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# The Dynamic Mundell-Fleming Model Reconsidered

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## abstract

In this paper we reconsider the dynamic Mundell-Fleming model of Sarno and Taylor (2002) by incorporating one of the recent New Keynesian ingredients. In an extended framework, we reconfirm that their results on the effects of an expansionary fiscal policy are robust. However, we also show that their results on the effects of an expansionary monetary policy should be modified.

## 1. Introduction

In their recent book, Sarno and Taylor (2002) presented an interesting extension of the original Mundell-Fleming model into the dynamic framework, and called it, in their words, the perfect-foresight Mundell-Fleming model.<sup>1)</sup> Taking into account the perfect foresight and the dynamic quantitative adjustment, Sarno and Taylor (2002) obtained the following interesting conclusions. (i) Without price stickiness, exchange rate overshooting *à la* Dornbusch (1976) emerges necessarily as a result of an expansionary monetary policy. (ii) Even under the flexible exchange rate system, an expansionary fiscal policy is effective in stimulating the real economy, and the exchange rate shows an initial undershooting phenomenon on the adjustment process to the new steady-state equilibrium.

These two conclusions of Sarno and Taylor (2002) are interesting to us who are familiar with the conventional wisdom of the original Mundell-Fleming model and the Dornbusch (1976) model. The exchange rate overshooting without price stickiness is particularly interesting. Generally, price stickiness is taken as the fundamental source of exchange rate overshooting since Dornbusch (1976) finding. However, they obtained an overshooting phenomenon of the exchange rate under the assumptions of price fixity and

production sluggishness. Sarno and Taylor (2002) actively applied the analytical concepts of perfect foresight and saddle-point stability to the original Mundell-Fleming model. These analytical concepts were unavailable techniques when Mundell and Fleming wrote their original works in the early 1960s. Using up-to-date techniques developed in the 1970s, therefore, Sarno and Taylor (2002) obtained such an exchange rate overshooting under the price fixity.

By the way, in recent development of the New Keynesian macroeconomics, a group of researchers extend an analysis by the reduced-form macroeconomic models.<sup>2)</sup> According to them, a slight modification of the reduced-form model becomes available as an approximation of fully micro-founded model. That is, a reduced-form macroeconomic model has an implicit microeconomic foundation. In this paper we apply this approach and extend the dynamic Mundell-Fleming model by Sarno and Taylor (2002). By incorporating one of the New Keynesian ingredients, we reconsider the results of Sarno and Taylor (2002). On the effects of an expansionary fiscal policy, we reconfirm the robustness of their results. On the effects of an expansionary monetary policy, however, we show their results to be modified.

The rest of the paper is organized as follows. In Section 2 we present the model. In Section 3, we examine the equilibrium of the model and its dynamic properties, and investigate the effects of expansionary monetary and fiscal policies in the case where real money balance is deflated by the general price index. On the other hand, in Section 4, we explore the equilibrium of the model and its dynamic properties, and investigate the effects of expansionary monetary and fiscal policies in the case where real money balance is deflated only by domestic price index. Finally, in Section 5, concluding remarks are given.

## 2. The Model

In this section, we present the model, which is an extension of Sarno and Taylor (2002) and reformulated in the discrete-time framework. Consider a small open economy under the flexible exchange rate system and the perfect capital mobility. The economy is described by the following equations (1) - (5). The model is log-linear and all variables except for interest rates are in logarithm.

$$i_t = i_t^* + E_t s_{t+1} - s_t . \quad (1)$$

Equation (1) is the uncovered interest parity condition.  $i_t$  is the domestic interest rate,  $i_t^*$  is the world interest rate,  $s_t$  is the nominal exchange rate, and  $E_t s_{t+1}$  is the expected future exchange rate. Equation (1) means that the domestic interest rate is equal to the world interest rate plus the expected rate of depreciation in exchange rate. Under the perfect foresight assumption we impose below, the expected rate of depreciation in exchange rate is equal to the actual rate of depreciation.<sup>3)</sup>

$$m_t - q_t = -\lambda i_t + \phi y_t , \quad (2)$$

where  $\lambda > 0$  and  $\phi > 0$ . Equation (2) is the money market equilibrium condition. The left-hand side of equation (2) is real money supply where nominal money  $m_t$  is deflated by the general price index  $q_t$ . On the other hand, the right-hand side of equation (2) is real money demand. Real money demand is assumed to depend negatively on the domestic interest rate  $i_t$  and positively on real income  $y_t$ .

$$y_t^d = E_t y_{t+1} + \delta (s_t + p_t^* - p_t) - \sigma i_t + u , \quad (3)$$

where  $\delta > 0$  and  $\sigma > 0$ . Equation (3) means that the goods demand depends on the expected future income level  $E_t y_{t+1}$ , real exchange rate  $s_t + p_t^* - p_t$ , and interest rate  $i_t$ .  $u$  is a shift parameter, including government expenditures. Containing the expected future income  $E_t y_{t+1}$  is the property of the current New Keynesian macroeconomics. Incorporating one of the New Keynesian ingredients into to the IS relation, the forward-looking property is introduced to the static, original Mundell-Fleming model.

$$\Delta y_{t+1} = \eta (y_t^d - y_t) , \quad (4)$$

where  $0 < \eta < 1$ . Equation (4) describes the dynamic quantity adjustment.  $\eta$  is an adjustment coefficient explaining the sensitivity of economy's supply side. If the current demand exceeds the current supply, then the future output increases. Real income is assumed to be demand-determined. This reflects the Keynesian view on macro economy.  $\eta < 1$  means that the dynamic quantity adjustment is incomplete, i.e., the current excess aggregate demand is not fulfilled in the next period. This expresses the sluggishness of production side. As  $\eta < 1$  also guarantees the stability of the model, we assume this for the analysis below.

$$q_t \equiv \theta(s_t + p_t^*) + (1 - \theta)p_t, \quad (5)$$

where  $0 \leq \theta < 1$ . Equation (5) is a definition of general price index, which is a weighted average of foreign goods prices evaluated by domestic currencies and home goods prices.  $\theta$  is a weight assigned to the foreign goods prices evaluated by domestic currencies. When  $\theta = 0$ , the general price index is equivalent to the home goods prices. In the original Mundell-Fleming model this case was examined, and we also investigate this case in Section 4.

For simplicity, we impose several assumptions. Foreign interest rate and nominal money supply are given constant, except for once-and-for-all monetary expansion that we consider in the following sections. So we omit time subscripts from them and denote  $i^*$  and  $m$  below. Following the original Mundell-Fleming model, we also assume price fixity and set  $p_t = p_t^* = 0$  for simplicity. This means that real exchange rate  $s_t + p_t^* - p_t$  moves parallel to nominal exchange rate  $s_t$ . We further assume the perfect foresight in the model. So we have  $E_t s_{t+1} = s_{t+1}$  and  $E_t y_{t+1} = y_{t+1}$ .

### 3. Real Money Balance with Exchange Rate Effect

In this section we consider the case for  $0 < \theta < 1$  where the general price index is constituted by a weighted average of foreign goods price evaluated by domestic currencies and home goods price. In this case the fluctuation of exchange rate affects the real money balance, and influences the effects of monetary and fiscal policies. Sarno and Taylor (2002) examined this case.

First of all, let us examine the equilibrium of the economy and its dynamic properties. Substituting equations (1) and (5) into equation (2), together with the perfect foresight assumption  $E_t s_{t+1} = s_{t+1}$ , we get the following equation (6) describing the motion of exchange rate.

$$\Delta s_{t+1} = \frac{\theta}{\lambda} s_t + \frac{\phi}{\lambda} y_t - \frac{1}{\lambda} m - i^*. \quad (6)$$

Next, combining equations (2), (3), and (5) with equation (4), together with the perfect foresight assumption  $E_t y_{t+1} = y_{t+1}$ , we have the following equation (7) describing output adjustment.<sup>4)</sup>

$$\Delta y_{t+1} = \frac{\eta}{1-\eta} \left[ \left( \delta - \frac{\sigma\theta}{\lambda} \right) s_t - \frac{\sigma\phi}{\lambda} y_t + \frac{\sigma}{\lambda} m + u \right]. \quad (7)$$

The dynamics of the economy is described by equations (6) and (7). The  $\Delta s_{t+1} = 0$  locus is:

$$s_t = -\frac{\phi}{\theta} y_t + \frac{m + \lambda i^*}{\theta}. \quad (8)$$

On the other hand, the  $\Delta y_{t+1} = 0$  locus is:

$$s_t = \frac{1}{\delta - \sigma\theta/\lambda} \left( \frac{\sigma\phi}{\lambda} y_t - \frac{\sigma}{\lambda} m - u \right). \quad (9)$$

The equilibrium of the economy is the intersection of the  $\Delta s = 0$  and the  $\Delta y = 0$  loci, and described as point *A* in Figure 1.

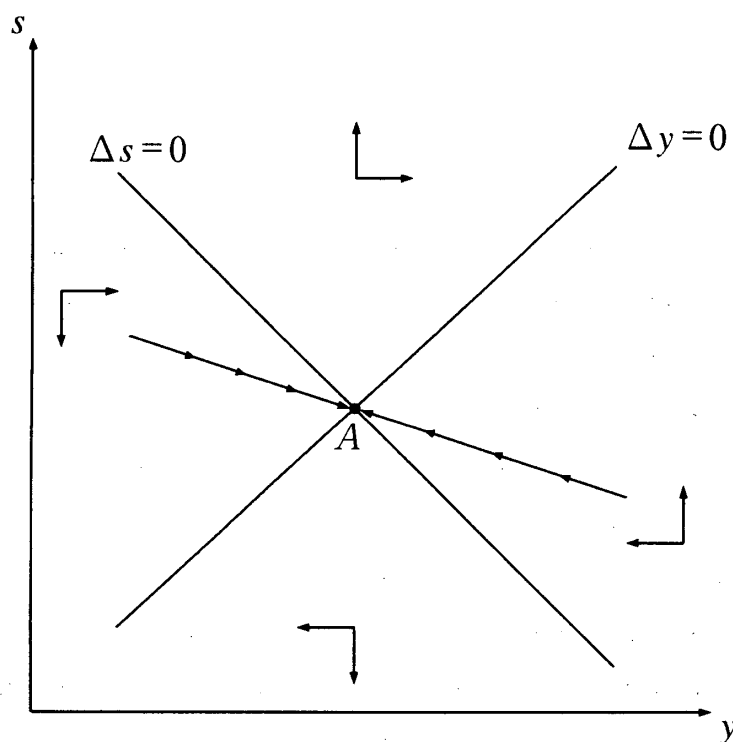


Figure 1. The Dynamic Mundell-Fleming Model for  $0 < \theta < 1$ .

To examine the dynamic properties of the equilibrium, let us present equations (6) and (7) in matrix form.

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta y_{t+1} \end{bmatrix} = J \begin{bmatrix} s_t \\ y_t \end{bmatrix} + \begin{bmatrix} -\frac{1}{\lambda} m - i^* \\ \frac{\eta}{1-\eta} \left( \frac{\sigma}{\lambda} m + u \right) \end{bmatrix}, \quad (10)$$

where

$$J \equiv \begin{bmatrix} \frac{\theta}{\lambda} & \frac{\phi}{\lambda} \\ \frac{\eta}{1-\eta} \left( \delta - \frac{\sigma\theta}{\lambda} \right) & \frac{-\eta}{1-\eta} \left( \frac{\sigma\phi}{\lambda} \right) \end{bmatrix}.$$

The coefficient matrix  $J$  of the system has the negative determinant as follows.

$$\det J = \frac{-\eta\phi\delta}{(1-\eta)\lambda} < 0. \quad (11)$$

As the determinant of the coefficient matrix  $J$  has a negative sign, the equilibrium of the economy is saddle-point stable as shown in Figure 1. So there exists a saddle-point path convergent to point  $A$ .

### 3.1. Monetary Policy

In this subsection we examine the effects of an expansionary monetary policy on the economy. When an unanticipated monetary expansion is conducted, both the  $\Delta s=0$  and the  $\Delta y=0$  loci are affected. The  $\Delta s=0$  and the  $\Delta y=0$  loci shift rightward at the same magnitude.<sup>5)</sup> So when we have a monetary expansion, the equilibrium of the economy moves from point  $A$  to point  $C$  in Figure 2. Comparing new equilibrium with old one, real output increases while exchange rate remains unchanged.

Considering the dynamic adjustment process, we have an interesting phenomenon in the short run. When an expansionary monetary policy is conducted, the economy, initially located at point  $A$ , jumps upward to the point  $B$ . Point  $B$  is located on the new saddle-point path through the new equilibrium point  $C$ . At the same time, exchange rate largely depreciates instantaneously. Then, the economy moves from point  $B$  to point  $C$  over time along the new saddle-point path. On the adjustment process, real output increases while exchange rate appreciates and returns to its initial level. That is, we have no overshooting phenomenon of the nominal exchange rate. This result contrasts with the Sarno and Taylor (2002) result.

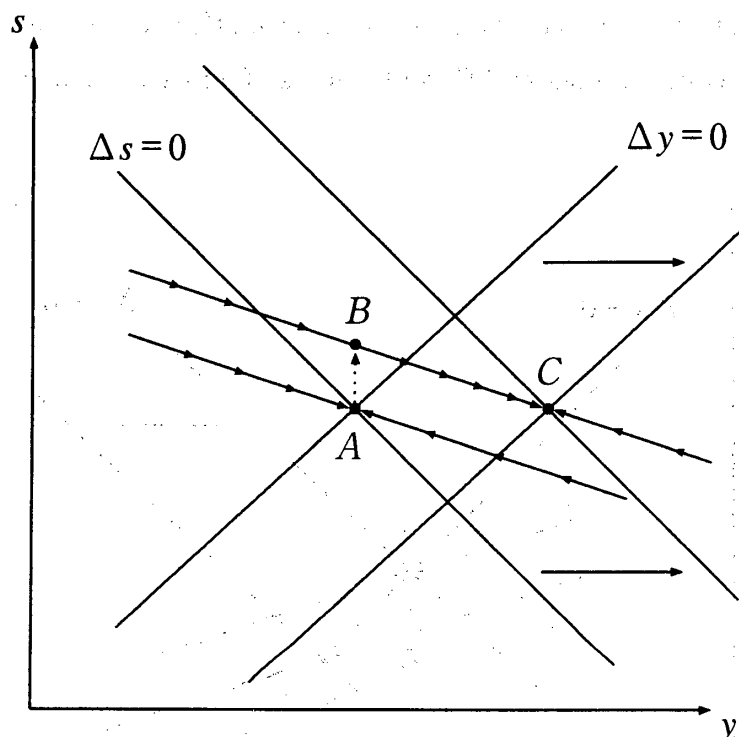


Figure 2. Monetary Policy for  $0 < \theta < 1$ .

Since Dornbusch (1976) finding, exchange rate overshooting is considered as a result of the presence of price stickiness. On the other hand, Sarno and Taylor (2002) shows the possibility of exchange rate overshooting under price fixity, which means that price stickiness is not a necessary condition for overshooting phenomenon of the exchange rate. In this paper, however, we show that their results should be modified if we incorporate one of the New Keynesian ingredients into the Sarno and Taylor (2002) model. The result that an expansionary monetary policy has no effect on the long-run level of exchange rate is a new result we obtained here.

### 3.2. Fiscal Policy

In this subsection we consider the effects of an expansionary fiscal policy on the economy. When an unanticipated fiscal expansion is performed, only the  $\Delta y = 0$  locus is affected.  $\Delta y = 0$  locus shifts rightward. On the other hand, the  $\Delta s = 0$  locus remains unchanged. As a result, the equilibrium of the economy moves from point A to point C in Figure 3. Comparing new equilibrium with old one, real output increases while exchange rate appreciates. As is well known, an expansionary fiscal policy in a small



open economy under the flexible exchange rate is ineffective in the original Mundell-Fleming model. However, we show different results of an expansionary fiscal policy here.

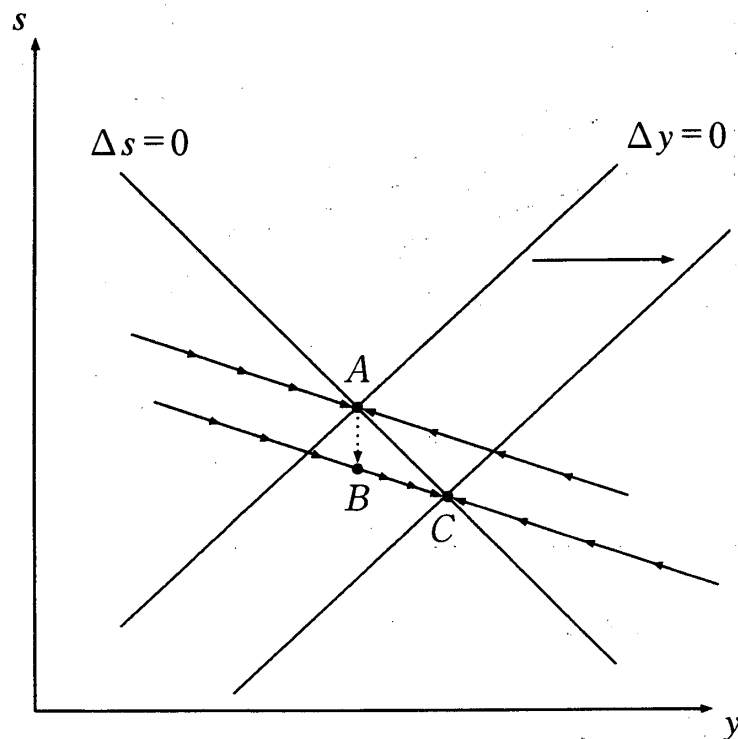


Figure 3. Fiscal Policy for  $0 < \theta < 1$ .

Examining the dynamic adjustment process, the economy jumps downward from point  $A$  to point  $B$  when a fiscal expansion is conducted, and at the same time exchange rate appreciates instantaneously. The point  $B$  is located on the new saddle-point path through the new equilibrium point  $C$ . After reaching point  $B$ , the economy moves toward new equilibrium point  $C$  over time along the new saddle-point path. On the adjustment process, real output increases while exchange rate appreciates. The remarkable feature in the case for a fiscal expansion is the emergence of exchange rate undershooting. This exchange rate undershooting phenomenon similarly emerges in Sarno and Taylor (2002). This undershooting of exchange rate is an important property of the dynamic Mundell-Fleming model.

In the original Mundell-Fleming model, it is well known that, as a comparative statics result, an expansionary fiscal policy generates exchange rate appreciation while keeps real output unchanged. However, in a dynamic framework with perfect foresight and quantitative adjustment, it becomes clear that, even under the flexible exchange rate

system, an expansionary fiscal policy is effective to stimulate the real economy. Also, it is shown that exchange rate undershooting emerges on the adjustment process.

#### 4. Real Money Balance without Exchange Rate Effect

In this section we examine the case for  $\theta=0$ .<sup>6)</sup> In this case we have  $q_t=p_t$ , so real money balance is deflated only by domestic price, and independent of the effect of exchange rate fluctuation. The original Mundell-Fleming model investigated this case. Sarno and Taylor (2002) refer to this case briefly. But we examine this case in detail here.

In the case of  $\theta=0$ , equations (6) and (7), describing the motion of exchange rate and real output respectively, must be modified. Assuming  $\theta=0$  in equations (6) and (7), we have:

$$\Delta s_{t+1} = \frac{\phi}{\lambda} y_t - \frac{1}{\lambda} m - i^* \quad (12)$$

and

$$\Delta y_{t+1} = \frac{\eta}{1-\eta} \left( \delta s_t - \frac{\sigma\phi}{\lambda} y_t + \frac{\sigma}{\lambda} m + u \right) \quad (13)$$

Therefore, in the case of  $\theta=0$ , the working of the economy is described by equations (12) and (13). The  $\Delta s_{t+1}=0$  locus is:

$$y_t = \frac{1}{\phi} m + \frac{\lambda}{\phi} i^* \quad (14)$$

On the other hand, the  $\Delta y_{t+1}=0$  locus is:

$$s_t = \frac{1}{\delta} \left( \frac{\sigma\phi}{\lambda} y_t - \frac{\sigma}{\lambda} m - u \right) \quad (15)$$

When  $\theta=0$ , the  $\Delta s=0$  locus becomes vertical. Meanwhile, the  $\Delta y=0$  locus is still upward-sloping as before and becomes steeper. Similarly to the previous section, the equilibrium of the economy is the intersection of both the  $\Delta s=0$  and the  $\Delta y=0$  loci as shown in Figure 4.

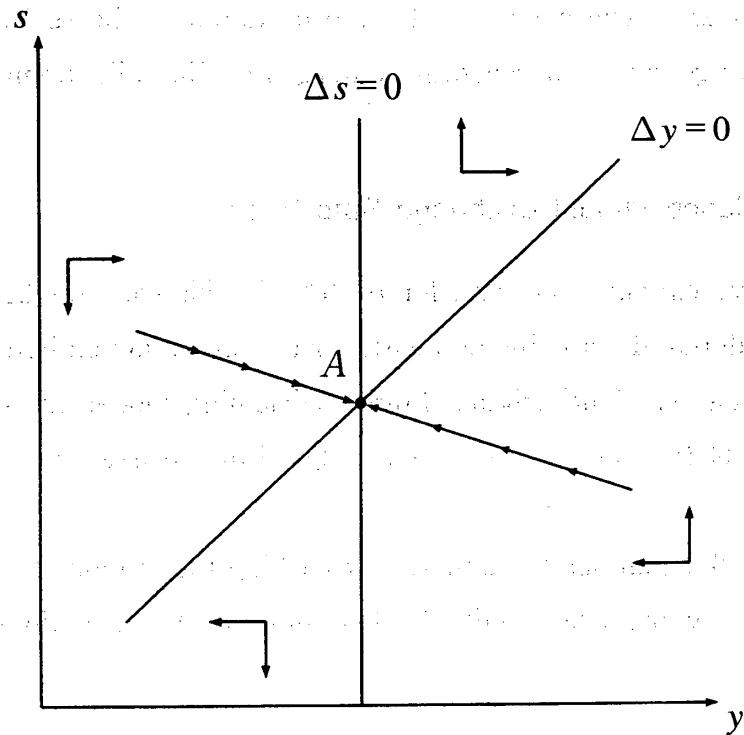


Figure 4. The Dynamic Mundell-Fleming Model for  $\theta=0$ .

To examine the dynamic properties of the equilibrium, representing equations (12) and (13) in matrix form, we have:

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta y_{t+1} \end{bmatrix} = H \begin{bmatrix} s_t \\ y_t \end{bmatrix} + \begin{bmatrix} -\frac{1}{\lambda} m - i^* \\ \frac{\eta}{1-\eta} \left( \frac{\sigma}{\lambda} m + u \right) \end{bmatrix}. \quad (16)$$

where

$$H \equiv \begin{bmatrix} 0 & \frac{\phi}{\lambda} \\ \frac{\eta \delta}{1-\eta} & \frac{-\eta}{1-\eta} \left( \frac{\sigma \phi}{\lambda} \right) \end{bmatrix}.$$

The coefficient matrix  $H$  of the system has the negative determinant as follows.

$$\det H = \frac{-\eta \phi \delta}{(1-\eta)\lambda} < 0. \quad (17)$$

As the determinant of the coefficient matrix  $H$  has a negative sign, the equilibrium of the economy, point  $A$ , is saddle-point stable as shown in Figure 4. Thus, there exists a

unique convergent saddle-point path through point A.

#### 4.1. Monetary Policy

Consider first the effects of an expansionary monetary policy on the economy. When we have an unanticipated monetary expansion, both the  $\Delta s=0$  and the  $\Delta y=0$  loci are affected. The  $\Delta s=0$  and the  $\Delta y=0$  loci simultaneously shift rightward at the same magnitude.<sup>7)</sup> When an expansionary monetary policy is executed, the equilibrium of the economy moves from point A to point C in Figure 5. Comparing before and after monetary expansion, real output increases while exchange rate remains unchanged. Monetary policy is effective in the sense that monetary expansion stimulates the real economy.

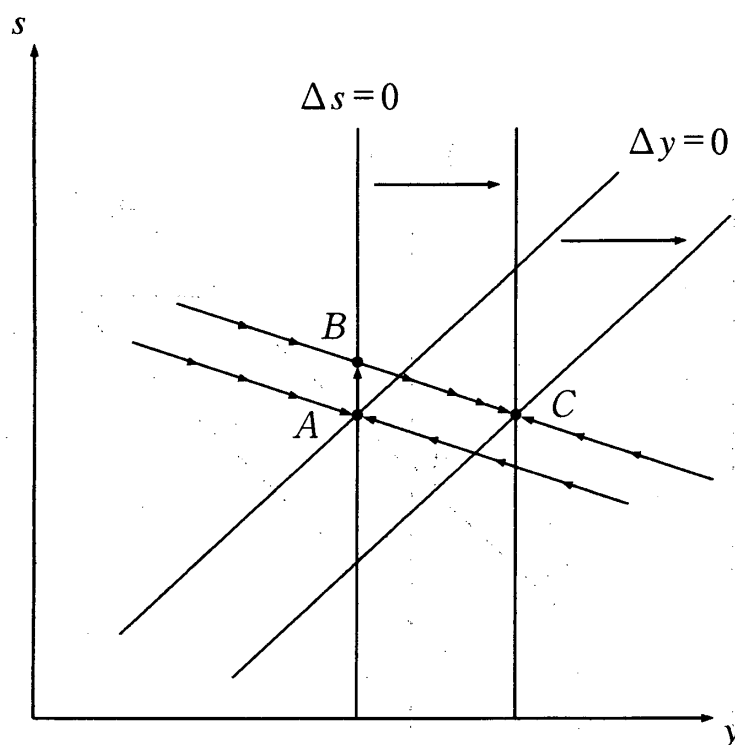


Figure 5. Monetary Policy for  $\theta=0$ .

On the adjustment process, real output increases gradually. On the other hand, the exchange rate depreciates first, and then appreciates and returns to its initial level. That is, in the case of  $\theta=0$ , we have no exchange rate overshooting. This result is similar to that of the previous section in the case of  $0 < \theta < 1$ .

In spite of initial large depreciation, the equilibrium level of exchange rate is unaffected by a monetary expansion.

#### 4.2. Fiscal Policy

Consider next the effects of an expansionary fiscal policy on the economy. When we have an unanticipated fiscal expansion, the  $\Delta y=0$  locus is affected while the  $\Delta s=0$  locus is unaffected. When an expansionary fiscal policy is conducted, the  $\Delta y=0$  locus shifts rightward and the equilibrium of the economy moves from point  $A$  to point  $C$  in Figure 6. Comparing before and after fiscal expansion, real output remains unchanged while exchange rate appreciates. As the fiscal expansion does not stimulate the economy, the original Mundell-Fleming result that a fiscal policy is ineffective in a small open economy under the flexible exchange rate system is restored in this case.

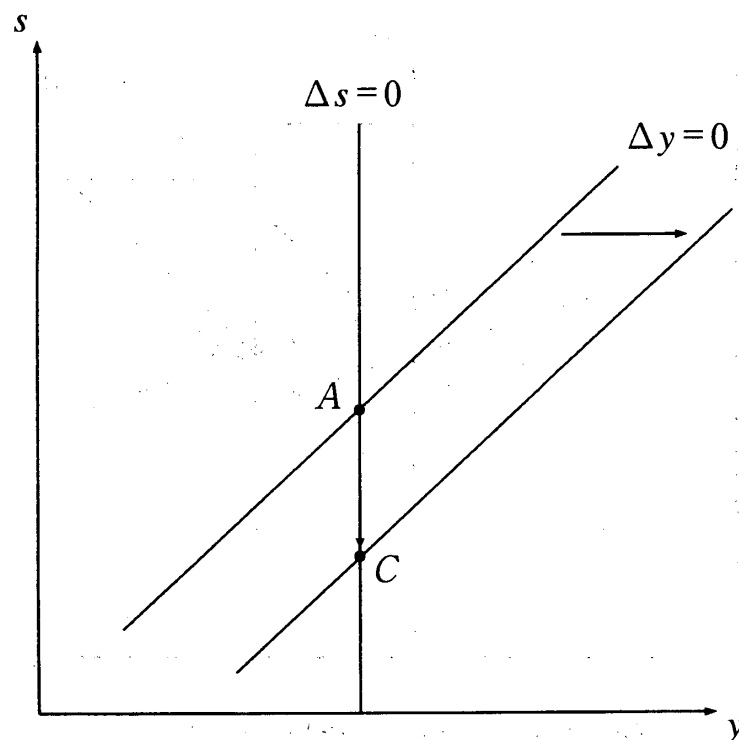


Figure 6. Fiscal Policy for  $\theta=0$ .

The remarkable feature of this case is that exchange rate jumps down from point  $A$  to point  $C$  instantaneously. So there is no transitional time in this case.

## 5. Concluding Remarks

In this paper we reconsider the dynamic Mundell-Fleming model of Sarno and Taylor (2002) by incorporating one of the recent New Keynesian ingredients. In an extended framework, we reconfirm that their results on the effects of an expansionary fiscal policy are robust. When we have a fiscal expansion, real output increases while exchange rate appreciates in the long-run, with initial undershooting phenomenon. However, we also show that their results on the effects of an expansionary monetary policy should be modified. When we have a monetary expansion, real output increases while exchange rate remains unchanged and shows no overshooting phenomenon.

## Notes

- 1) By the term "the original Mundell-Fleming model," we mean an extension of the IS-LM model to a small open economy, where price fixity, perfect capital mobility, and static expectation on the future exchange rate are assumed. Under the flexible exchange rate system, an expansionary monetary policy is effective and the equilibrium exchange rate depreciates. On the contrary, an expansionary fiscal policy is ineffective and the equilibrium exchange rate appreciates. As analyzed in a static framework, the details of the adjustment process are not clear in the original Mundell-Fleming model.
- 2) See, e.g., Clarida, Gali, and Gertler (1999), McCallum and Nelson (1999), and Walsh (2003).
- 3) In the original Mundell-Fleming model, the static expectation of the future exchange rate is assumed. So the domestic interest rate is always equal to the world interest rate, and thus exchange rate dynamics is concealed. On the other hand, in this paper we assume perfect foresight on the future exchange rate, and so we can analyze the exchange rate dynamics as below.
- 4) The influence of incorporating the expected future income  $E_t y_{t+1}$  into the model appears as the coefficient  $\frac{\eta}{1-\eta}$  in equation (7). However, the incorporation does not modify the qualitative properties of the equilibrium significantly. If we do not incorporate  $E_t y_{t+1}$  into the model,  $\frac{\eta}{1-\eta}$  should be replaced with  $\eta$ .

5) As easily confirmed from equations (8) and (9), we have:

$$\left. \frac{\partial y}{\partial m} \right|_{\Delta s=0} = \left. \frac{\partial y}{\partial m} \right|_{\Delta y=0} = \frac{1}{\phi} > 0.$$

This implies that both the  $\Delta s=0$  and the  $\Delta y=0$  loci shift at the same magnitude.

6) Blanchard and Fischer (1989, pp.540-542.) examine the effects of monetary policy in the similar framework. Their model is an extension of Dornbusch (1976) with output level adjustment. The difference is that we incorporate one of the recent New Keynesian ingredients into the model. Further, they did not examine the effects of fiscal policy there.

7) Similarly to the case for  $0 < \theta < 1$ , both the  $\Delta s=0$  and the  $\Delta y=0$  loci shift at the same magnitude. From equations (14) and (15), we have:

$$\left. \frac{\partial y}{\partial m} \right|_{\Delta s=0} = \left. \frac{\partial y}{\partial m} \right|_{\Delta y=0} = \frac{1}{\phi} > 0.$$

See also note 5) above.

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