Government Spending in an Economy with Habit Formation

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Abstract

This paper examines the effects of a permanent increase in government spending in an economy with habit formation. For this purpose we apply the continuous-time version of the utility function originally suggested by Abel (1990) to the framework of the modified Ramsey model *à la* Djajić (1987) and Ihori (1990). We show that the effects of a permanent increase in government spending already obtained in the modified Ramsey model are still robust in an economy with habit formation. We also show that the interaction between the inverse of intertemporal elasticity of substitution and the habit parameter is an important factor for the shadow prices of capital and habit stocks in the long-run.

Keywords: Government Spending; Habit Formation; Time-Nonseparable Preferences; Edgeworth Substitutes

JEL Classification: E21; E62; O40

1. Introduction

Recently the time-nonseparable preferences are extensively used in the macroeconomic analysis. Among them, the preferences with habit formation are the front runner. Examples include Abel (1990), Mansoorian (1996), Carroll, Overland and Weil (1997, 2000), Fuhrer (2000), Alvarez-Cuadrado, Monteiro and Turnovsky (2004) and so on. Except for Mansoorian (1996), the cited literatures commonly use the utility function originally suggested by Abel (1990).

Capturing the consumption externalities in a tractable way, Abel (1990) formulates the time-nonseparable utility function, which includes a standard time-separable preference as a special case.[1] Along the line of time-nonseparable preferences, in this paper we apply the continuous-time version of the utility function originally suggested by Abel (1990) to the framework of the modified Ramsey model extended by Djajić (1987) and Ihori (1990).

In Djajić (1987) and Ihori (1990), the benefit of government spending is taken as a variable giving direct utility to consumers. However, the modified Ramsey model by Djajić (1987) and Ihori (1990) assumes the time-separable preferences. The time-nonseparable preferences are not considered in their framework. Incorporating the habit formation structure *à la* Abel (1990) into their model implies introducing the time-nonseparable preferences therein.

We show that the effects of a permanent increase in government spending already obtained in the modified Ramsey model are still robust in an economy with habit formation. We also show that the interaction between the inverse of intertemporal elasticity of substitution and the habit parameter is an important factor for the shadow prices of capital and habit stocks in the long-run.

The rest of the paper is organized as follows. In Section 2 we present the model. In Section 3 we study the steady-state equilibrium of the economy with habit formation. In Section 4 we investigate the long-run effects of a permanent increase in government spending and the influences of habit parameter on the economy. Finally, in Section 5, concluding remarks are given.

2. The Model

Let us consider a closed economy consisting of a representative agent and a government. The representative agent seek to maximize the following lifetime utility over the infinite horizon.

$$\int_0^{\infty} U(C,\ G,\ S)e^{-\rho t}dt. \tag{1}$$

Lifetime utility is an integral of discounted instantaneous utility from time zero to infinity. The instantaneous utility $U(C, G, S)$ depends on private consumption C, the flow of government spending G and the habit stock S. Habit stock is reflecting past consumption experience of the representative agent and defined in equation (4) below. ρ is the subjective rate of time preference. To obtain closed-form solutions, we apply the following instantaneous utility function, which is the continuous-time version of the utility function originally suggested by Abel (1990).

$$U(C, G, S) = \begin{cases} \dfrac{[C+\eta G/S^{\gamma}]^{1-\sigma}}{1-\sigma} & \text{for } \sigma > 0 \text{ and } \sigma \neq 1, \\ \log(C+\eta G) - \gamma \log S & \text{for } \sigma = 1. \end{cases} \quad (2)$$

In equation (2) σ is the inverse of intertemporal elasticity of substitution. η is a parameter describing the substitutability between private consumption and government spending. As we assume that private consumption and government spending are Edgeworth substitutes, η is always positive. γ is a parameter reflecting the influences of the habit stock on the instantaneous utility and assumed to be positive. Hereafter, we call it the habit parameter. If habit parameter is zero, the utility function in equation (2) degenerates into the time-separable preference. By the definition of natural logarithm, S is always positive. Differing from Abel (1990), the instantaneous utility is defined over the effective consumption.[2)]

$$\dot{K} = Y - T - C, \quad (3)$$

where the initial value of K is given. Equation (3) is the flow budget constraint of a representative agent, or equation of the capital accumulation. Real output is consumed or invested after taxation. Output is produced by the well-behaved neoclassical production function such that $F'(K) > 0$ and $F''(K) < 0$. To obtain the closed-form solutions, we assume a Cobb-Douglas production function, $Y = AK^{\alpha}$ where $A > 0$ and $0 < \alpha < 1$. Labor is assumed to be supplied inelastically.

Government is assumed to impose a lump-sum tax T on a representative agent, and immediately expends as a government spending. We suppose that government's budget is always balanced, i.e., $G = T$ at any point of time.

$$\dot{S} = \theta[(C+\eta G) - S], \quad (4)$$

where $\theta > 0$ and the initial value of S is given. Equation (4) describes how habit stock evolves over time. If the current effective consumption exceeds habit stock, then the habit stock will increase in the next instance. The coefficient θ reflects the degree of such an adjustment process.

The representative agent maximizes equation (1), together with equation (2), subject to equations (3) and (4). The current-value Hamiltonian for the intertemporal optimization problem can be formulated as

$$H = \frac{[(C+\eta G)/S^{\gamma}]^{1-\sigma}}{1-\sigma} + \lambda[AK^{\alpha} - G - C] + \mu\theta[(C+\eta G) - S], \quad (5)$$

where λ and μ are the co-state variables with respect to K and S, respectively. Taking the sequence of government spending G as exogenous, the first-order conditions for optimality become as follows.

$$\frac{\partial H}{\partial C} = (C+\eta G)^{-\sigma}/S^{\gamma(1-\sigma)} - \lambda + \mu\theta = 0, \quad (6)$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} + \rho\lambda = \lambda\alpha A K^{\alpha-1}, \quad (7)$$

$$\frac{\partial H}{\partial S} = -\dot{\mu} + \rho\mu = -\gamma(C+\eta G)^{1-\sigma}/S^{\gamma(1-\sigma)+1} - \mu\theta, \quad (8)$$

and equations (3) and (4). Transversality conditions are:

$$\lim_{t\to\infty} e^{-\rho t}\lambda K = 0 \text{ and } \lim_{t\to\infty} e^{-\rho t}\mu S = 0. \quad (9)$$

Equations (3), (4), (6), (7) and (8) describes the behavior of the economy.

3. The Steady-State Analysis

In this section we investigate the steady-state equilibrium and its dynamic properties. As the system has five variables C, K, S, λ and μ, it is hard to solve analytically. However, by eliminating C from the system, we do make the analysis easier. From equation (6), we have

$$C = S^{\gamma(1-1/\sigma)}(\lambda-\mu\theta)^{-(1/\sigma)} - \eta G. \quad (10)$$

Substituting equation (10) into equations (3), (4), (7) and (8), we have the following dynamic system of four variables K, S, λ, and μ.

$$\dot{K} = AK^{\alpha} - S^{\gamma(1-1/\sigma)}(\lambda-\mu\theta)^{-(1/\sigma)} - (\eta-1)G, \quad (11)$$

$$\dot{S} = \theta[S^{\gamma(1-1/\sigma)}(\lambda-\mu\theta)^{-(1/\sigma)} - S], \quad (12)$$

$$\dot{\lambda} = (\rho - \alpha A K^{\alpha-1})\lambda, \quad (13)$$

$$\dot{\mu} = (\rho+\theta)\mu + \gamma S^{\gamma(1-1/\sigma)-1}(\lambda-\mu\theta)^{1-(1/\sigma)}. \quad (14)$$

Consider first the steady-state of the economy described by equations (11)-(14). Setting $\dot{K} = \dot{S} = \dot{\lambda} = \dot{\mu} = 0$ in equations (11)-(14), we have the following steady-state values of K, S, λ and μ expressed with bars.

$$\bar{K} = (\alpha A/\rho)^{\frac{1}{1-\alpha}}, \tag{15}$$

$$\bar{S} = A^{\frac{1}{1-\alpha}}(\alpha/\rho)^{\frac{\alpha}{1-\alpha}} + (\eta-1)G, \tag{16}$$

$$\bar{\lambda} = \frac{\rho/(\rho+\theta)}{\left[A^{\frac{1}{1-\alpha}}(\alpha/\rho)^{\frac{\alpha}{1-\alpha}} + (\eta-1)G\right]^{\gamma+\sigma(1-\gamma)}}, \tag{17}$$

$$\bar{\mu} = \frac{-1/(\rho+\theta)}{\left[A^{\frac{1}{1-\alpha}}(\alpha/\rho)^{\frac{\alpha}{1-\alpha}} + (\eta-1)G\right]^{\gamma+\sigma(1-\gamma)}}. \tag{18}$$

From equation (15) and the postulated Cobb-Douglas production function, we obtain

$$\bar{Y} = A^{\frac{1}{1-\alpha}}(\alpha/\rho)^{\frac{\alpha}{1-\alpha}}. \tag{19}$$

Substituting equations (16), (17) and (18) into equation (10), we have

$$\bar{C} = A^{\frac{1}{1-\alpha}}(\alpha/\rho)^{\frac{\alpha}{1-\alpha}} - G. \tag{20}$$

From the results above, we have the following relationships at the steady-state. We use these relationships, especially equation (22), in the dynamic analysis below.

$$\bar{C} + \eta G = \bar{S} = A^{\frac{1}{1-\alpha}}(\alpha/\rho)^{\frac{\alpha}{1-\alpha}} + (\eta-1)G, \tag{21}$$

$$\bar{\lambda} - \bar{\mu}\theta = \frac{1}{\left[A^{\frac{1}{1-\alpha}}(\alpha/\rho)^{\frac{\alpha}{1-\alpha}} + (\eta-1)G\right]^{\gamma+\sigma(1-\gamma)}} = \frac{1}{\bar{S}^{\gamma+\sigma(\gamma-1)}}. \tag{22}$$

Equation (21) means that the effective consumption at the steady-state depends on G and is independent of γ and σ. Equation (22) implies that the marginal utility of private consumption at the steady-state depends on G, γ and σ. How does the habit parameter γ influence the economy? We consider this problem in Section 4 below.

Consider next the dynamic properties of the economy described by equations (11)-(14). As the dynamic system is non-linear and consists of four variables, it is desirable to approximate the system around the steady-state equilibrium solved above. Linearizing equations (11)-(14) around the steady-state, we have the following dynamic system expressed in matrix notation.

$$\begin{bmatrix} \dot{K} \\ \dot{S} \\ \dot{\lambda} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ a_{31} & 0 & 0 & 0 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} K-\bar{K} \\ S-\bar{S} \\ \lambda-\bar{\lambda} \\ \mu-\bar{\mu} \end{bmatrix}, \tag{23}$$

where the elements of the coefficient matrix are given as follows.

$$a_{11} \equiv \rho,$$
$$a_{12} \equiv (1/\sigma - 1)\gamma\bar{S}^{2(\gamma-1)},$$
$$a_{13} \equiv (1/\sigma)\bar{S}^{2\gamma+(1+\sigma)(\gamma-1)},$$
$$a_{14} \equiv -(1/\sigma)\theta\bar{S}^{2\gamma+(1+\sigma)(\gamma-1)},$$
$$a_{22} \equiv [(1-1/\sigma)\gamma\bar{S}^{2(\gamma-1)} - 1]\theta,$$
$$a_{23} \equiv -(1/\sigma)\theta\bar{S}^{2\gamma+(1+\sigma)(\gamma-1)},$$
$$a_{24} \equiv (1/\sigma)\theta^2\bar{S}^{2\gamma+(1+\sigma)(\gamma-1)},$$
$$a_{31} \equiv (1-\alpha)\alpha\rho A\bar{K}^{\alpha-2}/(\rho+\theta)\bar{S}^{\gamma+\sigma(1-\gamma)},$$
$$a_{42} \equiv \gamma[\gamma(1-1/\sigma)-1)]/\bar{S}^{2+(1-\sigma)(\gamma-1)},$$
$$a_{43} \equiv \gamma(1-1/\sigma)\bar{S}^{2(\gamma-1)},$$
$$a_{44} \equiv \rho+\theta[1+\gamma(1-1/\sigma)\bar{S}^{2(1-\gamma)}].$$

$\bar{K}$ and $\bar{S}$ are given in equations (15) and (16), respectively. In the case of $\sigma = 1$, we have the simpler expression of the dynamic system.

We can analytically solve the characteristic equation of the coefficient matrix in equation (23) by using what is called "Ferrari's method" for solving a quartic. However, the solved expressions are so complicated and it does not seem appropriate and useful for the analysis below. Instead, we use the simple numerical experiment in this paper. For the specified parameters, we calculate the characteristic roots and evaluate the dynamic properties of the steady-state equilibrium.

As a numerical example, we specify the parameters as follows. The production parameters are $A = 1$ and $\alpha = 0.35$. On the other hand, the preference parameters are $\rho = 0.04$, $\sigma = 2.5$, $\gamma = 0.5$ and $\theta = 0.2$. These numbers are plausible and borrowed from Alvarez-Cuadrado, Monteiro and Turnovsky (2004). Furthermore, we assume $\eta = 1$, which means that the size of government spending is initially at the Samuelson level.[3] For these parameters, we have four characteristic roots: 0.0540, 0.4327, -0.1809 and -0.0143. As we have obtained two positive and two negative characteristic roots, this implies that the steady-state equilibrium is saddle-point stable.

Based on the saddle-point stability of the steady-state equilibrium, hereafter, we restrict our analysis only to the long-run effects of a permanent increase in government spending and the influences of habit parameter on the economy. We leave the detailed numerical analysis of the transitional dynamics of the economy for the future research.

4. Long-Run Effects of Government Spending and Habit Parameter

In this section we study the long-run effects of a permanent increase in government spending and the influences of habit parameter on the economy.

Consider first the long-run effects of a permanent increase in government spending. As easily recognized from equations (15) and (19), $\bar{K}$ and $\bar{Y}$ are independent of G. Thus, in the long run, the steady-state levels of K and therefore Y remains unchanged. On the other hand, equation (20) shows that private consumption C at the steady-state depends negatively on G. Thus, in the long run, a permanent increase in government spending decreases private consumption and the perfect crowding out emerges at the steady-state. These results are the same as in the modified Ramsey model *à la* Djajić (1987) and Ihori (1990).[4] We reconfirm that the effects of a permanent increase in government spending already obtained in the modified Ramsey model are still robust in our analytical framework with habit formation.

Habit stock at the steady-state $\bar{S}$ depends on G in equation (16). However, the effect of a permanent increase in government spending on $\bar{S}$ is ambiguous in this case. As you can see easily in equation (16), the effect depends on the parameter of substitutability between private consumption and government spending η. If $0 < \eta < 1$, then the habit stock at the steady-state increases in response to a permanent increase in government spending. Meanwhile, if $\eta > 1$, then the habit stock at the steady-state decreases. Furthermore, if $\eta = 1$ which means the size of government spending is initially at the Samuelson level, then the habit stock remains unchanged and shows no response to a permanent increase in government spending.

Consider next the influences of habit parameter on the economy. The steady-state levels of K, Y, C and S are independent of the habit parameter γ in equation (15), (16), (19) and (20). Thus, the variation in the habit parameter does not affect the real side of the economy. On the other hand, the shadow prices at the steady-state $\bar{\lambda}$ and $\bar{\mu}$, and thus the marginal utility of private consumption $\bar{\lambda}-\bar{\mu}\theta$, depend on the habit parameter γ in equations (17), (18) and (22). However, as the exponent of $\bar{S}$ shows, the influences of a variation in γ depend on the relationship with the inverse of intertemporal elasticity of substitution σ. If $0 < \sigma < 1$, then a rise in γ increases $\gamma+\sigma(1-\gamma)$ and so enhances the effects of $\bar{S}$. On the other hand, if $\sigma > 1$, then a rise in γ decreases $\gamma+\sigma(1-\gamma)$ and so represses the effects of $\bar{S}$. Furthermore, if $\sigma = 1$, then $\gamma+\sigma(1-\gamma)$ becomes unity and a rise in γ does not affect $\bar{S}$. This case has the same effect as the case of $\gamma = 1$ for any value of $\sigma > 0$ does. Thus, the interaction between the inverse of intertemporal elasticity of substitution and the habit parameter is an important factor for the shadow prices of capital stock and habit stock at the steady-state.

5. Concluding Remarks

This paper has examined the effects of a permanent increase in government spending in an economy with habit formation. For this purpose we have applied the continuous-time version of the utility function originally suggested by Abel (1990) to the framework of the modified Ramsey model *à la* Djajić (1997) and Ihori (1990).

We have shown that the effects of a permanent increase in government spending already obtained in the modified Ramsey model are still robust in an economy with habit formation. Even if we take habit formation into account, private consumption is perfectly crowded out while capital stock and therefore real output remains unchanged in the long run. We have also shown that the interaction between the inverse of intertemporal elasticity of substitution and the habit parameter is an important factor for the shadow prices of capital stock and habit stock at the steady-state.

In this paper we restrict our analysis only to the long-run effects of a permanent increase in government spending and the influences of habit parameter on the economy. The remaining task for the future research is the detailed numerical analysis of the transitional dynamics of the economy.

Notes

1) Habit formation is one of the consumption externalities. Another one is "Keeping up with the Joneses" preference. See, e.g., Abel (1990) and Alvarez-Cuadrado, Monteiro and Turnovsky (2004).
2) See Barro (1981) and Ihori (1990).
3) See Ihori (1990, pp. 66-68).
4) The results in the modified Ramsey model *à la* Djajić (1987) and Ihori (1990) are also the same as those in the standard Ramsey model. For the effects of an expansionary fiscal policy in the standard Ramsey model, see, e.g., Heijdra and van der Ploeg (2002, pp. 440-443) and Barro and Sala-i-Martin (2004, pp. 147-149).

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