<Article>Government Spending, Private Consumption, and Edgeworth Substitutes

Government Spending, Private Consumption, and Edgeworth Substitutes

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abstract

In this paper we extend Djajić (1987)-Ihori (1990) model of neoclassical optimal growth. In an extended model, (i) private consumption and government spending are assumed to be Edgeworth substitutes, and (ii) the work effort disutility is implicitly incorporated. Using this analytical framework, the effects of a permanent increase in government spending on private consumption, capital accumulation, and real output are examined. We show that the parameter of substituability between private consumption and government spending in the instantaneous utility function plays a crucial role in determining the long-run effects of a permanent increase in government spending.

1. Introduction

The macroeconomic effects of a government spending is an important theoretical issue in a neoclassical one-sector optimal growth model. Until now, many analytical frameworks have been suggested. Among them, Djajić (1987) presented a model where the benefit of government spending is incorporated into the instantaneous utility function. Using such a model, he analyzed the effects of a temporary increase in government spending. Later, Ihori (1990) extended Djajić (1987) model and examined the effects of temporary and permanent increases in government spending in detail.

A common drawback in Djajić (1987) and Ihori (1990) models is an assumption of inelastic labor supply. An exogenous labor supply is assumed in both models. Palivos and Yip (1996) explicitly incorporated the labor-leisure choice into Djajić (1987)-Ihori (1990) model. And they analyzed the macroeconomic effects of government spending.

In Kaneko (2001), we implicitly incorporated the work effort disutility into Djajić

(1987)-Ihori (1990) model, and examined the effects of various external shocks, including a permanent increase in government spending. However, for an analytical simplicity, Edgeworth independence between private consumption and government spending in the instantaneous utility function was assumed there.

In this paper we extend Djajić (1987)-Ihori (1990) model of neoclassical optimal growth. In an extended model, (i) private consumption and government spending are assumed to be Edgeworth substitutes, and (ii) the work effort disutility is implicitly incorporated. Using this analytical framework, the effects of a permanent increase in government spending on private consumption, capital accumulation, and real output are examined. We show that the parameter of substituability between private consumption and government spending in the instantaneous utility function plays a crucial role in determining the long-run effects of a permanent increase in government spending.

The remainder of the paper is organized as follows. In section 2 we present the model and show the dynamic properties of the economy. In section 3 we examine the effects of a permanent increase in government spending on private consumption, capital stock and real output. In section 4 the optimal size of government spending and its relation to the parameter for substituability are discussed. Finally, in section 5, concluding remarks are given.

2. The Model

In this section, we extend Djajić (1987)-Ihori (1990) model of neoclassical optimal growth. The remarkable feature of their model is an incorporation of the benefit of government spending in the instantaneous utility function. In addition, here we incorporate the work effort disutility implicitly, and assume Edgeworth substitutes between private consumption and government spending.

Consider a closed economy consisting of a representative agent and a government. The representative agent produces a single output with the production technology. The output can be used for consumption, capital accumulation, and tax payment.

To examine the influence of Edgeworth substituability between private consumption and government spending in a tractable way, we assume the following instantaneous utility function.

$$U(C,G,Y)=\log(C+\eta G)-\theta Y, \tag{1}$$

where $\eta>0$ and $\theta>0$.

Equation (1) means that instantaneous utility depends positively on effective consumption $C+\eta G$ and negatively on real output Y.[1] The effective consumption is a linear combination of private consumption C and government spending G. By the assumption of logarithm, the weighted sum of private consumption and government spending is always positive. Both private consumption and government spending positively affect the utility. The parameter η describes the substituability between private consumption C and government spending G. Since the parameter η is supposed to be positive, private consumption and government spending are Edgeworth substitutes. In the following, we investigate three cases of Edgeworth substitutes depending on the parameter η.

On the other hand, real output Y in the instantaneous utility function captures disutility of work effort. The level of work effort is related with that of real output through the production function. Real output negatively affects the utility. For simplicity, marginal disutility is supposed to be constant.

The objective of the representative agent is assumed to maximize his lifetime utility over the infinite horizon with perfect foresight:

$$\int_0^\infty U(C,G,Y)e^{-\rho t}dt, \tag{2}$$

where ρ is the constant, subjective rate of time preference ($\rho>0$). For simplicity, time subscript for all variables are omitted.

The government is assumed to levy a lump-sum tax T on the representative agent, and immediately expends as a government spending G which is an equal unit of the public good. For simplicity, the balanced budget $G=T$ is assumed at any time.

Under the balanced budget, real output net of private consumption and government spending contributes to capital accumulation. Equation (3) describes this, and is the flow budget constraint for the representative agent.

$$\dot{K}=Y-C-G. \tag{3}$$

The production function is given by equation (4) which implicitly captures labor supply.[2)]

$$Y=F(K),\quad F'(K)>0,\quad F''(K)<0. \tag{4}$$

And we also assume the following Inada conditions on production function.

$$\lim_{K\to 0} F(K)=0,\quad \lim_{K\to 0} F'(K)=\infty,\quad \lim_{K\to\infty} F'(K)=0. \tag{5}$$

The representative agent is supposed to maximize the lifetime utility in equation (2), together with equation (1), under the flow budget constraint in equation (3) and the production function in equation (4), given initial capital stock $K_0(>0)$, and government spending $G(>0)$.

The intertemporal optimization problem is summarized to maximize the following current-value Hamiltonian in equation (6) with respect to C, K, and λ, where λ is the co-state variable associated with the capital stock.

$$H=\log(C+\eta G)-\theta F(K)+\lambda[F(K)-C-G]. \tag{6}$$

The first-order conditions are

$$\frac{\partial H}{\partial C}=(C+\eta G)^{-1}-\lambda=0, \tag{7}$$

$$\frac{\partial H}{\partial K}=-\dot{\lambda}+\rho\lambda=\theta F'(K)+\lambda F'(K), \tag{8}$$

$$\frac{\partial H}{\partial \lambda}=\dot{K}=F(K)-C-G, \tag{9}$$

and the transversality condition,

$$\lim_{t\to\infty} \lambda K e^{-\rho t}=0. \tag{10}$$

Combining equations (7) and (8), we get the following Euler equation describing the motion of private consumption C.

$$\dot{C}=(C+\eta G)[F'(K)-\rho]-(C+\eta G)^2\theta F'(K). \tag{11}$$

Equations (9) and (11) describes the working of the economy. To investigate the steady-state equilibrium and its dynamic properties, we use the phase diagram technique.

First, examine the $\dot{C}=0$ locus. From equation (11), the $\dot{C}=0$ locus is described as the relation between C and K satisfying the following equation (12).

$$C=\frac{F'(K)-\rho}{\theta F'(K)}-\eta G. \tag{12}$$

Note that equation (12) depends on G and η. This implies that (i) an increase in government spending affects the $\dot{C}=0$ locus, and (ii) the degree of a change depends on the magnitude of the parameter η. The degree of substituability between private consumption and government spending in an Edgeworth sense is described by the size of the parameter η.

Next, consider the $\dot{K}=0$ locus. The $\dot{K}=0$ locus expresses the relation between C and K satisfying the following equation (13).

$$C=F(K)-G. \tag{13}$$

The phase diagram of this economy is described in Figure 1. The $\dot{C}=0$ locus has a negative slope, the vertical intercept on the C-axis at $\overline{C}$, and the horizontal intercept on the K-axis at $\overline{K}$. $\overline{C}$ is equal to $(1/\theta)-\eta G$ and $\overline{K}$ is satisfying $F'(\overline{K})=\rho/(1-\theta\eta G)$. The $\dot{K}=0$ locus has a positive slope and the horizontal intercept on the K at $\underline{K}$. $\underline{K}$ is satisfying $F(\underline{K})=G$.

The long-run equilibrium of the economy is described as the intersection of the $\dot{C}=0$ and $\dot{K}=0$ loci. Point E_0 in Figure 1 is the long-run steady-state equilibrium. Without population growth and technological progress, the steady-state values of C and K are constant. We denote the steady-state values as C^* and K^* in Figure 1.

As easily shown, the long-run equilibrium E_0 is saddle-point stable. Let us take a linear approximation of equations (9) and (11) around the steady-state equilibrium. From the linearized dynamic system, we can easily confirm that the determinant of the Jacobian matrix evaluated at the steady-state is always negative. This means that the steady-state equilibrium is a saddle-point. Since the steady-state equilibrium is a saddle-point, there exists one stable, convergent saddle-path through point E_0 in Figure 1.

Since capital stock K is a predetermined variable and private consumption C is a jumpable varible, C responds immediately when an external shock generates. If the economy faces an external shock, then C jumps up to the new saddle-path, given the initial value of capital stock. Afterward, the economy approaches the new steady-state value K^* and C^* gradually over time.

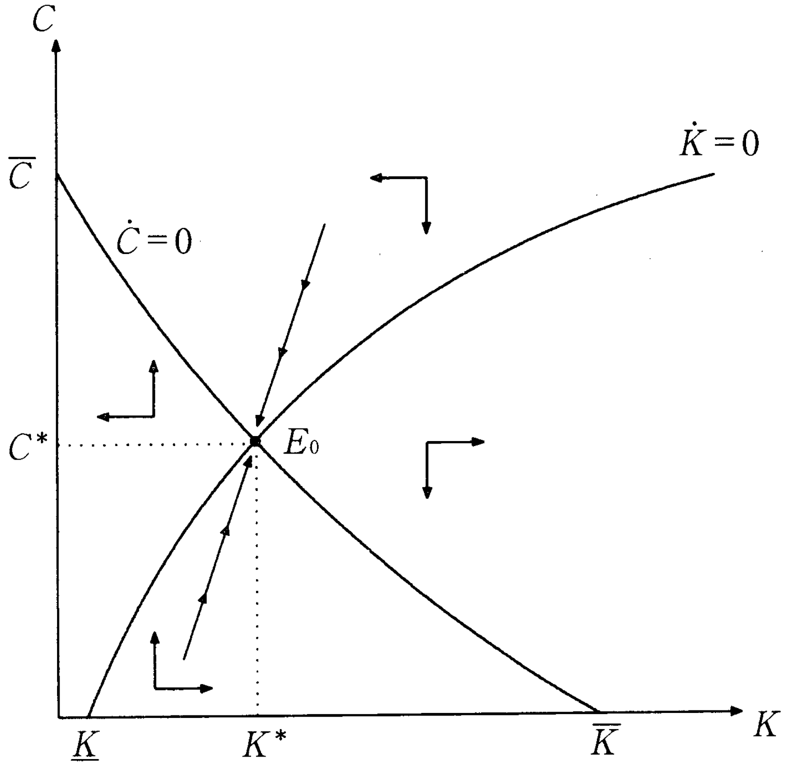

Figure 1.

3. A Permanent Increase in Government Spending

In this section we examine the effects of a permanent increase in government spending on private consumption, capital stock, and real output. Through all cases, we assume an unanticipated change in government spending.

Case 1: $0<\eta<1$

First, we examine the case for $0<\eta<1$. Suppose that a government spending G increases permanently at $t=t_0$. Then the $\dot{K}=0$ locus shifts downward in Figure 2. On the other hand, the $\dot{C}=0$ locus similarly shifts downward, but the magnitude of a shift is limited by the condition of $0<\eta<1$. The shift of the $\dot{C}=0$ locus is less than that of the $\dot{K}=0$ locus. Therefore, the long-run equilibrium moves from point E_0 to point E_1 in Figure 2.

Comparing new steady-state levels of K and C with old ones, K^* increases while C^* decreases. And as K^* increases, the steady-state level of output Y^* also does as a result.

Therefore, under the condition of $0<\eta<1$, we have no crowding-out effect on private investment. The increase of the steady-state level K by a permanent increase in G shows this.

An adjustment process from the old equilibrium point to new one is as follows. When G increases permanently at $t=t_0$, C jumps downward to the new saddle-point path, given the initial capital stock level. After reaching the new saddle-point path, both K and C increases gradually over time, along the new saddle-point path, toward new equilibrium point E_1. On the adjustment process, private consumption C which is a jumpable variable shows an overshooting phenomenon.

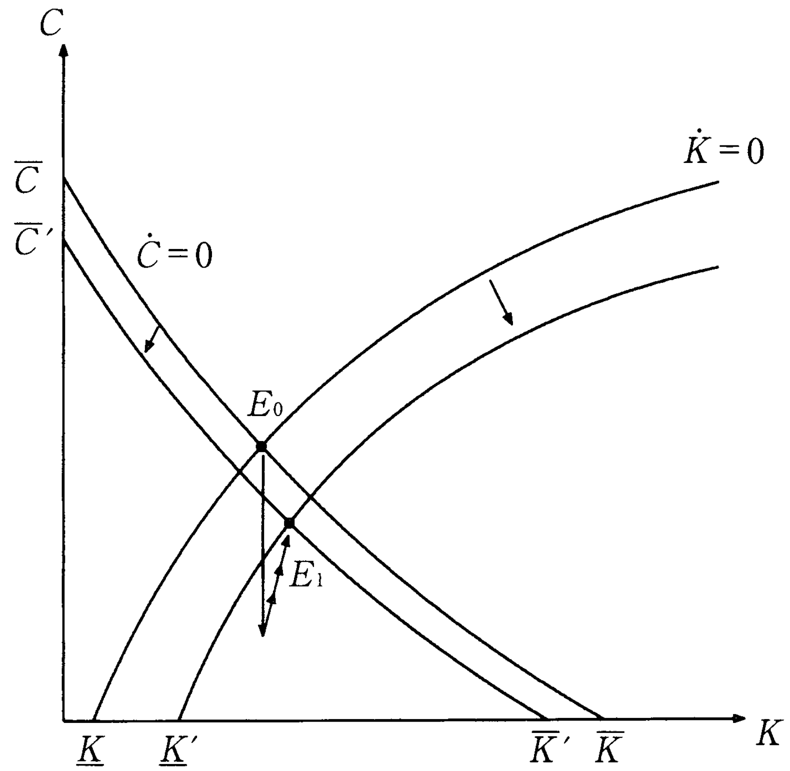

Figure 2.

Case 2: $\eta=1$

Next, we investigae the case for $\eta=1$. Consider a permanent increase in G at $t=t_0$. Under the assumption of $\eta=1$, both the $\dot{K}=0$ and $\dot{C}=0$ loci shift downward equivalently. Therefore, the long-run equilibrium moves from point E_0 to point E_1 in Figure 3.

Comparing two long-run equilibrium points before and after a fiscal change, the steady-state level of C decreases while that of K remains unchanged in the case of $\eta=1$. As K is still constant, the steady-state level of Y is also unchanged. Therefore, as

Y and K have no response to a permanent increase in G, the Keynesian-type, balanced budget multiplier proposition is not established. A permanent increase in G always decreases private consumption C at the steady-state. This implies that there exists a crowding-out phenomenon in the sense that an increase in public demand leads to a decrease in private demand, especially private consumption.

An adjustment process from point E_0 to point E_1 is as follows. When a government spending G permanently increases at $t=t_0$, a jumpable variable C immediately moves downward to the new long-run equilibrium, given the initial capital stock level. We can understand this adjustment mechanism as follows. Under the given assumption of effective consumption, a permanent increase in government spending G lowers the marginal utility of private consumption. This decrease in marginal utility, however, leads to a violation of the first-order conditions. So a representative agent decreases private consumption C, which is a perfect substitute in an Edgeworth sense, in order to recover the initial first-order conditions. In this way, a temporarily decresed marginal utility of private consumption C recovers the original level.

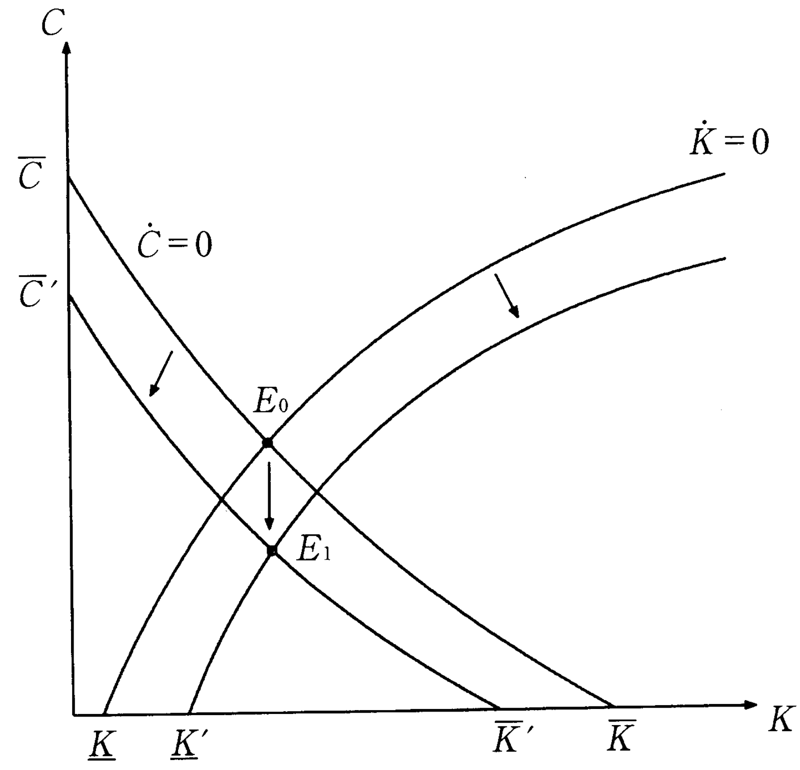

Figure 3.

Case 3: $\eta > 1$

Finally, we consider the case for $\eta > 1$. Supposed that at $t = t_0$ a government spending increses permanently. Similarly to the previous cases 1 and 2, the $\dot{K} = 0$ and $\dot{C} = 0$ loci shift downward. When $\eta > 1$, however, the $\dot{C} = 0$ locus shifts downward more than the $\dot{K} = 0$ locus. Therefore, the long-run equilibrium moves from point E_0 to point E_1 in Figure 4.

Comparing old and new long-run equilibriums, both K and C decrease at the steady-state. Also, in case of $\eta > 1$, as capital stock K decreases, the steady-state level of Y similarly decreases. Therefore, when $\eta > 1$, the crowding-out effect of private investment emerges as the steady-state K decreases in terms of a permanent increase in government spending. Moreover, similar to case 2 above, the crowding-out effect of private consumption generates.

Theoretically, the contractionary effects of a permanent increase in government spending may emerge as examined above. However, does such an effect emerge in an actual world ? Aschauer (1985) and Ihori (1987) showed that the estimated value of η is between 0 and 1, and that the possibility of $\eta \geq 1$ is not statistically supported. However, as explained in Ihori (1990), the parameter η is not naturally constant and may be variable depending on the magnitude of government spending G. Therefore, we should note the possibility of $\eta \geq 1$ in response to the level of government spending.

As in the previous cases, let us consider the transitional dynamics from the initial equilibrium point E_0 to the new point E_1. In Figure 4, private consumption C jumps downward immediately and reaches the new saddle-point path when a government spending G increases permanently at $t = t_0$. At that time, capital stock remains the same as the its initial level. Afterward, both K and C continue to decrease toward the new long-run equilibrium gradually over time. Also, as K^* decreases, the steady-state level of output Y^* does as well. In this case, the undershooting phenomenon of private consumption C appears on the adjustment process as shown in Figure 4.

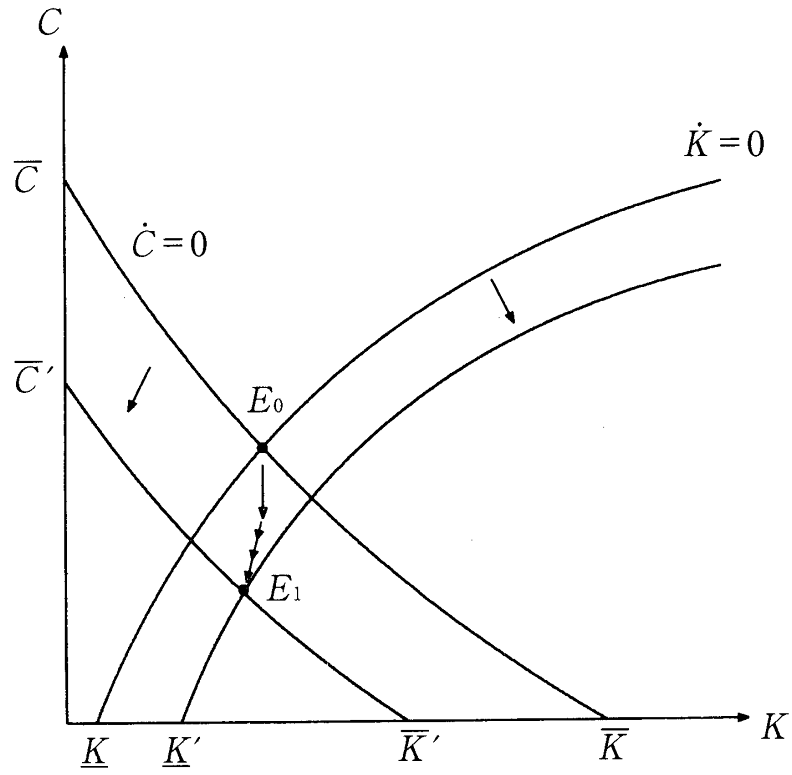

Figure 4.

4. The Optimal Size of Government Spending

In this section we briefly discuss the optimal size of government spending of the economy and its relation to the parameter for substituability between private consumption and government spending. Optimality condition is, in addition to equations (7), (8), and (9), the following famous Samuelson condition.

$$U_C(C, G, Y) = U_G(C, G, Y), \tag{14}$$

As described in Ihori (1990), the magnitude of η relative to unity implies that the relationship between the actual level and the Samuelson level of government spending. First, if $0<\eta<1$, then actual level G is a priori greater than the Samuelson level. Next, if $\eta=1$, then actual level G is a priori equal to the Samuelson level. Finally, if $\eta>1$, then actual level G is a priori less than the Samuelson level. Therefore, our results obtained in the previous section can be reworded as follows. The initial discrepancy between the actual and the Samuelson level of government spending plays a crucial role in the effects of a permanent increase in government spending.

5. Concluding Remarks

In this paper we have extended Djajić (1987)-Ihori (1990) model. Using this extended model, we have examined the effects of a permanent increase in government spending. Ihori (1990) showed that there is no effect of a permanent increase in government spending on capital accumulation. However, our extended model has offered different results. Table 1 summarizes our results. Comparing our results with Ihori (1990), optimal response of a representative agent on capital stock (and real output) depends on the parameter expressing substituability between private consumption and government spending in the effective consumption. For all cases on η, private consumption always decreases in response to a permanent increse in government spending. On the other hand, capital stock and real output may increase or decrease or unchange, depending on the magnitute of the parameter η.

Table 1. The Effects of a Permanent Increase in Government Spending

	This paper			Ihori (1990)	Kaneko (2001)
	$0<\eta<1$	$\eta=1$	$\eta>1$		
C^*	−	−	−	−	−
K^*	+	0	−	0	+
Y^*	+	0	−	0	+

Footnotes

1) For effective consumption, see, e.g., Aschauer (1985), Barro (1981), and Ihori (1990).

2) In Djajić (1987) and Ihori (1990), inelastic labor supply is assumed. On the other hand, in this paper, labor supply is supposed to be efficiently chosen with capital stock. In other words, we implicitly assume that the capital-labor ratio is always efficient. One interpretation for our assumption is as follows. Let us consider the following production function.

$$Y=F(Z), \text{ where } Z=\min(K, \phi L), \ \phi>0.$$

The production function $F(Z)$ satisfies that $F'(Z)>0$ and $F''(Z)<0$. Sub-production function Z is a fixed-coefficient type. On the efficient locus, we have: $K=\phi L$. The capital-labor ratio is $K/L=\phi$, which is always constant. Under the efficient production, we can rewrite the production as follows. $Y=F(K)$, where $F'(K)>0$ and $F''(K)<0$.

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