, k, d']_q$ code $\mathcal{C}'$ with $d' \ge d - q^u$ exists.*

The code $\mathcal{C}'$ in Lemma 6 can be constructed from $\mathcal{C}$ by removing the u-space U from the multiset $\mathcal{G}$, that is, by deleting the θ_u columns of G corresponding to U. This construction method is called *geometric puncturing* [8].

III. Proof of Theorem 3

Lemma 7. *There exists a quasi-twisted $[170, 4, 143]_7$ code.*

Proof. Let $\mathcal{C}$ be the quasi-twisted $[170, 4, 143]_7$ code with generator matrix

$$[3362^{10}] + 0011^{10} + 0041^{10} + 0051^{10} + 0401^{10} + 0301^{10} \\ +0221^{10} + 0121^{10} + 0411^{10} + 0361^{10} + 0511^{10} + 0321^{10} \\ +0631^{10} + 2001^{10} + 3001^{10} + 6041^{10} + 5061^{10},$$

where 3362 defines the polynomial $g(x) = x^4 - (3 + 3x + 6x^2 + 2x^3)$ dividing $x^{10} - 3$ and 0011 stands for the point $\mathbf{P}(0, 0, 1, 1)$ in $\mathrm{PG}(3, 7)$. The weight distribution of $\mathcal{C}$ is

$$0^1 143^{720} 144^{540} 145^{480} 149^{360} 151^{180} 156^{120}.$$

□

Lemma 8. *There exist a $[206, 4, 175]_7$ code.*

Proof. Let $\mathcal{C}$ be the $[94, 4, 77]_7$ code with generator matrix

$$[3050^6] + 1000^6 + 0011^6 + 0011^6 + 0111^6 + 0531^6 + 0361^6 \\ +0141^6 + 0621^6 + 6661^6 + 1121^6 + 6651^6 + 4141^6 + 5331^6 \\ +4151^3 + 1351^2 + 1351^2 + 5621^1 + 5621^1 + 4361^1.$$

Then $\mathcal{C}$ is 7-divisible since the weight distribution of $\mathcal{C}$ is $0^1 77^{1170} 84^{1224} 91^6$. As a projective dual of $\mathcal{C}$, we obtain a $[206, 4, 175]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 175^{1926} 182^{384} 189^{90}$. □

Lemma 9. *There exist a $[214, 4, 182]_7$ code.*

Proof. Let $\mathcal{C}$ be the 7-div $[86, 4, 70]_7$ code generated by the matrix

$$[3050^6] + 1000^6 + 0061^6 + 0441^6 + 0441^6 + 0261^6 \\ +0561^6 + 0151^6 + 3321^6 + 2461^6 + 2431^6 + 2621^6 \\ +2621^6 + 5341^2 + 5341^2 + 4631^2 + 4631^2$$

with weight distribution $0^1 70^{1152} 77^{1212} 84^{36}$. As a projective dual of $\mathcal{C}$, we get a $[214, 4, 182]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 182^{2016} 189^{252} 196^{132}$. □

Lemma 10. *There exist a $[222, 4, 189]_7$ code.*

Proof. Let $\mathcal{C}$ be the 7-div $[78, 4, 63]_7$ code generated by the matrix

$$[3050^6] + 0011^6 + 0011^6 + 0531^6 + 0531^6 + 0131^6 + 0351^6 + 0311^6 + 5551^6 + 1121^6 + 2461^6 + 5531^3 + 0301^2 + 0301^2 + 5621^1 + 5621^1 + 5621^1 + 4361^1 + 4361^1$$

with weight distribution $0^1 63^{1068} 70^{1332}$. As a projective dual of $\mathcal{C}$, we obtain a $[222, 4, 189]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 189^{2034} 196^{270} 203^{90} 210^6$. □

Lemma 11. *There exist a* $[255, 4, 217]_7$ *code.*

Proof. Let $\mathcal{C}$ be the 7-div $[95, 4, 77]_7$ code with generator matrix

$$[1106^6] + 0041^6 + 0021^6 + 0441^6 + 0241^6 + 0641^6 + 0141^6 + 0351^6 + 0631^6 + 0541^6 + 0161^6 + 4051^6 + 3661^6 + 0561^3 + 4041^3 + 0111^2 + 4551^2 + 5661^2 + 4631^1 + 4631^1 + 4631^1 + 4631^1 + 4631^1$$

with weight distribution $0^1 77^{870} 84^{1530}$. As a projective dual of $\mathcal{C}$, we get a $[255, 4, 217]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 217^{1854} 224^{540} 252^6$. □

Lemma 12. *There exists a* $[263, 4, 224]_7$ *code.*

Proof. Let $\mathcal{C}$ be the 7-div $[87, 4, 70]_7$ code generated by the matrix

$$[1106^6] + 0051^6 + 0441^6 + 0441^6 + 0221^6 + 0421^6 + 0151^6 + 0251^6 + 0251^6 + 2021^6 + 4051^6 + 0561^3 + 0231^3 + 2431^3 + 5141^3 + 5661^2 + 5661^2 + 2651^1 + 2651^1 + 2651^1 + 2651^1 + 4631^1$$

with weight distribution $0^1 70^{876} 77^{1470} 84^{54}$. As a projective dual of $\mathcal{C}$, one can obtain a $[263, 4, 224]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 224^{1944} 231^{390} 238^{66}$. □

Lemma 13. *There exists a* $[271, 4, 231]_7$ *code.*

Proof. Let $\mathcal{C}$ be the 7-div $[79, 4, 63]_7$ code with generator matrix

$$[1106^6] + 0041^6 + 0041^6 + 0051^6 + 0061^6 + 0361^6 + 0641^6 + 0461^6 + 3661^6 + 2461^6 + 0231^3 + 0341^3 + 3051^3 + 3641^3 + 3641^3 + 6001^1 + 6001^1 + 6001^1 + 2651^1$$

with weight distribution $0^1 63^{774} 70^{1626}$. As a projective dual of $\mathcal{C}$, we get a $[271, 4, 231]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 231^{1992} 238^{348} 245^{54} 252^6$. □

Lemma 14. *There exists a* $[279, 4, 238]_7$ *code.*

Proof. Let $\mathcal{C}$ be the 7-div $[71, 4, 56]_7$ code generated by the matrix

$$[1106^6] + 0041^6 + 0041^6 + 0131^6 + 0641^6 + 0141^6 + 0141^6 + 0421^6 + 0521^6 + 0341^3 + 4151^3 + 5141^3 + 0111^2 + 1221^1 + 1221^1 + 1221^1 + 1221^1 + 2651^1 + 2651^1$$

with weight distribution $0^1 56^{762} 63^{1602} 70^{36}$. As a projective dual of $\mathcal{C}$, one can obtain a $[279, 4, 238]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 238^{2070} 245^{246} 252^{78} 266^6$. □

Lemma 15. *There exist* $[467, 4, 399]_7$ *and* $[459, 4, 392]_7$ *codes.*

Proof. Let $\mathcal{C}$ be the $[83, 4, 63]_7$ code with generator matrix

$$[1106^6] + 0051^6 + 0021^6 + 0501^6 + 0301^6 + 0661^6 + 0361^6 + 3031^6 + 1021^6 + 4051^6 + 4161^6 + 0451^3 + 4041^3 + 3051^3 + 5141^3 + 4551^2 + 5661^2 + 1221^1.$$

Then $\mathcal{C}$ is 7-divisible since the weight distribution of $\mathcal{C}$ is $0^1 63^{30} 70^{1938} 77^{432}$. As a projective dual of $\mathcal{C}$, we obtain a $[467, 4, 399]_7$ code $\mathcal{C}^*$ whose weight distribution is $0^1 399^{1902} 406^{498}$. It can be checked with the aid of a computer that the multiset for $\mathcal{C}^*$ contains the line

$$\langle 0001, 0110 \rangle = \{0001, 0110, 0111, 0112, \ldots, 0116\}.$$

Hence, we get a $[459, 4, 392]_7$ code by Lemma 6. □

Now, Theorem 3 follows from Lemmas 7-15 since $g_7(4, 143) = 168$, $g_7(4, 175) = 205$, $g_7(4, 182) = 213$, $g_7(4, 189) = 221$, $g_7(4, 217) = 254$, $g_7(4, 224) = 262$, $g_7(4, 231) = 270$, $g_7(4, 238) = 278$, $g_7(4, 392) = 458$, $g_7(4, 399) = 467$. □

References

[1] N. Bono, M. Fujii, T. Maruta, "On optimal linear codes of dimension 4," J. Algebra Comb. Discrete Appl., in press.

[2] I.G. Bouyukliev, "Classification of Griesmer codes and dual transform," Discrete Math. vol. 309, pp. 4049–4068, 2009.

[3] A.E. Brouwer, M. van Eupen, "The correspondence between projective codes and 2-weight codes," Des. Codes Cryptogr., vol. 11, pp. 261–266, 1997.

[4] M. Grassl, Linear code bound [electronic table; online], `http://www.codetables.de/`.

[5] R. Hill, "Optimal linear codes," in *Cryptography and Coding II*, C. Mitchell, Ed., Oxford Univ. Press, Oxford, 1992, pp. 75–104.

[6] K. Kumegawa and T. Maruta, "Non-existence of some 4-dimensional Griesmer codes over finite fields," J. Algebra Comb. Discrete Appl., vol. 5, pp. 101–116, 2018.

[7] T. Maruta, "On the minimum length of q-ary linear codes of dimension four," Discrete Math. vol. 208/209, pp. 427–435, 1999.

[8] T. Maruta, "Construction of optimal linear codes by geometric puncturing," Serdica J. Computing, vol. 7, pp. 73–80, 2013.

[9] T. Maruta, Griesmer bound for linear codes over finite fields," `http://mars39.lomo.jp/opu/griesmer.htm.`

[10] T. Maruta, Y. Oya, "On optimal ternary linear codes of dimension 6," Adv. Math. Commun., vol. 5, pp. 505–520, 2011.

[11] T. Maruta, M. Shinohara, M. Takenaka, "Constructing linear codes from some orbits of projectivities," Discrete Math., vol. 308, pp. 832–841, 2008.

[12] M. Takenaka, K. Okamoto, T. Maruta, "On optimal non-projective ternary linear codes," Discrete Math., vol. 308, pp. 842–854, 2008.

TABLE I

VALUES AND BOUNDS FOR $n = n_7(4, d)$ WITH $g = g_7(4, d)$

d	g	n	d	g	n	d	g	n
1	4	4	64	77	0-1	127	150	1-2
2	5	5	65	78	0-1	128	151	1-2
3	6	6	66	79	0-1	129	152	1-2
4	7	7	67	80	0-1	130	153	1-2
5	8	8	68	81	0-1	131	154	1-2
6	9	10	69	82	83	132	155	1-2
7	10	11	70	83	84	133	156	1-2
8	12	12	71	85	0-1	134	158	1-2
9	13	13	72	86	0-1	135	159	1-2
10	14	14	73	87	0-1	136	160	1-2
11	15	16	74	88	0-1	137	161	1-2
12	16	17	75	89	90	138	162	1-2
13	17	18	76	90	91	139	163	1-2
14	18	19	77	91	92	140	164	1-2
15	20	20	78	93	0-1	141	166	1-2
16	21	0-1	79	94	0-1	142	167	1-2
17	22	0-1	80	95	96	143	168	1-2
18	23	24	81	96	97	144	169	1-3
19	24	25	82	97	98	145	170	1-3
20	25	26	83	98	99	146	171	1-3
21	26	27	84	99	100	147	172	1-3
22	28	0-1	85	101	102	148	175	0-2
23	29	0-1	86	102	103	149	176	0-2
24	30	0-1	87	103	104	150	177	0-2
25	31	32	88	104	105	151	178	0-2
26	32	33	89	105	106	152	179	0-2
27	33	1-2	90	106	107	153	180	0-2
28	34	1-2	91	107	108	154	181	0-2
29	36	0-1	92	109	110	155	183	0-1
30	37	0-1	93	110	111	156	184	0-1
31	38	39	94	111	112	157	185	0-1
32	39	40	95	112	113	158	186	0-1
33	40	41	96	113	114	159	187	0-1
34	41	42	97	114	115	160	188	0-1
35	42	43	98	115	116	161	189	0-1
36	44	44	99	118	0-1	162	191	0-2
37	45	45	100	119	0-1	163	192	0-2
38	46	46	101	120	0-1	164	193	0-2
39	47	47	102	121	0-1	165	194	0-2
40	48	48	103	122	0-1	166	195	0-2
41	49	49	104	123	0-1	167	196	0-2
42	50	50	105	124	0-2	168	197	0-3
43	52	53	106	126	0-1	169	199	200
44	53	54	107	127	0-1	170	200	201
45	54	55	108	128	0-1	171	201	202
46	55	56	109	129	0-1	172	202	203
47	56	57	110	130	0-1	173	203	204
48	57	58	111	131	0-1	174	204	205
49	58	1-2	112	132	0-1	175	205	206
50	61	61	113	134	0-2	176	207	208
51	62	0-1	114	135	0-2	177	208	209
52	63	0-1	115	136	0-2	178	209	210
53	64	0-1	116	137	0-2	179	210	211
54	65	0-1	117	138	0-2	180	211	212
55	66	0-1	118	139	0-2	181	212	213
56	67	68	119	140	0-2	182	213	214
57	69	0-1	120	142	0-2	183	215	216
58	70	0-1	121	143	0-2	184	216	217
59	71	0-1	122	144	0-2	185	217	218
60	72	0-1	123	145	0-2	186	218	219
61	73	0-1	124	146	1-2	187	219	220
62	74	0-1	125	147	1-2	188	220	221
63	75	0-2	126	148	1-2	189	221	222

d	g	n	d	g	n	d	g	n
190	223	224	253	297	0-1	316	370	370
191	224	225	254	298	0-1	317	371	371
192	225	226	255	299	0-1	318	372	372
193	226	227	256	300	0-1	319	373	373
194	227	228	257	301	302	320	374	374
195	228	229	258	302	303	321	375	375
196	229	230	259	303	304	322	376	376
197	232	0-1	260	305	0-1	323	378	378
198	233	0-1	261	306	0-1	324	379	379
199	234	0-1	262	307	0-1	325	380	380
200	235	0-1	263	308	0-1	326	381	381
201	236	0-1	264	309	310	327	382	382
202	237	0-1	265	310	311	328	383	383
203	238	0-2	266	311	312	329	384	384
204	240	0-1	267	313	314	330	386	386
205	241	0-1	268	314	315	331	387	387
206	242	0-1	269	315	316	332	388	388
207	243	0-1	270	316	317	333	389	389
208	244	0-1	271	317	318	334	390	390
209	245	0-2	272	318	319	335	391	391
210	246	0-2	273	319	320	336	392	392
211	248	249	274	321	322	337	394	394
212	249	250	275	322	323	338	395	395
213	250	251	276	323	324	339	396	396
214	251	252	277	324	325	340	397	397
215	252	253	278	325	326	341	398	398
216	253	254	279	326	327	342	399	399
217	254	255	280	327	328	343	400	400
218	256	257	281	329	330	344	404	404
219	257	259	282	330	331	345	405	405
220	258	260	283	331	332	346	406	406
221	259	261	284	332	333	347	407	407
222	260	262	285	333	334	348	408	408
223	261	263	286	334	335	349	409	409
224	262	263	287	335	336	350	410	410
225	264	265	288	337	337	351	412	412
226	265	266	289	338	338	352	413	413
227	266	267	290	339	339	353	414	414
228	267	268	291	340	340	354	415	415
229	268	269	292	341	341	355	416	416
230	269	270	293	342	342	356	417	417
231	270	271	294	343	343	357	418	418
232	272	273	295	346	346	358	420	420
233	273	274	296	347	347	359	421	421
234	274	275	297	348	348	360	422	422
235	275	276	298	349	349	361	423	423
236	276	277	299	350	350	362	424	424
237	277	278	300	351	351	363	425	425
238	278	279	301	352	352	364	426	426
239	280	281	302	354	354	365	428	428
240	281	282	303	355	355	366	429	429
241	282	283	304	356	356	367	430	430
242	283	284	305	357	357	368	431	431
243	284	285	306	358	358	369	432	432
244	285	286	307	359	359	370	433	433
245	286	287	308	360	360	371	434	434
246	289	289	309	362	362	372	436	436
247	290	290	310	363	363	373	437	437
248	291	291	311	364	364	374	438	438
249	292	292	312	365	365	375	439	439
250	293	293	313	366	366	376	440	440
251	294	294	314	367	367	377	441	441
252	295	295	315	368	368	378	442	442

d	g	n	d	g	n	d	g	n
379	444	444	442	518	518	505	591	0-1
380	445	445	443	519	519	506	592	0-1
381	446	446	444	520	520	507	593	0-1
382	447	447	445	521	521	508	594	0-1
383	448	448	446	522	522	509	595	0-1
384	449	449	447	523	523	510	596	0-1
385	450	450	448	524	524	511	597	0-1
386	452	453	449	526	526	512	599	0-1
387	453	454	450	527	527	513	600	0-1
388	454	455	451	528	528	514	601	0-1
389	455	456	452	529	529	515	602	0-1
390	456	457	453	530	530	516	603	0-1
391	457	458	454	531	531	517	604	0-1
392	458	459	455	532	532	518	605	0-1
393	461	461	456	534	534	519	607	608
394	462	462	457	535	535	520	608	609
395	463	463	458	536	536	521	609	610
396	464	464	459	537	537	522	610	611
397	465	465	460	538	538	523	611	612
398	466	466	461	539	539	524	612	613
399	467	467	462	540	540	525	613	614
400	469	0-1	463	542	0-1	526	615	616
401	470	0-1	464	543	0-1	527	616	617
402	471	0-1	465	544	0-1	528	617	618
403	472	0-1	466	545	0-1	529	618	619
404	473	0-1	467	546	0-1	530	619	620
405	474	0-1	468	547	0-1	531	620	621
406	475	0-1	469	548	0-1	532	621	622
407	477	0-1	470	550	551	533	623	624
408	478	0-1	471	551	552	534	624	625
409	479	0-1	472	552	553	535	625	626
410	480	0-1	473	553	554	536	626	627
411	481	0-1	474	554	555	537	627	628
412	482	0-1	475	555	556	538	628	629
413	483	0-1	476	556	557	539	629	630
414	485	0-1	477	558	559	540	632	632
415	486	0-1	478	559	560	541	633	633
416	487	0-1	479	560	561	542	634	634
417	488	0-1	480	561	562	543	635	635
418	489	0-1	481	562	563	544	636	636
419	490	0-1	482	563	564	545	637	637
420	491	0-1	483	564	565	546	638	638
421	493	0-1	484	566	567	547	640	640
422	494	0-1	485	567	568	548	641	641
423	495	0-1	486	568	569	549	642	642
424	496	0-1	487	569	570	550	643	643
425	497	0-1	488	570	571	551	644	644
426	498	0-1	489	571	572	552	645	645
427	499	0-1	490	572	573	553	646	646
428	501	502	491	575	575	554	648	648
429	502	503	492	576	576	555	649	649
430	503	504	493	577	577	556	650	650
431	504	505	494	578	578	557	651	651
432	505	506	495	579	579	558	652	652
433	506	507	496	580	580	559	653	653
434	507	508	497	581	581	560	654	654
435	509	510	498	583	583	561	656	656
436	510	511	499	584	584	562	657	657
437	511	512	500	585	585	563	658	658
438	512	513	501	586	586	564	659	659
439	513	514	502	587	587	565	660	660
440	514	515	503	588	588	566	661	661
441	515	516	504	589	589	567	662	662