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Abstract

The mixed-integer linear programming (MILP) method has been applied widely to optimal design of energy supply systems. A hierarchical MILP method has been proposed to solve such optimal design problems efficiently. In addition, a method of reducing model by time aggregation has been proposed to search design candidates accurately and efficiently at the upper level. In this paper, the hierarchical MILP method and model reduction by time aggregation are applied to the multiobjective optimal design. The methods of clustering periods by the order of time series, by the k-medoids method, and based on an operational strategy are applied for the model reduction. As a case study, the multiobjective optimal design of a gas turbine

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cogeneration system is investigated by adopting the annual total cost and primary energy consumption as the objective functions, and the clustering methods are compared with one another in terms of the computation efficiency. It turns out that the model reduction by any clustering method is effective to enhance the computation efficiency when importance is given to minimizing the first objective function, but that the model reduction only by the k-medoids method is effective very limitedly when importance is given to minimizing the second objective function.

Keywords: Energy supply, Multiobjective optimal design, Mixed-integer linear programming, Hierarchical optimization, Time aggregation, Clustering

1. Introduction

For the purpose of attaining the highest performance of energy supply systems, it is important to rationally select equipment from many alternatives, and determine capacities and numbers of selected equipment in consideration of their operational strategies such as on/off status of operation and load allocation corresponding to seasonal and hourly variations in energy demands. As one of the ways to achieve this purpose, mathematical programming methods, and especially the mixed-integer linear programming (MILP) method have been utilized widely [1]. This is because the MILP method can consider discrete characteristics for selection and on/off status of operation of equipment, and can also treat nonlinear performance characteristics of equipment by piecewise linear approximations. This is also because commercial MILP solvers have become more efficient, which has enabled one to solve large scale practical optimization problems.

In many cases, equipment capacities have been treated relatively simply to solve the optimal design problems relatively easily, which cannot express real situations regarding performance characteristics and capital costs of equipment. For example, only the types of equipment have been determined under fixed capacities [2, 3]; the types and numbers of equipment have been determined under fixed capacities [4–6]; the types and capacities of equipment have been determined, but the capacities have been treated as continuous variables [7–11]; similar models have been used, but the dependence of performance characteristics of equipment on their capacities or part load levels have not been taken into account [12, 13]. On the other hand, an optimal design method has been proposed in consideration of discreteness of equipment capacities [14–16]. Although this method can treat real situations regarding performance characteristics and capital costs of equipment, it makes optimal design problems more complex, and even commercial MILP solvers which are recently available may not derive the optimal solutions in practical computation times.

On one hand, an approach to solve optimal design problems with large numbers of periods, which are set to consider variations in energy demands, efficiently is to utilize structural features of the problems. An MILP method utilizing the hierarchical relationship between design and operation variables has been proposed to solve optimal design problems efficiently [17]. In addition, for the purpose of enhancing the computation efficiency especially for the multiobjective optimal design, some strategies have been proposed to reduce the number of design candidates generated at the upper level and the number of optimal operation problems solved at the lower level [18].

This method has also been extended to search K-best solutions efficiently in the optimal design [19], and many design alternatives have been evaluated by searching K-best solutions in the multiobjective optimal design [20]. Through case studies in these works, it has turned out that the multiobjective optimal design problem is much more difficult and its computation time is much longer as compared with the single-objective optimal design problem.

On the other hand, an approach to solve optimal design problems with large numbers of periods efficiently is to derive approximate optimal solutions by reducing the numbers of periods. This time aggregation approach has been used widely [21], and some approximate solution methods for reducing the numbers of periods have been proposed by selecting representative days [22–28] and aggregating periods [29–33] based on energy demands using clustering methods such as k-means, k-medoids, etc. In some of these works, several time aggregation methods including the clustering ones have been compared [25, 27, 29]. In some of the works, values of some variables have first been determined with time aggregation, and then values of the other variables have been determined without time aggregation. A two level approach has been proposed: discrete design decision has first been made with time aggregation, and then the other decision has been made without time aggregation [28]. A similar approach has been adopted to bound the error and evaluate upper and lower bounds for the optimal value of the objective function [30, 31]. These bounding methods have been extended so that it can be applied to the optimal design of energy supply systems with energy storage units for short- and long-term storage cycles [32, 33]. Many of the models for optimal design used in these works have adopted discrete and continuous design variables as well as discrete and continuous operation variables [24–26, 30–33].

However, simpler models have also been applied by considering discrete and continuous design variables as well as only continuous operation variables [28, 29], and by considering only continuous design and operation variables [27]. In addition, energy storage units have been included in addition to energy conversion units in many of the models [25–29, 32, 33]. Although these approximate solution methods are applicable to any optimal design problem regardless of MILP solvers, they sacrifice solution exactness and affect both design and operation solutions. Thus, it is necessary to investigate how the optimal solutions of the optimal design problems with reduced numbers of periods are close to those with the original ones. However, it is essentially impossible to clarify this because these approximate solution methods are used since it is difficult to derive the optimal solutions of the optimal design problems with the original numbers of periods for the reference.

The combination of the aforementioned two approaches may be feasible to solve optimal design problems with large numbers of periods efficiently. As a novel strategy to enhance the computation efficiency in the aforementioned hierarchical MILP method, a method of reducing model by time aggregation has been proposed to search design candidates efficiently at the upper level. In addition, the previous strategies to enhance the computation efficiency have been modified in accordance with this novel strategy. This method has been realized only by clustering periods and averaging energy demands for clustered periods, while it guarantees to derive the optimal solutions because of features of the model reduction. As an initial trial, a simple method of clustering periods by the order of time series has been applied, and its effectiveness of the method has been clarified in a case study on the single-objective optimal design [34]. In addition, an method of clustering periods based on an operational strategy has also been proposed and applied to the same case study, and it has turned out that this method is more effective than the simple one [35]. However, these methods have not been applied to the multiobjective optimal design.

In this paper, the aforementioned hierarchical MILP method and model reduction by time aggregation are applied to the multiobjective optimal design. In applying the model reduction, the methods of clustering periods by the order of time series and based on an operational strategy are applied as conventional ones. In addition, a method of clustering periods by the k-medoids method is also applied as a novel one. As a case study, the multiobjective optimal design of a gas turbine cogeneration system with a practical configuration is investigated, and the clustering methods are compared with one another in terms of the computation efficiency.

2. Summary of formulation of multiobjective optimal design problem [18]

To consider seasonal and hourly variations in energy demands, a typical year is divided into M periods, and energy demands are estimated at each period. Each period is identified by the subscript or argument m $(m = 1, 2, \cdots, M)$. Energy demands $\boldsymbol{y}(m)$ are estimated certainly at each period. A super structure for an energy supply system is created to match energy demand requirements. The super structure is composed of all the units of equipment considered as candidates for selection, and a real structure is created by selecting some units of equipment from the candidates. Furthermore, some units of equipment are operated to satisfy energy demands at each period. Here, it is assumed that there are no temporally coupling constraints on the equipment operation. The selection, capacities, and numbers of equipment are considered as design variables. Maximum demands of utilities such as purchased electricity and city gas are also considered as design variables, and it is assumed that they are selected among discrete values. These are expressed by binary and integer variables $\boldsymbol{\eta}$. The number of equipment at the on status of operation as well as the load allocation of equipment and consumptions of utilities are considered as operation variables. The number of equipment at the on status of operation is expressed by integer variables $\boldsymbol{\delta}(m)$, and the load allocation of equipment and consumptions of utilities by continuous ones $\boldsymbol{x}(m)$. The annual total cost z_1 and the annual primary energy consumption z_2 are adopted as the objective functions to be minimized.

For simplicity, the weighting method is used for the multiobjective optimization, and the weighted sum of the two objective functions is adopted as the combined objective function to be minimized. Under the aforementioned definition, the multiobjective optimal design problem is formulated as

$$\left.\begin{array}{ll} \text{min.} & z = f_0(\boldsymbol{\eta}) + \displaystyle\sum_{m=1}^{M} f_m(\boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \Delta t(m) \\ \text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\eta}) \leq \mathbf{0} \\ & \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \leq \mathbf{0} \quad (m = 1, 2, \cdots, M) \\ & \boldsymbol{h}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) = \mathbf{0} \quad (m = 1, 2, \cdots, M) \\ & \boldsymbol{\eta} \in \mathbb{Z}^{n_1} \\ & \boldsymbol{\delta}(m) \in \mathbb{Z}^{n_2} \quad (m = 1, 2, \cdots, M) \\ & \boldsymbol{x}(m) \in \mathbb{R}^{n_3} \quad (m = 1, 2, \cdots, M) \end{array}\right\} \qquad (1)$$

where f_0 and f_m denote the terms composed of the design and operation variables, respectively, in the combined objective function, and are expressed as

$$
\left.\begin{aligned}
&f_0 = (w_1/z_1^\circ)\Gamma(\boldsymbol{\eta}) \\
&f_m = (w_1/z_1^\circ)\Psi_m(\boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \\
&\qquad + (w_2/z_2^\circ)\Phi_m(\boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \quad (m = 1, 2, \cdots, M)
\end{aligned}\right\} \tag{2}
$$

where Γ is the annual capital cost of equipment plus the annual demand charge of utilities, and Ψ and Φ are the energy charge and primary energy consumption of utilities per hour at each period. w_1 and w_2 are weights for z_1 and z_2, respectively, which are related with each other by $w_1 + w_2 = 1$, and z_1° and z_2° are reference values of z_1 and z_2, respectively. In Eq. (1), $\Delta t(m)$ is the duration per year of each period. $\boldsymbol{g}_0$ denotes the inequality constraints which relate the design variables, and means that the restrictions concerning the selection, capacities, and numbers of equipment are considered as constraints to be satisfied. $\boldsymbol{g}_m$ and $\boldsymbol{h}_m$ denote the inequality and equality constraints, respectively, which relate the design and operation variables, and mean that performance characteristics of equipment, relationships between maximum demands and consumptions of utilities, and energy balance relationships are considered as constraints to be satisfied. In addition, n_1, n_2, and n_3 are the numbers of the variables in $\boldsymbol{\eta}$, $\boldsymbol{\delta}(m)$, and $\boldsymbol{x}(m)$, respectively.

Refer to reference [17] for a concrete optimal design problem formulated for a simple energy supply system.

3. Summary of solution by hierarchical MILP method and model reduction [34]

3.1. Solution method

The optimal design problem of Eq. (1) has the hierarchical relationship between the design and operation variables, as shown in Fig. 1. Namely, if the values of the design variables $\boldsymbol{\eta}$ are assumed tentatively, the constraints $\boldsymbol{g}_m$ and $\boldsymbol{h}_m$ become independent at each period, and the values of the operation variables $\boldsymbol{\delta}(m)$ and $\boldsymbol{x}(m)$ can be optimized independently at each period. Thus, a hierarchical MILP method is used together with model reduction, as shown in Fig. 2.

At the upper level, the integer operation variables $\boldsymbol{\delta}(m)$ are first relaxed to continuos ones. The resultant problem is named the relaxed optimal design problem, although it is not shown here. In addition, M periods are categorized into L clusters, and energy demands are averaged in each cluster for model reduction by time aggregation as follows:

$$\boldsymbol{y}'(l) = \sum_{m \in A_l} \boldsymbol{y}(m) \Delta t(m) \Bigg/ \sum_{m \in A_l} \Delta t(m) \quad (l = 1, 2, \cdots, L) \tag{3}$$

where A_l is the set which includes the indices of periods in the lth cluster. The resultant problem is named the relaxed and reduced optimal design problem, and is expressed as follows:

$$\left.\begin{array}{ll}
\text{min.} & z = f_0(\boldsymbol{\eta}) + \displaystyle\sum_{l=1}^{L} f'_l(\boldsymbol{\delta}'(l), \boldsymbol{x}'(l), \boldsymbol{y}'(l)) \sum_{m \in A_l} \Delta t(m) \\
\text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\eta}) \leq \mathbf{0} \\
 & \boldsymbol{g}'_l(\boldsymbol{\eta}, \boldsymbol{\delta}'(l), \boldsymbol{x}'(l), \boldsymbol{y}'(l)) \leq \mathbf{0} \quad (l = 1, 2, \cdots, L) \\
 & \boldsymbol{h}'_l(\boldsymbol{\eta}, \boldsymbol{\delta}'(l), \boldsymbol{x}'(l), \boldsymbol{y}'(l)) = \mathbf{0} \quad (l = 1, 2, \cdots, L) \\
 & \boldsymbol{\eta} \in \mathbb{Z}^{n_1} \\
 & \boldsymbol{\delta}'(l) \in \mathbb{R}^{n_2} \quad (l = 1, 2, \cdots, L) \\
 & \boldsymbol{x}'(l) \in \mathbb{R}^{n_3} \quad (l = 1, 2, \cdots, L)
\end{array}\right\} \tag{4}$$

where $(\ )'$ denotes the variables, objective function, and constraints after model reduction by time aggregation. The binary and integer design variables $\boldsymbol{\eta}$ are selected as branching variables, and their values are assumed tentatively. Then, a design candidate is generated. This process is conducted by searching a feasible solution in the relaxed and reduced optimal design problem of Eq. (4).

At the lower level, under the values of the design variables $\boldsymbol{\eta}$, the values of the operation variables $\boldsymbol{\delta}(m)$ and $\boldsymbol{x}(m)$ can be determined by solving the following optimal operation problems:

$$\left.\begin{array}{ll} \text{min.} & f_m(\boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \\ \text{sub. to} & \boldsymbol{g}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) \leq \boldsymbol{0} \\ & \boldsymbol{h}_m(\boldsymbol{\eta}, \boldsymbol{\delta}(m), \boldsymbol{x}(m), \boldsymbol{y}(m)) = \boldsymbol{0} \\ & \boldsymbol{\delta}(m) \in \mathbb{Z}^{n_2} \\ & \boldsymbol{x}(m) \in \mathbb{R}^{n_3} \end{array}\right\} (m = 1, 2, \cdots, M) \tag{5}$$

The value of the combined objective function z is assessed based on the values of f_0 and f_m, which are evaluated based on the values of the design and operation variables assumed tentatively and determined optimally, respectively. A design candidate can be an incumbent solution, and the corresponding value of the combined objective function can be an upper bound for the optimal value of the combined objective function. Thus, it is used for the bounding procedure in searching other design candidates at the upper level. If the value of the combined objective function at a branching node at the upper level exceeds the upper bound, the corresponding branch can be cut.

3.2. Features of method

The purpose of the relaxed optimal design problem is not to find the optimal design solution and evaluate the optimal value of the combined objective function, but to search design candidates and evaluate lower bounds for the values of the combined objective function for the design candidates. Even the relaxed and reduced optimal design problem can be used for this purpose in place of the relaxed one. This is why a lower bound obtained by a continuous relaxation or linear programming problem of Eq. (4) at each branching node in the branch and bound method is smaller than or equal to that of the relaxed optimal design problem, which has been mathematically proved previously [34]. This means that Eq. (4) is a relaxation of the relaxed optimal design problem. As a result, even the use of the relaxed and reduced optimal design problem in place of the relaxed one never cut the optimal design solution, and the optimal design solution can be obtained certainly. This is a very valuable feature for model reduction by time aggregation. In addition, this model reduction by time aggregation can be used in combination with any clustering method. However, smaller lower bounds increase not only the number of design candidates generated at the upper level but also the number of optimal operation problems solved at the lower level. This may deteriorate the computation efficiency at both the levels. Thus, it is important to reduce decreases in the lower bounds to avoid an excessive increase in the number of the design candidates for a high computation efficiency. Decreases in the lower bounds, and thus the computation efficiency depend on the clustering method.

4. Clustering of periods

As aforementioned, the hierarchical MILP method with model reduction can be combined with any clustering method. However, it is important to reduce decreases in the lower bounds for the values of the combined objective function. For this purpose, it is important to cluster periods appropriately. In this paper, the following three methods are applied, and it is investigated how they affect the computation efficiency in the multiobjective optimal design. Figure 3 shows the process flows in the clustering methods.

4.1. Clustering by order of time series

A simple clustering is conducted by categorizing M periods into L clusters by the same number of periods per cluster, or $J = M / L$ regularly in time series [34]. Then, the set A_l is expressed as follows:

$$A_l = \{(l-1)J+1,\ (l-1)J+2,\ \cdots,\ lJ\} \quad (l = 1,\ 2,\ \cdots,\ L) \tag{6}$$

Although this clustering does not require any computation time for clustering, it is necessary to give the number of clusters L in advance. This clustering has a higher possibility of aggregating periods for which energy demands differ largely with an increase in J. Then, the lower bounds for the values of the combined objective function obtained by Eq. (4) are likely to be much smaller than those by the relaxed optimal design problem, which may deteriorate the computation efficiency.

4.2. Clustering by k-medoids method

The k-medoids method has been proposed as a general clustering method [36], and it has been applied to clustering periods for the optimal design of energy supply systems [22, 25–27, 29]. In this paper, the k-medoids method is selected among similar ones to cluster periods in consideration of energy demands. A representative element is chosen as a medoid, the distance between the medoid and another element is evaluated in each cluster, and the clustering is conducted to minimize the sum of the distances for all the clusters and elements.

The distance between the periods i and j is calculated by the following Euclid distance based on energy demands:

$$\Delta d_{ij} = \| \boldsymbol{y}(i) - \boldsymbol{y}(j) \|_2 \quad (i, j = 1, 2, \cdots, M) \tag{7}$$

Based on the values of the distances, the following optimization problem is solved to categorize M periods into L clusters:

$$\left.\begin{array}{ll} \text{min.} & D = \sum_{i=1}^{M}\sum_{j=1}^{M} \Delta d_{ij} v_{ij} \\ \text{sub. to} & \sum_{i=1}^{M} u_i = L \\ & v_{ij} \leq u_i \quad (i, j = 1, 2, \cdots, M) \\ & \sum_{i=1}^{M} v_{ij} = 1 \quad (j = 1, 2, \cdots, M) \\ & u_i \in \{0, 1\} \quad (i = 1, 2, \cdots, M) \\ & v_{ij} \in \{0, 1\} \quad (i, j = 1, 2, \cdots, M) \end{array}\right\} \tag{8}$$

where u_i is the binary variable whose value is set at 1 only when period i is chosen as a medoid in any cluster, and at 0 otherwise, v_{ij} is the binary variable whose value is set at 1 only when period j is included in the cluster in which period i is included as a medoid, and at 0 otherwise, and D is the sum of all the distances, or the objective function to be minimized. The clustering is conducted based on the following equation using the values of u_i and v_{ij} obtained by the optimization calculation:

$$j \in A_{\sum_{n=1}^{i} u_n} \quad (u_i = 1;\ v_{ij} = 1;\ i, j = 1, 2, \cdots, M) \tag{9}$$

It takes an extra computation time to solve the optimization problem of Eq. (8), it is necessary to give the number of clusters L in advance, and it is necessary to normalize the values of different types of energy demands properly to evaluate the distances rationally. However, since the clustering is conducted in consideration of energy demands, the lower bounds for the values of the combined objective function obtained by Eq. (4) are likely to be close to those by the relaxed optimal design problem, which may not deteriorate the computation efficiency. In addition, since this clustering is independent from design candidates, it may be robust in any case.

4.3. Clustering based on operational strategy

A method of clustering periods has been proposed to avoid a large difference between the lower bounds for the values of the combined objective function by Eq. (4) and the relaxed optimal design problem without model reduction [35]. Periods are categorized into clusters based on the number of equipment at the on status of operation obtained as the optimal solution by solving the relaxed optimal design problem in advance. The reason for using this method is as follows: if the optimal basic solution by Eq. (4) does not change from that by the relaxed optimal design problem, the optimal value of the objective function by Eq. (4) is equal to that by the relaxed optimal design problem; the basic solution is considered to be related closely with the number of equipment at the on status of operation; although many design candidates different from the optimal solution are generated, they are considered to be close to the optimal solution. The concrete procedure to cluster periods is shown below.

The integer operation variables for the numbers of all the types of equipment at the on status of operation at each period are expressed as follows:

$$\boldsymbol{\delta}(m) = (\delta_1(m), \delta_2(m), \cdots, \delta_{n_2}(m))^{\mathrm{T}} \quad (m = 1, 2, \cdots, M) \tag{10}$$

where δ is the number of each type of equipment at the on status of operation at each period. Since the integer operation variables are relaxed into continuous ones, their optimal values are not necessarily integer ones. Thus, these optimal values are utilized to cluster periods after they are converted into integer values by ceiling functions as follows:

$$\lceil \boldsymbol{\delta}(m) \rceil = (\lceil \delta_1(m) \rceil, \lceil \delta_2(m) \rceil, \cdots, \lceil \delta_{n_2}(m) \rceil)^{\mathrm{T}} \quad (m = 1,\, 2,\, \cdots,\, M) \tag{11}$$

where $\lceil (\ ) \rceil$ denotes the ceiling function. If the numbers of all the types of equipment at the on status of operation are the same at two periods m and m', these periods are categorized into the same cluster. This condition for clustering is expressed as follows:

$$\lceil \boldsymbol{\delta}(m) \rceil = \lceil \boldsymbol{\delta}(m') \rceil \quad (\forall m \in A_l, \forall m' \in A_l, m \neq m',\, l = 1,\, 2,\, \cdots,\, L) \tag{12}$$

Else, if the numbers of any type of equipment at the on status of operation are different at two periods m and m', these periods are categorized into different clusters. This condition for clustering is expressed as follows:

$$\lceil \boldsymbol{\delta}(m) \rceil \neq \lceil \boldsymbol{\delta}(m') \rceil \quad \begin{array}{l} (\forall m \in A_l, \forall m' \in A_{l'}, l = 1,\, 2,\, \cdots,\, L-1; \\ l' = l+1,\, l+2,\, \cdots,\, L) \end{array} \tag{13}$$

Although it takes an extra computation time to solve the relaxed optimal design problem, it is possible to determine the number of clusters L uniquely and automatically. This clustering is based on the reason why the lower bounds for the values of the combined objective function obtained by Eq. (4) are likely to be close to those by the relaxed optimal design problem, which may not deteriorate the computation efficiency. However, if many design candidates generated are far from the optimal solution obtained by the relaxed optimal design problem, this clustering may not be effective.

5. Case study

5.1. Input data

A gas turbine cogeneration system for district energy supply shown in Fig. 4 is investigated in a case study. The super structure for the cogeneration system is defined for the optimal design. It is composed of four gas turbine generators (GT), four waste heat recovery boilers (BW), four gas-fired auxiliary boilers (BG), four electric compression refrigerators (RE), four steam absorption refrigerators (RS), a device for receiving electricity (EP). It is assumed that when multiple units are installed for each type of equipment, their capacities are same, and that the gas turbine generators and waste heat recovery boilers are selected together as cogeneration units. Pumps for supplying cold water (PC) are common to all the possible structures, and only their power consumption is considered. A part of the formulation of the optimal design problem is shown in Appendix A.

Table 1 shows the capacities and performance characteristic values of candidates of equipment for selection. These values are based on the data prepared by an industrial association. Although the performance characteristic values are shown only at the rated load level, changes in efficiencies and coefficients of performance at part load levels are also taken into account as shown by the formulation in Appendix A. The maximum demands of electricity and city gas purchased from outside utility companies are also determined in the design, and are selected among discrete values by 1.0 MW and 0.5×10^3 m^3/h, respectively. The capacity of the device for receiving electricity is also selected among discrete values by 1.0 MW correspondingly. Table 2 shows the capital unit costs of equipment as well as the unit costs for demand and energy charges of electricity and city gas. The equipment costs are also based on the data prepared by an industrial association. In evaluating the annual total cost, the capital recovery factor is set at 0.964 with the interest rate 0.05, and the life of equipment 15 y. In evaluating the annual primary energy consumption, the coefficients for converting electricity and city gas consumptions to their primary energy ones are set at 2.58 kWh/kWh and 11.57 kWh/m^3, respectively.

Two hotels and four office buildings with the total floor area of 383.7×10^3 m^2 are selected as the buildings which are supplied with electricity, cold water, and steam by the cogeneration system. To take account of seasonal and hourly variations in energy demands, a typical year is divided into three representative days in winter, mid-season, and summer whose numbers of days per year are set at 122, 121, and 122 d/y, respectively, and each day is further divided into 24 sampling time intervals of 1 h. Thus, the year is divided into M = 72 periods. The energy demands used here are based on the data estimated actually for the target buildings in a district energy supply project, and the time discretization is also based on this estimation.

The weight for the annual total cost w_1 is changed in the range from 0.2 to 1.0. This is because in the case of the weight w_1 < 0.2, the computation time becomes too long to obtain the K-best solutions shown below for reference. In addition, the reference values of the annual total cost and primary energy consumption are set at z_1° = 1.4512×10^9 yen/y and z_2° = 257.59 GWh/y, respectively, which are obtained by minimizing z_1 and z_2, respectively.

All the optimization calculations by the hierarchical MILP method are conducted in combination of the previous strategies to enhance the computation efficiency: additional bounding procedures at both the upper and lower levels, and ordering of optimal operation problems at the lower level. To investigate the effect of each of the three clustering methods, the optimization calculations are conducted without the model reduction and with the model reduction by the corresponding clustering method. When the clustering by the order of time series or the k-medoids method is used, the number of clusters L is given in advance and is changed as a parameter. When the clustering based on the operational strategy is used, the number of clusters L is determined uniquely and automatically.

All the optimization calculations are conducted using a commercial MILP solver IBM ILOG CPLEX Optimization Studio Ver. 12.6.1 on a MacBook Pro with Intel Core i7 processor (4 cores and 2.4 GHz) [37]. To confirm the effectiveness of the hierarchical MILP method with and without the model reduction, the conventional optimization calculation is also conducted using a commercial MILP solver GAMS/CPLEX Ver. 12.6.0 directly on the same computer [38].

5.2. Results and discussion

Figure 5 shows the trade-off relationship between the two objective functions, or the annual total cost and primary energy consumption, obtained by the hierarchical MILP method. This figure include not only the Pareto optimal solutions denoted by red points but also the corresponding K-best solutions placed in the upper right-hand region, which have been obtained in the previous paper by setting the maximum number of the K-best solutions and the maximum relative difference in the value of the combined objective function between the K-best solutions and optimal one at 1 000 and 1.0 %, respectively [20]. In the case of the weight $w_1 \geq 0.6$, the numbers of the K-best solutions are small, and the values of the two objective functions are concentrated in narrow regions. However, in the case of $w_1 < 0.6$, the numbers of the K-best solutions increase drastically, and the values of the two objective functions are dispersed in wide regions. Therefore, discussion is focused on the cases of $w_1 = 0.6$, 0.5, and 0.4 in the following. Table 3 shows the optimal values of the design variables in these cases. The numbers of gas turbine cogeneration units and gas-fired auxiliary boilers as well as the maximum demand of purchased electricity change between cases $w_1 = 0.5$ and 0.4.

Figures 6 (a) to (c) show the convergence characteristics of the upper and lower bounds for the optimal value of the combined objective function in the cases of the weight $w_1 = 0.6$, 0.5, and 0.4, respectively. Note that the combined objective function is converted by multiplying the first equation in Eq. (1) by z_1°. These figures include the results obtained by the conventional optimization calculation using the commercial solver directly, and by the hierarchical MILP method without and with the model reduction. The number of clusters $L = 36$ is selected for the clustering by the order of time series or the k-medoids method, while the number of clusters $L = 9$ is determined uniquely and automatically for the clustering based on the operational strategy. In the conventional optimization calculation, the upper and lower bounds do not coincide with each other even in 7 200 s. The gaps between them are 0.23, 0.93, and 2.03 % in the cases of $w_1 = 0.6$, 0.5, and 0.4, respectively. The hierarchical MILP method makes the coincidence of the upper and lower bounds in shorter computation times. In the case of $w_1 = 0.6$, the performance of the hierarchical MILP method with the model reduction is much higher than that without the model reduction. In the case of w_1 = 0.5, the performance of the hierarchical MILP method with the model reduction is still higher than that without the model reduction. In the case of w_1 = 0.4, the performance of the hierarchical MILP method with the clustering by the k-medoids method is slightly higher than that without the model reduction. However, the performance of the hierarchical MILP method with the clustering by the order of time series becomes lower than that without the model reduction, and that with the clustering based on the operational strategy becomes much lower than that without the model reduction. This is because with a decrease in w_1, the number of the K-best solutions becomes large drastically, and decreases in lower bounds for the values of the combined objective function by the model reduction increase the number of design candidates generated at the upper level. This means that the clustering based on the operational strategy is effective to evaluate lower bounds for design candidates close to the optimal solution, but is not for design candidates far from the optimal solution.

Figures 7 (a) to (c) show the computation time in relation to the number of clusters for periods L in the cases of w_1 = 0.6, 0.5, and 0.4, respectively. For the clustering by the order of time series, the case of L = 36 is the best in the cases of w_1 = 0.6 and 0.5, which is different from the result obtained in the single-objective optimal design, where the case of L = 18 is the best [34]. However, even the case of L = 36 is worse than that without the model reduction in the case of w_1 = 0.4. This is also because of the aforementioned features of the number of the K-best solutions and decreases in lower bounds. For the clustering by the k-medoids method, the cases of $L \leq 36$ are better than that without the model reduction in the cases of w_1 = 0.6 and 0.5. In addition, the cases of $L \leq 36$ are slightly better than that without the model reduction in the case of w_1 = 0.4. However, the computation time is unstable in the cases of $L \leq 36$, and thus it may be difficult to determine the number of clusters for periods L properly in advance. For the clustering based on the operational strategy, the computation time is comparable to that by the k-medoids method in the cases of w_1 = 0.6 and 0.5. However, it becomes longer drastically in the case of w_1 = 0.4.

Figures 8 (a) to (c) show the average decrease in lower bounds for the values of the combined objective function between the cases with and without the model reduction in relation to the number of clusters for periods L in the cases of w_1 = 0.6, 0.5, and 0.4, respectively. Note that the combined objective function is converted by multiplying the first equation in Eq. (1) by z_1°. Since the design candidates generated at the upper level differs in both the cases, this average decrease in lower bounds is obtained by averaging the decreases in lower bounds for all the design candidates common to both the cases. For the clustering by the order of time series, the average decrease in lower bounds increases largely with a decrease in L. This is because periods for which energy demands differ largely may be clustered. However, the average decrease in lower bounds in the case of L = 18 is smaller than that in the case of L = 24. This is because the former case is more suitable than the latter one from the viewpoint of minimizing the decreases in lower bounds. It is easily understood that this result affects that shown in Fig. 7. For the clustering by the k-medoids method, the average decrease in lower bounds remains vary small in the cases of $L \geq 24$. It is also easily understood that this result affects that shown in Fig. 7. However, the average decrease in lower bounds increases in the cases of $L < 24$, which makes the computation time unstable, as shown in Fig. 7. For the clustering based on the operational strategy, although L is smaller than those by the k-medoids method, the average decrease in lower bounds is comparable to those by the k-medoids method in the cases of w_1 =

0.6 and 0.5. However, the average decrease in lower bounds increases drastically in the case of w_1 = 0.4. This is because the clustering based on the operational strategy uses only the optimal solution of the relaxed optimal design problem, which increases the decreases in lower bounds for many design candidates far from the optimal solution. This result reflects that shown in Fig. 7.

In consideration of this result, the clustering based on the operational strategy is tried again using not only the optimal solution but also another design candidate. Since the influence of gas turbine cogeneration units on the operational strategy may be the largest, the operational strategies are assessed not only for the optimal solution but also for the 2nd best solution with different specifications for gas turbine cogeneration units. Then, the clustering of periods is conducted for each operational strategy, and both the clusters for periods are merged with each other. The computation time and the average decrease in lower bounds obtained by this procedure are included in Figs. 7 (c) and 8 (c), respectively. They become comparable to those by the k-medoids method.

6. Conclusions

In this paper, the hierarchical MILP method and the model reduction by time aggregation for the optimal design of energy supply systems have been applied to the multiobjective optimal design. In applying the model reduction, the methods of clustering periods by the order of time series, by the k-medoids method, and based on the operational strategy have been applied. As a case study, the multiobjective optimal design of a gas turbine cogeneration system with a practical configuration has been investigated, and the clustering methods have been compared with one another in terms of the computation efficiency. The following main results have been obtained:

- When importance is given to minimizing the annual total cost, the model reduction with any clustering method is effective to enhance the computation efficiency. The clustering by the order of time series is the least effective. The clustering by the k-medoids method is effective, but its effectiveness depends on the number of clusters. Although the number of clusters is determined uniquely and automatically for the clustering based on the operational strategy, it is effective similarly to that by the k-medoids method.
- When importance is given to minimizing the annual primary energy consumption, the model reduction is effective very limitedly to enhance the computation efficiency. The clustering by the order of time series is not effective. The clustering by the k-medoids method is slightly effective only for large numbers of clusters. The clustering based on the operational strategy deteriorates the computation efficiency drastically. This may be recovered using multiple operational strategies, but its effectiveness may be very limited.

As a future work, it is important to make the model reduction effective to enhance the computation efficiency in all the cases. To attain this objective, it is obviously necessary to reduce decreases in lower bounds for the values of the combined objective function for the design candidates evaluated by the relaxed and reduced optimal design problem at the upper level. This may not be so easy, and may be a challenging work.

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Appendix A

A part of the formulation of the optimal design problem is given for the gas turbine cogeneration system investigated in the case study.

For gas turbine generators, for example, the restrictions concerning the selection, capacities, and numbers are expressed by

$$\left.\begin{array}{l} \eta_{\mathrm{GT}k}/N_{\mathrm{GT}k} \le \gamma_{\mathrm{GT}k} \le \eta_{\mathrm{GT}k} \quad (k=1,\,2,\,\cdots,\,K_{\mathrm{GT}}) \\ \sum_{k=1}^{K_{\mathrm{GT}}} \gamma_{\mathrm{GT}k} \le 1 \\ \gamma_{\mathrm{GT}k} \in \{0,\,1\} \quad (k=1,\,2,\,\cdots,\,K_{\mathrm{GT}}) \\ \eta_{\mathrm{GT}k} \in \{0,\,1,\,\cdots,\,N_{\mathrm{GT}k}\} \quad (k=1,\,2,\,\cdots,\,K_{\mathrm{GT}}) \end{array}\right\} \tag{A1}$$

where the capacity of the gas turbine generators is selected from its K_{GT} candidates. In addition, the number of the kth capacity is determined within its maximum $N_{\mathrm{GT}k}$. The selection and number of the kth capacity are designated by the binary variable $\gamma_{\mathrm{GT}k}$ and the integer variable $\eta_{\mathrm{GT}k}$, respectively. The inequalities in Eq. (A1) constitute a part of $\boldsymbol{g}_0$ in Eq. (1).

For example, the performance characteristics for gas turbine generators are expressed by

$$
\left.\begin{array}{l}
E_{\mathrm{GT}}(m) = \sum_{k=1}^{K_{\mathrm{GT}}} p_{\mathrm{GT}k} \gamma_{\mathrm{GT}k} F_{\mathrm{GT}}(m) + \sum_{k=1}^{K_{\mathrm{GT}}} q_{\mathrm{GT}k} \gamma_{\mathrm{GT}k} \delta_{\mathrm{GT}}(m) \\
Q_{\mathrm{GT}}^{\mathrm{x}}(m) = \sum_{k=1}^{K_{\mathrm{GT}}} p_{\mathrm{GT}k}^{\mathrm{x}} \gamma_{\mathrm{GT}k} F_{\mathrm{GT}}(m) + \sum_{k=1}^{K_{\mathrm{GT}}} q_{\mathrm{GT}k}^{\mathrm{x}} \gamma_{\mathrm{GT}k} \delta_{\mathrm{GT}}(m) \\
E_{\mathrm{GT}}^{\mathrm{a}}(m) = \sum_{k=1}^{K_{\mathrm{GT}}} p_{\mathrm{GT}k}^{\mathrm{a}} \gamma_{\mathrm{GT}k} F_{\mathrm{GT}}(m) + \sum_{k=1}^{K_{\mathrm{GT}}} q_{\mathrm{GT}k}^{\mathrm{a}} \gamma_{\mathrm{GT}k} \delta_{\mathrm{GT}}(m) \\
\sum_{k=1}^{K_{\mathrm{GT}}} \underline{F}_{\mathrm{GT}k} \gamma_{\mathrm{GT}k} \delta_{\mathrm{GT}}(m) \leq F_{\mathrm{GT}}(m) \leq \sum_{k=1}^{K_{\mathrm{GT}}} \overline{F}_{\mathrm{GT}k} \gamma_{\mathrm{GT}k} \delta_{\mathrm{GT}}(m) \\
\delta_{\mathrm{GT}}(m) \leq \sum_{k=1}^{K_{\mathrm{GT}}} \eta_{\mathrm{GT}k} \\
\delta_{\mathrm{GT}}(m) \in \{0,\ 1,\ \cdots,\ \max_{1 \leq k \leq J_{\mathrm{GT}}} N_{\mathrm{GT}k}\}
\end{array}\right\} \quad (m = 1,\ 2,\ \cdots,\ M) \qquad \text{(A2)}
$$

where flow rates of input and output energy of the gas turbine generators are related linearly. E_{GT} is the electric power generated, $Q_{\mathrm{GT}}^{\mathrm{x}}$ is the exhaust heat generated, $E_{\mathrm{GT}}^{\mathrm{a}}$ is the auxiliary power consumption, and F_{GT} is the city gas consumption. These are continuous variables. δ_{GT} is the integer variable for the number of equipment at the on status of operation. Here, it is assumed that δ_{GT} units are operated at the same load level, and the sums of the flow rates of input and output energy are expressed by the continuous variables. The quadratic terms $\gamma_{\mathrm{GT}k} F_{\mathrm{GT}}$ and $\gamma_{\mathrm{GT}k} \delta_{\mathrm{GT}}$ are linearized accurately with additional continuous variables and inequality constraints by considering $\gamma_{\mathrm{GT}k}$ is binary [39]. $p_{\mathrm{GT}k}$ and $q_{\mathrm{GT}k}$, etc. are the slopes and intercepts, respectively, of the linear relationships between the flow rates of input and output energy for a unit at the on status of operation, and $\underline{F}_{\mathrm{GT}k}$ and $\overline{F}_{\mathrm{GT}k}$ are the lower and upper limits for city gas consumption, respectively, for a unit at the on status of operation. The performance not only at the rated load but also at the part load can be expressed by adjusting the values of $p_{\mathrm{GT}k}$ and $q_{\mathrm{GT}k}$, etc. The inequalities and equalities in Eq. (A2) constitute a part of $\boldsymbol{g}_m$ and $\boldsymbol{h}_m$ in Eq. (1), respectively.

For example, the relationship between maximum demand and consumption, and the energy balance relationship for electricity are expressed by

$$\left.\begin{aligned} &E_{\mathrm{elec}}(m) \le \bar{E}_{\mathrm{elec}} \\ &E_{\mathrm{elec}}(m) + E_{\mathrm{GT}}(m) = E_{\mathrm{RE}}(m) + E_{\mathrm{PC}}(m) + E^{\mathrm{a}}_{\mathrm{GT}}(m) + E^{\mathrm{a}}_{\mathrm{BW}}(m) \\ &\qquad\qquad + E^{\mathrm{a}}_{\mathrm{BG}}(m) + E^{\mathrm{a}}_{\mathrm{RE}}(m) + E^{\mathrm{a}}_{\mathrm{RS}}(m) + E_{\mathrm{dem}}(m) \end{aligned}\right\} \quad (m = 1, 2, \cdots, M) \tag{A3}$$

where E_{elec} is the electric power purchased from an electric power company, $\bar{E}_{\mathrm{elec}}$ is its maximum demand, E_{dem} is the electricity demand, E_{RE} and E_{PC} are the electric power consumed by electric compression refrigerators and pumps, respectively, and E^{a} is the electric power consumed by auxiliary machinery for the equipment shown by the subscript. The inequality and equality in Eq. (A3) also constitute a part of $\boldsymbol{g}_m$ and $\boldsymbol{h}_m$ in Eq. (1), respectively.

The annual total cost and annual primary energy consumption as the objective functions to be minimized are expressed by

$$\left.\begin{aligned} z_1 = {} & R\left(\sum_{k=1}^{K_{\mathrm{GT}}} C_{\mathrm{GT}k}\eta_{\mathrm{GT}k} + \sum_{k=1}^{K_{\mathrm{BW}}} C_{\mathrm{BW}k}\eta_{\mathrm{BW}k} + \sum_{k=1}^{K_{\mathrm{BG}}} C_{\mathrm{BG}k}\eta_{\mathrm{BG}k} \right. \\ & \left. + \sum_{k=1}^{K_{\mathrm{RE}}} C_{\mathrm{RE}k}\eta_{\mathrm{RE}k} + \sum_{k=1}^{K_{\mathrm{RS}}} C_{\mathrm{RS}k}\eta_{\mathrm{RS}k} + c_{\mathrm{EP}}\bar{E}_{\mathrm{EP}}\right) \\ & + 12\vartheta_{\mathrm{elec}}\bar{E}_{\mathrm{elec}} + 12\vartheta_{\mathrm{gas}}\bar{F}_{\mathrm{gas}} \\ & + \sum_{m=1}^{M}\Big(\psi_{\mathrm{elec}}E_{\mathrm{elec}}(m) + \psi_{\mathrm{gas}}F_{\mathrm{gas}}(m)\Big)\Delta t(m) \\ z_2 = {} & \sum_{m=1}^{M}\Big(\varphi_{\mathrm{elec}}E_{\mathrm{elec}}(m) + \varphi_{\mathrm{gas}}F_{\mathrm{gas}}(m)\Big)\Delta t(m) \end{aligned}\right\} \tag{A4}$$

where $\bar{E}_{\mathrm{EP}}$ is the capacity of the receiving device for purchased electricity, F_{gas} is the amount of city gas purchased from a gas company, $\bar{F}_{\mathrm{gas}}$ is its maximum demand, C and c are the capital cost and capital unit cost of each type of equipment, respectively, R is the capital recovery factor, ϑ and ψ are the unit costs for demand and energy charges of each utility, respectively, and φ is the conversion coefficient from the purchased amount of each utility to the primary energy consumption. The first to third lines in Eq. (A4) constitute the first line in Eq. (2), while the fourth and fifth lines in Eq. (A4) constitute the second and third lines in Eq. (2).

Nomenclature

A : set for indices of periods in cluster

C : capital cost of equipment, yen

c : capital unit cost of equipment, yen/kW

D : sum of Euclid distances

Δd : Euclid distance

E : electric power, kWh/h

F : city gas consumption, m^3/h

f : part of objective function

$\boldsymbol{g}$: vector for inequality constraints

$\boldsymbol{h}$: vector for equality constraints

J : number of periods per cluster

K : number of candidates for capacities

L : number of clusters for periods

M : number of periods

N : maximum number of equipment

p : slope of linear relationship between flow rates of input and output energy, kWh/m^3

Q : heat flow rate, kWh/h

q : intercept of linear relationship between flow rates of input and output energy, kWh/h

R : capital recovery factor

Δt : duration per year of period, h/y

u : binary variable for expressing selection of period as medoid

v : binary variable for expressing selection of period and medoid in same cluster

w : weight for objective functions

$\boldsymbol{x}$: vector for continuous operation variables

$\boldsymbol{y}$: vector for energy demands

z : objective function, yen/y, kWh/y

Γ : annual capital cost of equipment plus the annual demand charge of utilities yen/y

γ : binary design variable for selection of equipment

δ : number of equipment at on status of operation

$\boldsymbol{\delta}$: vector for integer operation variables

η : integer design variable for number of equipment

$\boldsymbol{\eta}$: vector for binary and integer design variables

ϑ : unit cost for demand charge of utilities, yen/(kW·month), yen/(m^3/h·month)

Φ : primary energy consumption of utilities per hour, kWh/h

φ : conversion coefficient for primary energy consumption, kWh/kWh, kWh/m^3

Ψ : energy charge of utilities per hour, yen/h

ψ : unit cost for energy charge of utilities, yen/kWh, yen/m^3

$( )'$: reduction by time aggregation

$( )^\circ$: reference value

$\overline{( )}$: upper limit, equipment capacity, or utility maximum demand

$\underline{( )}$: lower limit

$\lceil ( ) \rceil$: ceiling function

Subscripts and arguments

dem : demand

elec : purchased electricity

gas : purchased city gas

i, j, m : indices of periods (in part related with operation)

k : index of candidates for capacities

l : index of clusters for periods

0 : part related with design

$1, 2$: two objective functions

Superscripts

a : auxiliary machinery

n_1, n_2, n_3 : numbers of variables in $\boldsymbol{\eta}$, $\boldsymbol{\delta}$, and $\boldsymbol{x}$, respectively

T : transposition of vector

x : exhaust heat

Equipment symbols (subscripts)

BG : gas-fired auxiliary boiler

BW : waste heat recovery boiler

EP : device for receiving electricity

GT : gas turbine generator

PC : pump

RE : electric compression refrigerator

RS : steam absorption refrigerator

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Captions for tables and figures

Table 1 Capacities and performance characteristic values of candidates of equipment for selection

Table 2 Capital unit costs of equipment, and unit costs for demand and energy charges of utilities

Table 3 Optimal values of design variables

Fig. 1 Hierarchical relationship between design and operation variables

Fig. 2 Solution process by hierarchical MILP method and model reduction

Fig. 3 Process flows in clustering methods

(a) Clustering by order of time series

(b) Clustering by k-medoids method

(c) Clustering based on operational strategy

Fig. 4 Configuration of gas turbine cogeneration system

Fig. 5 Trade-off relationship between two objective functions

Fig. 6 Changes in upper and lower bounds for optimal value of combined objective

(a) w_1 = 0.6

(b) w_1 = 0.5

(c) w_1 = 0.4

Fig. 7 Effect of number of clusters on computation time

(a) w_1 = 0.6

(b) w_1 = 0.5

(c) w_1 = 0.4

Fig. 8 Effect of number of clusters on average decrease in lower bounds for values of combined objective function

(a) $w_1 = 0.6$

(b) $w_1 = 0.5$

(c) $w_1 = 0.4$

Table 1 Capacities and performance characteristic values of candidates of equipment for selection

Equipment	Capacity/performance*	Candidate			
Gas turbine cogeneration unit		#1	#2	#3	#4
	Max. power output MW	1.29	1.60	2.00	2.40
	Max. steam output MW	5.69	3.34	4.10	4.57
	Power generating efficiency	0.140	0.173	0.169	0.179
	Heat recovery efficiency	0.617	0.362	0.347	0.341
		#5	#6	#7	#8
	Max. power output MW	2.93	3.50	3.54	4.36
	Max. steam output MW	6.44	6.97	6.89	8.92
	Power generating efficiency	0.256	0.271	0.273	0.273
	Heat recovery efficiency	0.563	0.540	0.531	0.559
		#9	#10		
	Max. power output MW	5.23	5.32		
	Max. steam output MW	8.91	9.05		
	Power generating efficiency	0.301	0.306		
	Heat recovery efficiency	0.513	0.521		
Gas-fired auxiliary		#1	#2	#3	#4
	Max. steam output MW	5.24	6.55	7.86	9.82
	Thermal efficiency	0.92	0.92	0.92	0.92
Electric compression		#1	#2	#3	#4
	Max. cooling output MW	2.82	3.52	4.22	5.28
	Coefficient of performance	4.57	4.73	4.76	5.04
Steam absorption refrigerator		#1	#2	#3	#4
	Max. cooling output MW	3.46	5.18	6.91	8.64
	Coefficient of performance	1.20	1.20	1.20	1.20

*At rated load level

Table 2 Capital unit costs of equipment, and unit costs for demand and energy charges of utilities

Equipment/utility		Unit cost
Gas turbine generator		230.0×10^3 yen/kW
Waste heat recovery boiler		9.6×10^3 yen/kW
Gas-fired auxiliary boiler		6.6×10^3 yen/kW
Electric compression refrigerator		34.4×10^3 yen/kW
Steam absorption refrigerator		30.1×10^3 yen/kW
Receiving device		56.3×10^3 yen/kW
Electricity	Demand charge	1740 yen/(kW·month)
	Energy charge	10.77 yen/kWh (Summer) 9.79 yen/kWh (Others)
City gas	Demand charge	2033 yen/(Nm^3/h·month)
	Energy charge	30.88 yen/Nm^3

Table 3 Optimal values of design variables

Weight w_1	Candidate, number, and capacity of equipment					Utility maximum demand	
	GT, BW	BG	RE	RS	EP MW	Electricity MW	City gas $\times 10^3$ Nm^3/h
0.6	#10×3	#1×1	#1×1	#3×4	4.0	4.0	4.5
0.5	#10×3	#1×1	#1×1	#3×4	4.0	4.0	4.5
0.4	#10×2	#1×2	#1×1	#3×4	10.0	10.0	3.5

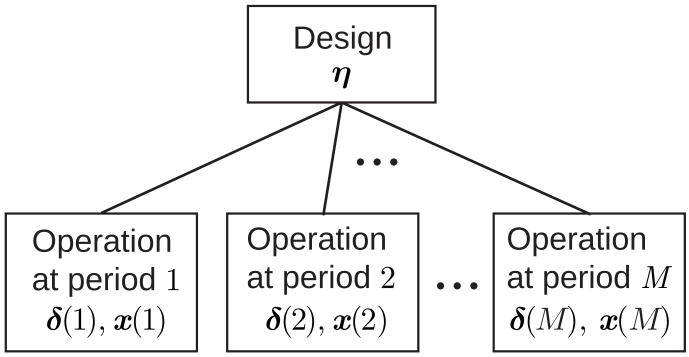


Fig. 1 Hierarchical relationship between design and operation variables

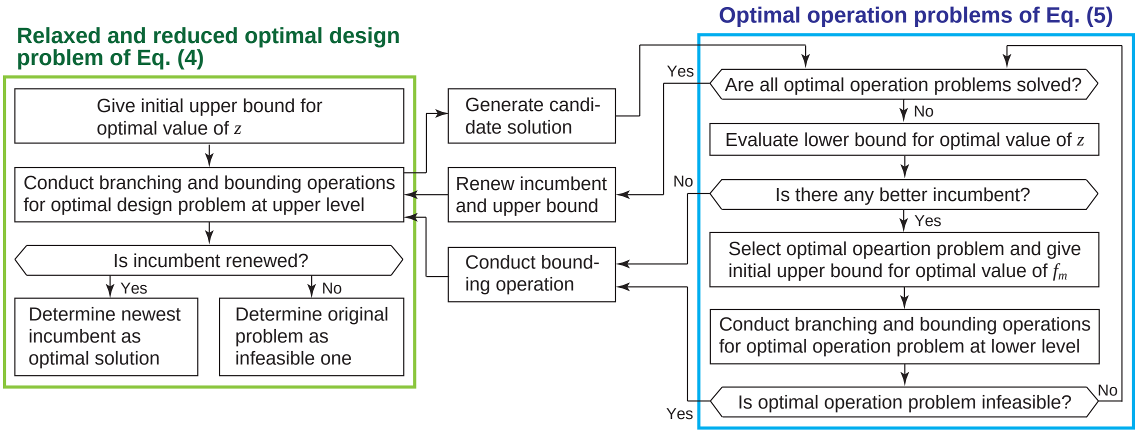
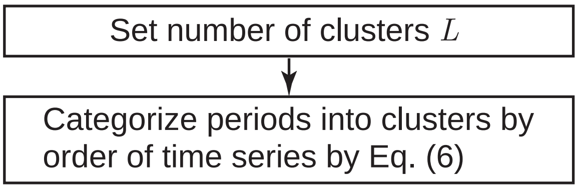
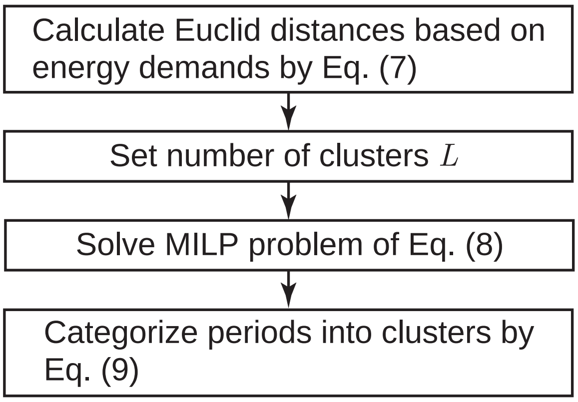


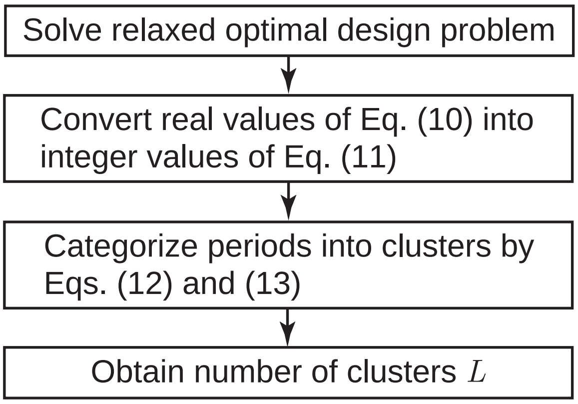
Fig. 2 Solution process by hierarchical MILP method and model reduction



(a) Clustering by order of time series



(b) Clustering by k-medoids method



(c) Clustering based on operational strategy

Fig. 3 Process flows in clustering methods

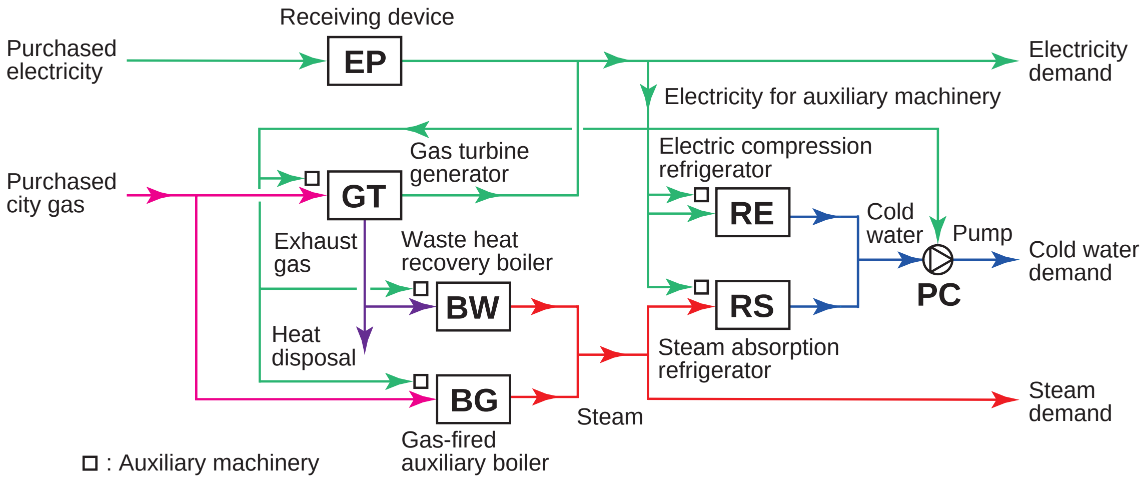


Fig. 4 Configuration of gas turbine cogeneration system

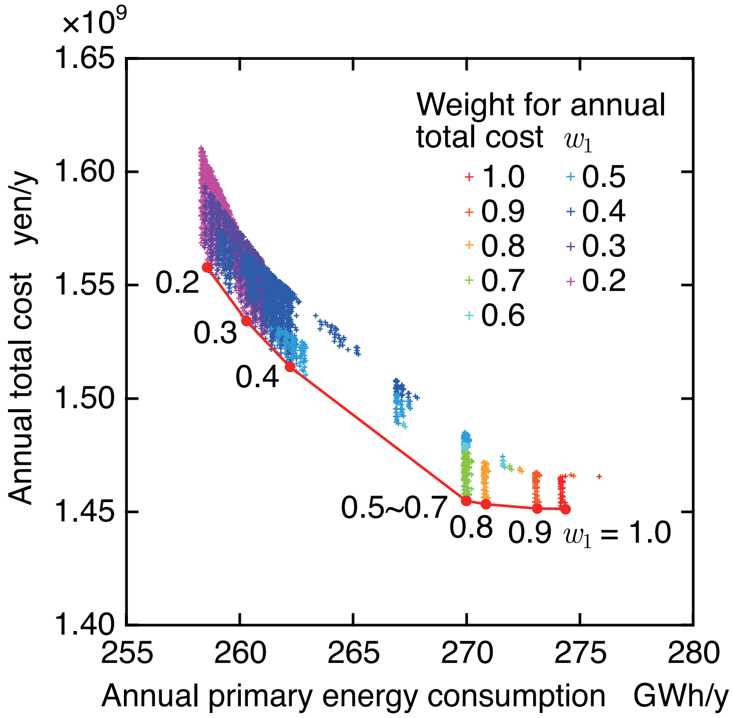
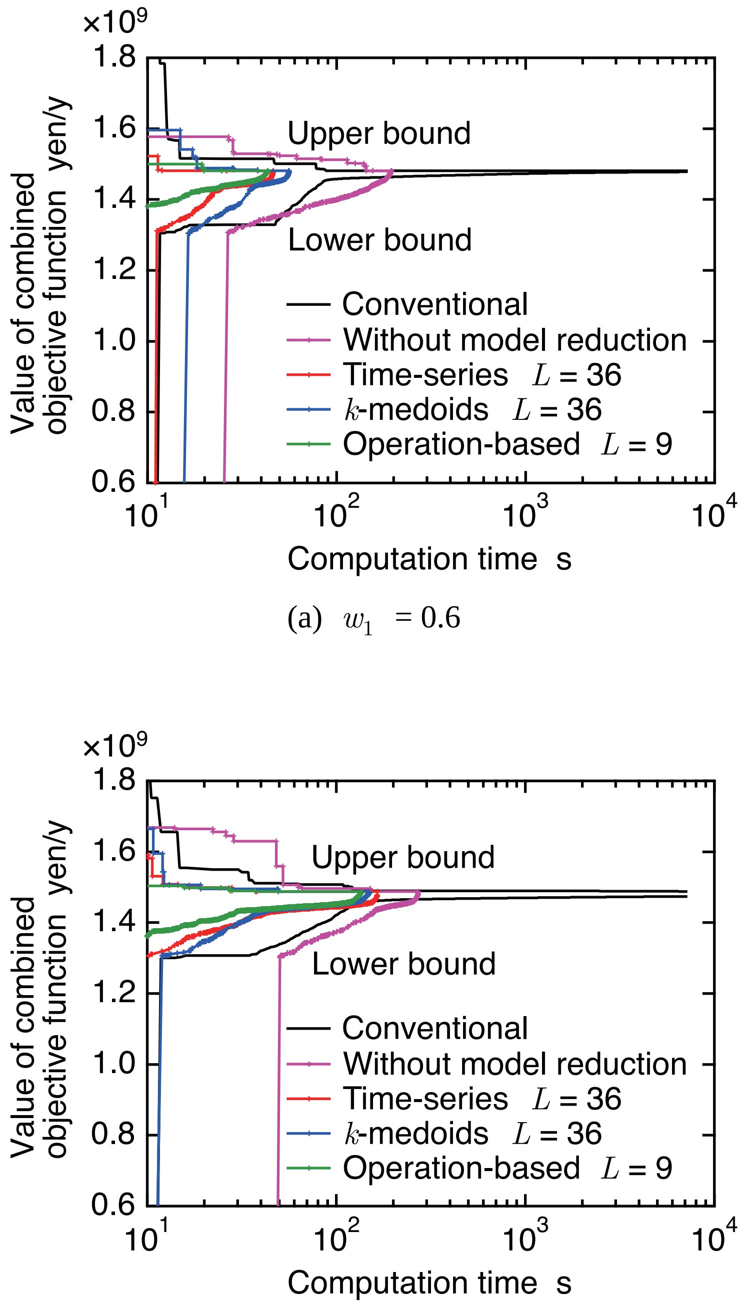


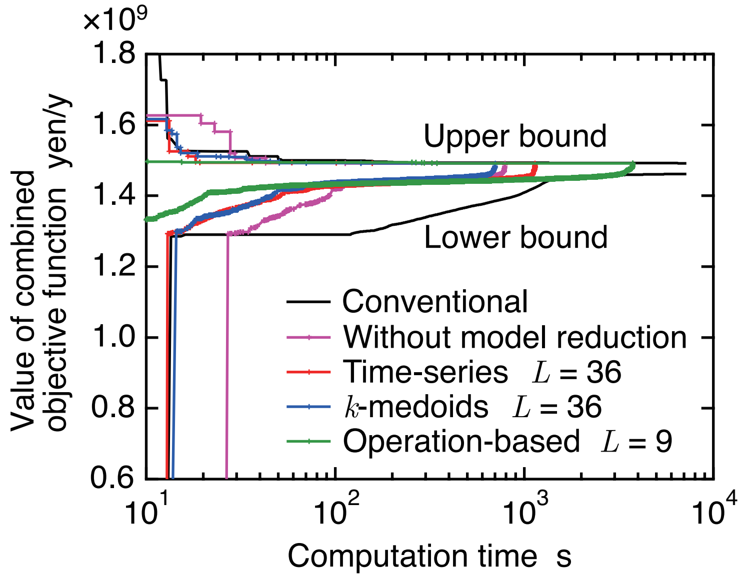
Fig. 5 Trade-off relationship between two objective functions



(a) w_1 = 0.6

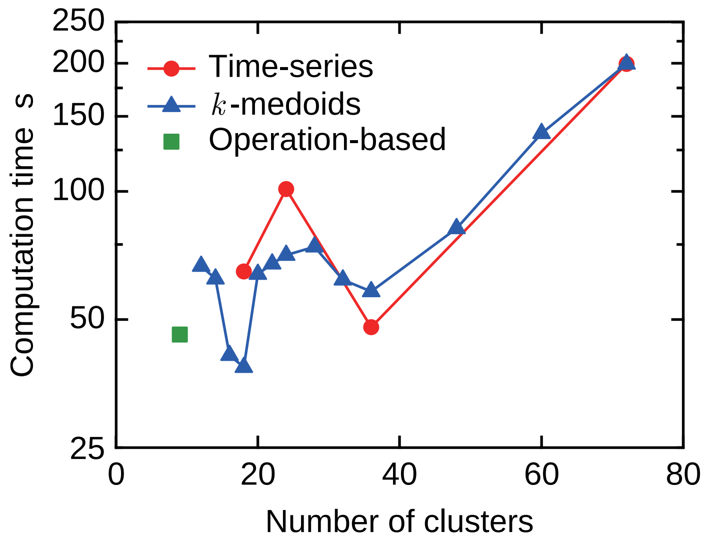
(b) w_1 = 0.5

Fig. 6 Changes in upper and lower bounds for optimal value of combined objective function

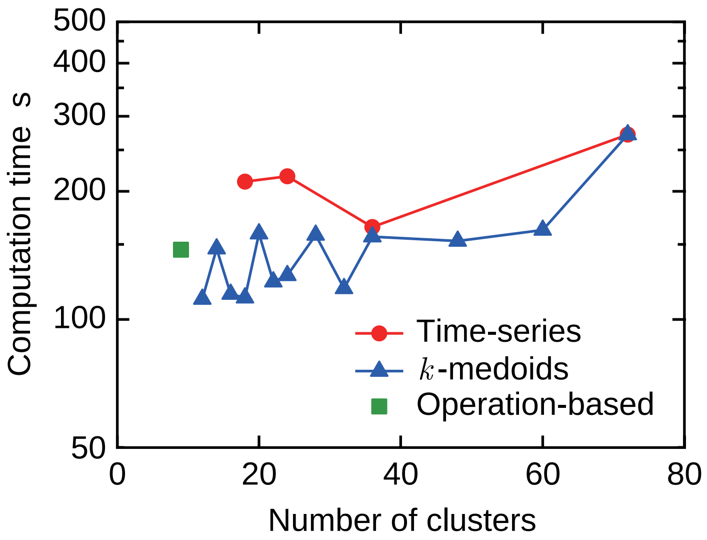


(c) w_1 = 0.4

Fig. 6 Changes in upper and lower bounds for optimal value of combined objective function

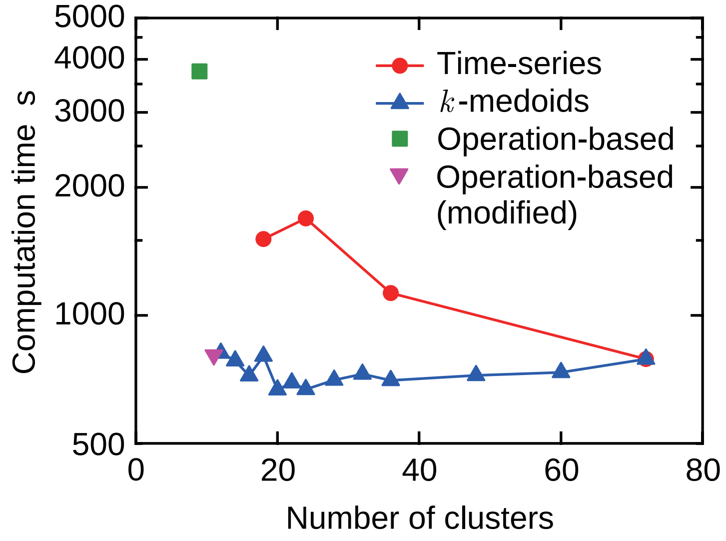


(a) w_1 = 0.6



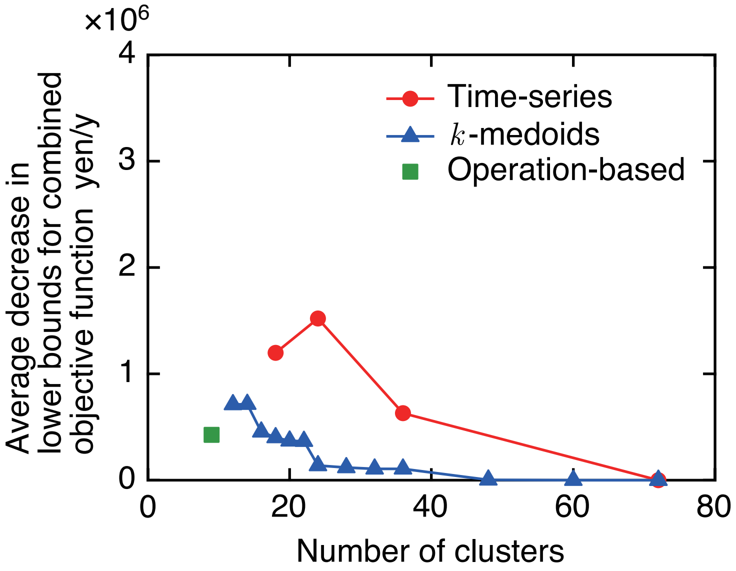
(b) w_1 = 0.5

Fig. 7 Effect of number of clusters on computation time

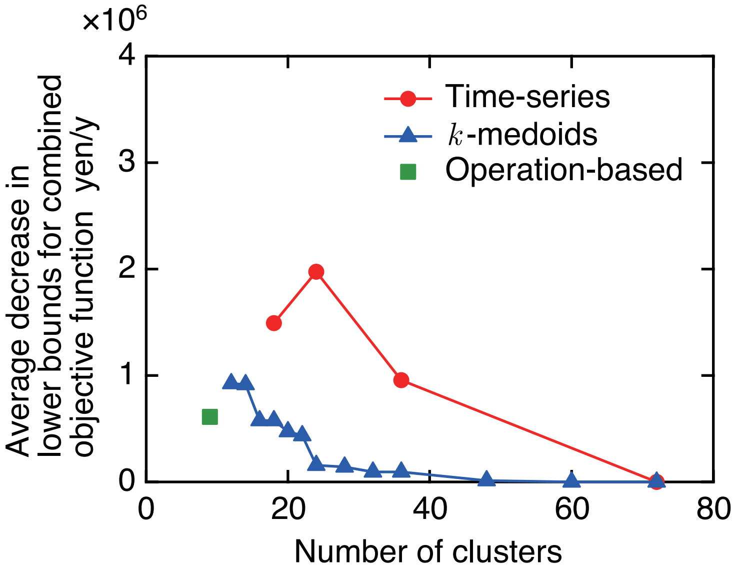


(c) w_1 = 0.4

Fig. 7 Effect of number of clusters on computation time

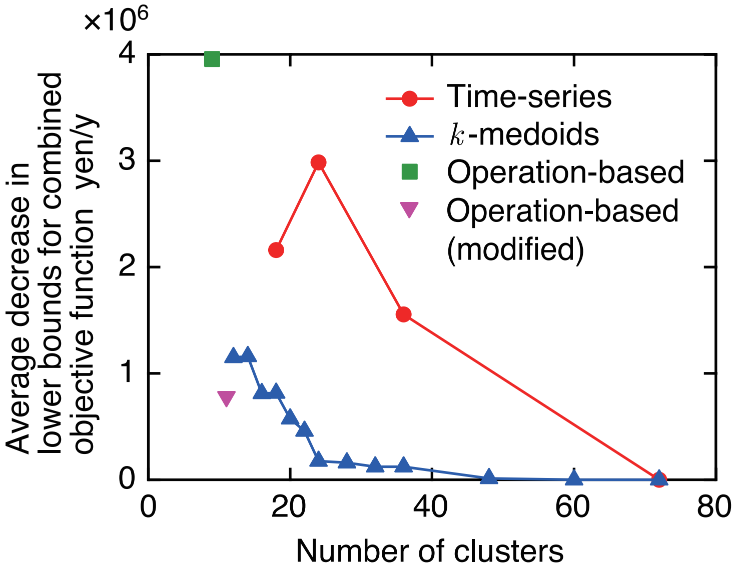


(a) w_1 = 0.6



(b) w_1 = 0.5

Fig. 8 Effect of number of clusters on average decrease in lower bounds for values of combined objective function



(c) w_1 = 0.4

Fig. 8 Effect of number of clusters on average decrease in lower bounds for values of combined objective function