Free Entry into a Co-opetitive Mixed Oligopoly

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Abstract

This study investigates the social desirability of free entry into a co-opetitive mixed market where firms compete in a homogeneous product market while investing in property resources to boost demand. Our findings indicate that the magnitude relation between the business stealing effect and the common property effect depends on the degree of privatization and the value of fixed cost, and that the magnitude is a critical factor determining whether the number of entrants under free entry is socially insufficient or excessive. Firstly, we consider a scenario where all firms in the market have to pay a fixed (set-up) cost. When there is a highly nationalized semi-public firm in the market, the number of entrants is socially insufficient under free entry. Meanwhile, when there is a highly privatized semi-public firm, the number of entrants is socially excessive under free entry. Secondly, we show that free entry yields excess entry in cases of no fixed (set-up) cost environment. Therefore, the degree of privatization and the value of fixed (set-up) cost may serve as criterions for determining whether the government would be better off regulating or deregulating on private firms' entry into the market.

Keywords: Co-opetition, Mixed Oligopoly, Excess Entry

JEL Code: C72, L13, L33

1 Introduction

Since the 1980s, we have observed a worldwide wave of the privatization of state-owned public enterprises. There are still many public and semi-public enterprises, i.e., firms owned by both public and private sectors, in various countries. They compete to increase their market share with private firms in a wide range of industries from airlines, rail, telecommunications, natural gas, electricity, steel, and overnight-delivery industries, to services such as banking, home loans, health care, life insurance, hospitals, broadcasting, and education, while investing for demand-boosting. Both public and private firms often attempt to control market demand by investing in common property resources, and this expands the

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market size and increases consumers' willingness to pay. Such a situation of that competition and cooperation are simultaneously performed is called "co-opetition," which was defined by Brandenburger and Nalebuff (1996).[1]

High-quality or high-standard common property resources produce non-exclusive benefits for firms in the form of "an expansion of the market size and an increase in consumers' willingness to pay." That is, investments from individual firms in common property resources are deemed as public goods. A good example of the above is the case of the mobile phone companies. An investment of a firm facilitates the expansion of the access area of Wi-Fi, which is the benefit of all the mobile phone companies, and increases the accessibility for all the mobile phone users, resulting in raised consumers' willingness to pay and increased profits for the all mobile phone companies. Another example is generic advertising for commodities such as tea, oranges, and dairy products. The image enhancement of a product generates non-excludable benefits for all the producers of the same product and improves the reputation of the whole industry. Firms' investments in product safety is also considered as co-opetitive behavior when they share an industry's collective reputation for product safety. According to Ngo and Okura (2008), the three mobile phone operators in the French market (Orange, SFR, and Bouygues Telecom) launched their cooperative project. Orange is a semi-public firm of which French government holds a part of the company's stock, SFR, and Bouygues Telecom are private firms. While competing in quantity of user contacts, they collaboratively established Wireless Link Association to offer prompt, inexpensive, and top-quality homogeneous nation-wide Wi-Fi service.

"Deregulation of market entry" often takes place in conjunction with "privatization of public firms." Owing to recent deregulation and liberalization, entry restrictions in mixed oligopolies have significantly weakened. As a result, private enterprises have newly entered into many mixed oligopoly markets. However, the policies of deregulation and privatization do not always bring about positive impacts on social welfare. As can be seen from the example of the British rail truck company that was privatized

[1]By conceptualizing co-opetition as co-operative agreements among potential competitors, Ray and Tirole (2017) showed that price caps allow the firms to solve Cournot's multiple marginalization problem when goods were complements. Chen and Rey (2016) considered co-opetition as the diversity of purchasing patterns and showed that, with asymmetric competition between retailers, loss leading allowed to better screen consumers according to their shopping costs. The conception of co-opetition in this study is a situation of firms' investing in common property resources which serve as a public good under no cooperative agreement.

in the mid-1990s. As a result of privatization, Railtrack was compelled to cut operating costs as an overriding concern over safety in operations such as track and signaling. Railtrack caused a succession of fatal rail accidents at 2000, and Potter's Bar in 2002.[2] Subsequent official inquiries exposed serious underinvestment in safety resulting from entry deregulation and privatization. Analyzing the effect of free entry in a co-opetitive mixed oligopoly model bring a better understanding of the effects of entry deregulation (liberalization) and privatization on social welfare.

According to Hattori and Yoshikawa (2016), there are two effects, the business stealing effect and the common property effect, of free entry on social welfare. The business stealing effect of entry creates production inefficiency and generally leads to socially excessive entry in the industry (Mankiw and Whinston 1986). The common property effect is the entry effect affecting the amount of the total investment in common property resources. The total investment in common property resources may increase with increase in the number of entrant firms. If this is the case, the number of entrants may result in socially insufficient under free entry because private entrant firms take no account of the positive external effect of their investments in common property resources on other firms. On the other hand, an increase in the number of private firms may also exacerbate underprovisions of common property resources, which may cause a tragedy of the commons situation. As observed in cases of tourist destinations such as Acapulco in Mexico, the French Riviera and Mallorca and Torremolinos in Spain, an increase in the number of firms running local businesses for tourists may increase the incentives of free ride on investment in environment preservation of sightseeing spots such as wild life parks and historic sights (Puppim de Oliveira, 2003). This leads to decreased total investment, resulting in a disrepair and a decline in local tourism. In these cases, the common property effect causes socially excessive entry. This raises the important question of whether government regulation or deregulation needs to be applied on private firms' entry into the co-opetitive mixed industry.

Applying the demand boosting strategy to the standard mixed oligopoly model, we analyze a model of a co-opetitive mixed oligopoly. A semi-public firm is a partially privatized public firm, which aims to maximize the weighted sum of social welfare and its profit while the private firms aim only to maximize

[2]Rail accidents repeatedly occurred at Southall in 1997, Ladbroke Grove in 1999, Hatfield in 2000, and Potter's Bar in 2002. For the details of these rail crashes, see Wolmer (2001).

their profits. The firms compete on the quantity of their output, while investing in common property resources. From a social perspective, we examine whether the number of entrant firms with free entry is excessive or insufficient by the comparison between the number of firms at free entry equilibrium and the number desired by a social planner. We found that the magnitude between the business stealing effect and the common property effect is the determinant factor for the number of entrants under free entry to be socially insufficient or excessive. As explained by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), the business stealing effect creates a wedge between the evaluation of the desirability of private firm's entry which depends on the firm's profit, and the evaluation by a social planner who aims to maximize social welfare. Since a marginal entrant firm takes no account of the negative impact on other firms' outputs by its entry, the business stealing effect causes socially excessive entry.[3] The common property effect varies depending on the effect of entry on the total investment in common property resources. As the number of entrants increases, the incentive to "free ride" on other firms' investments in common property resources (public goods) is increased, resulting in decreased individual investment. The total investment in common property resources may result in either increased or decreased. Entry generates positive external effects on other firms when the total investment increases by an increased number of entrants. In this case, the common property effect causes insufficient entry because private firms leave the positive externality out of consideration when deciding whether to enter a market. The marginal entry creates negative external effects on other firms when market entry results in a decreased total investment in common property resources. In this case, a negative common property effect brings about socially excessive entry.

The purpose of this study is to investigate the relationship between privatization and the effects of free entry on social welfare. Firstly, we focused on the case where all firms in the market have to pay a fixed (set-up) cost. When the semi-public firm is highly nationalized, the number of entrants is

[3]There is a rich literature on excess entry theorem. See also Wang and Chen (2010) for homogeneous goods oligopoly and Fujiwara (2007) for another formulation of product differentiation in free-entry mixed markets. Mukherjee and Mukherjee (2008) and Mukherjee (2012) show theoretically that free entry can result in socially insufficient entry in the presence of technology licensing or market leaders. Furthermore, in either cases that entry is ex-ante uncertain (Creane 2007) or that consumers care about social status (Woo 2013), the excess entry result does not always hold. Incorporating strategic cost-reducing R&D activities into the Cournot model, Okuno-Fujiwara and Suzumura (1993) show that the existence of R&D investment strengthens the tendency of excess entry in a free-entry equilibrium. The critical difference between R&D investment in their study and investment in common property resources in our study is that the former generates private benefits for the investing firm, while the latter generates public benefits for all the firms in the market.

socially insufficient under free entry. This is because a marginal entry of a private firm increases the total investment, and the common property effect of entry dominates the business stealing effect. When the semi-public firm is highly privatized, free entry yields social excessive entry. A marginal entry of a private firm decreases the total investment, and the common property effect and the business stealing effect work negatively in the same direction. Secondly, we examined the case where all firms in the market are not required to pay fixed (set-up) cost. Although the number of entrant firms under free entry is infinity, the socially optimal number of firms is positive finite. This indicates that the number of entrants at free-entry equilibrium is socially excessive. Thus, we elucidated that the degree of privatization and the value of fixed cost often serve as the key criterions for determining whether the government would be better off regulating or deregulating on private firms' entry into the market. Our findings make an important contribution in terms of the understanding of the effect of private firms' entry on social welfare and suggest policy implications. In a scenario where there is a highly nationalized semi-public firm in the market, the government can improve welfare by encouraging private firms' market entry, e.g., by subsidizing entry. Meanwhile, when there is a highly privatized semi-public firm, the government should regulate entry of private firms or subsidize investment to prevent the depletion of common property resources. In the following section, we give a brief review of the relevant literature. Section 3 presents the model, and Section 4 shows the equilibrium outcome. Section 5 provides the concluding remarks.

2 literature Review

The literature on mixed markets involving private and public enterprises has become rich and diverse in recent years.[4] Many studies assume that public firms maximize social welfare, whereas private firms maximize their profits. The pioneering work is De Fraja and Delbono (1989), in which they demonstrated that welfare in a Cournot-type oligopoly might be higher when a public firm acted to maximize profits rather than welfare. Allowing for the possibility of partial privatization, Matsumura (1998) focused on partial privatization and the relationship between partial private ownership and managers' choices. Nishimori and Ogawa (2002) and Cato (2010) focused R&D in mixed oligopoly. By comparing a public monopoly with a mixed oligopoly with no entry, Nishimori and Ogawa (2002) investigated the rela-

[4]For an extensive literature survey, see Matsumura and Shimizu (2010).

tionship the effect of private firm entry and the cost-reducing incentives of a public firm. Cato (2010) examined the effect of privatization on the cost-reducing incentives of the private firm. These studies assume that the number of firms is exogenous given. This assumption was very natural as many mixed markets used to be highly regulated, including both explicit and implicit restrictions on entry.

As mentioned above, these entry restrictions have, however, been significantly weakened.[5] Ino and Matsumura (2010) and Matsumura and Kanda (2005) demonstrated that the social welfare is maximized by the presence of a welfare-maximizing public firm in the mixed oligopoly market with free entry. By extending the framework of Matsumura and Kanda (2005), Brandão and Castro (2007) showed that the presence of a public firm could be an alternative to direct regulation to prevent the excess entry problem. These studies examined the benefits of public ownership in free entry mixed oligopoly. They found that a welfare-maximizing public firm reduces the welfare losses caused by excessive entry problems, because the presence of the public firm exerts deterrent effect on entry of private firms. This study shows that the above remarkable result that welfare-maximizing behavior by the public firm is always optimal in mixed markets with free entry does not hold true if we consider a co-opetitive situation such as demand boosting strategy.

As mentioned above, the firms in a market often attempt to control market demand through co-opetitive promotion activities such as advertisements, commercial messages, direct mail and other types of demand-boosting strategies. A series of marketing analyses indicates that such demand-boosting and creation strategy is undoubtedly an important instrument for firms in an oligopoly market (Krishnamurthy, 2000). There are some other studies focusing on demand-boosting strategies, such as Krishnamurthy (2000), Dearden and Lilien (2001), and Hattori and Yoshikawa (2015). Krishnamurthy (2000) and Dearden and Lilien (2001) examined the firms that advertised to expand the market size and capture the largest share of demand. Hattori and Yoshikawa (2015) showed that the number of entrant firms under free entry into co-opetitive markets always becomes socially excessive in the case of pre-commitment co-opetition (the investment has commitment power).

[5]Using a monopolistic competition model with free entry, Anderson et al. (1997) found that privatization of the public firm improved welfare if and only if the public firm has a deficit. Adopting the differentiated product approach, Fujiwara (2007) exhibited a monotonic relationship between the degree of product differentiation and the optimal degree of privatization in the free-entry market. Wang and Chen (2010) extended the framework of Matsumura and Kanda (2005) and showed that partial privatization is the best policy regardless of whether the entry of foreign private firms is restricted or not.

However, very few studies referred to co-operative mixed market. Applying the concept of co-opetition to mixed duopoly market which consists of a semi-public firm and a private firm, Ngo and Okura (2008) investigated the impact of privatization on the degree of cooperation and competition in the market. They argued that the competitive behavior level (to increase the market share) of a semi-public firm was smaller than that of a private firm. The critical differences between their study and ours are competition structure and market structure. Whereas they suppose mixed duopoly market where a semi-public firm and a private firm compete for market share, we assume mixed oligopoly market where a semi-public firm and private firms play á la Cournot competition. Han and Ogawa (2012) made an attempt to incorporate the effects of firms' behavior on consumer demand in the mixed oligopoly model. However, none of their studies has analyzed the effects of free entry into the market from the perspective of social welfare. By extending the framework of Hattori and Yoshikawa (2015), we aim to analyze the effect of deregulation of a cooperative mixed oligopoly market, which is of great importance in formulating a policy in terms of social welfare.

3 Basic Model

We consider a canonical model of a co-operative mixed oligopoly market with $n+1$ firms: a partially privatized public firm ($i=0$) and private firms ($i=1,2,\cdots,n$). These $n+1$ firms operate in a market with homogeneous good and an inverse demand function given by

$$P(X,G) = a + G - bX, \tag{1}$$

where $P(X,G)$ is the market price (or inverse demand) of the product, a and b are positive constants, $X \equiv \sum_{i=0}^{n} x_i$ is the industry output, $x_i \geq 0$ is firm i's output, $G \equiv \sum_{i=0}^{n} g_i$ is the total amount of investment (or total amount of a public good), and $g_i \geq 0$ is the amount of firm i's investment (or individual contribution to a public good).

Each private firm i ($i = 1,2,\cdots,n$) are profit-maximizing private firm, and firm 0 is a partially privatized public firm that maximizes a certain objective function, which is presented later on. The firms compete in their output in a market while investing in common property resources that serve as a public

good for all the competing firms. The total cost function of each firms is identical and defined as

$$TC(x_i, g_i) \equiv C(x_i) + D(g_i) + F,$$

Where $C(x_i)$ is the production cost function, $D(g_i)$ is the investment cost function, and F is fixed cost. Furthermore, we employ the constant marginal production cost and the quadratic investment cost function,

$$C(x_i) = cx_i, \quad \text{and} \quad D(g_i) \equiv \frac{d}{2} g_i^2. \tag{2}$$

where c and d are positive constants. The profit of firm i $(i = 0, 1, 2, \cdots, n)$ is defined by

$$\pi_i = P(X, G)x_i - C(x_i) - D(g_i) - F$$

Social welfare is defined as

$$\begin{aligned} W &\equiv \int_0^X P(s, G)ds - P(X, G) \cdot X + \sum_{i=0}^{n} \pi_i \\ &= \int_0^X P(s, G)ds - \sum_{i=0}^{n} C(x_i) - \sum_{i=0}^{n} D(g_i) - (n+1)F, \end{aligned} \tag{3}$$

Following Matsumura (1998), The manager of firm 0 will maximize the weighted average of social welfare and profit, defined by

$$U_0 \equiv \alpha W + (1 - \alpha)\pi_0.$$

We assume that the government owns a share of $\alpha \in [0, 1]$ of firm 0, that is, the government owns a share of α of firm 0; $\alpha = 1$ means that firm 0 is a complete public firm that maximizes social welfare; and $\alpha = 0$ means that firm 0 is a complete private firm that maximizes its profit. Thus, α can be used to measure the degree of privatization. The firm becomes more profit oriented as the degree of partial privatization increases.

The firms play the following three-stage game. In the first stage, each firm chooses whether to enter the market. In the second stage, they independently and simultaneously choose the levels of their investment which increases market size. That is, a private firm i maximizes π_i with respect to g_i, and a firm 0 maximizes U_0 with respect to g_0. In the third stage, in which G is observed, each firm simultaneously and independently chooses the levels of their output to maximize their payoff.

4 Equilibrium

In this section, we discuss the equilibrium of the model formulated above. We solve this game by backward induction, and the equilibrium concept is the subgame perfect Nash equilibrium.

In the third stage, a private firm i maximizes π_i with respect to x_i, and a firm 0 maximizes U_0 with respect to x_0. Each first-order conditions for products of firm 0 and private firm i ($i = 1, 2, \cdots, n$) are given by

$$P - C + (1-\alpha)P_X \cdot x_0 = 0, \tag{4}$$

$$P - C + P_X \cdot x_i = 0, \tag{5}$$

where a subscript denotes a partial derivative. Using the inverse demand function (1), Nash equilibrium of the third stage is characterized as

$$\tilde{x}_0 = \frac{a-c+G}{b\varphi}, \quad \tilde{x_i} = \tilde{x_j} = \tilde{x} = (1-\alpha)\tilde{x}_0, \quad \tilde{X} = \frac{\{(1-\alpha)n+1\}(a-c+G)}{b\varphi}. \tag{6}$$

From the stability of Nash equilibrium in the third stage, we have $\varphi \equiv n(1-\alpha)+(2-\alpha) > 0$. Then, We have

$$\begin{aligned}
\frac{\partial \tilde{x}_0}{\partial G} &= \frac{1}{b\varphi} > 0, \quad \frac{\partial \tilde{x}}{\partial G} = (1-\alpha)\frac{\partial \tilde{x}_0}{\partial G} > 0, \quad \frac{\partial \tilde{X}}{\partial G} = \frac{(1-\alpha)n+1}{b\varphi} > 0, \\
\frac{\partial \tilde{x}_0}{\partial n} &= -\frac{(a-c+G)(1-\alpha)}{b\varphi^2} < 0, \quad \frac{\partial \tilde{x}}{\partial n} = (1-\alpha)\frac{\partial \tilde{x}_0}{\partial n} < 0, \frac{\partial \tilde{X}}{\partial n} = \frac{(a-c+G)(1-\alpha)^2}{b\varphi^2} > 0,
\end{aligned}$$

which indicates that individual output decreases, whereas total output increase with the number of private firms.

In the second stage, the firms independently and simultaneously choose the level of their investment. The first-order conditions each of firm 0 and private firm i ($i = 1, 2, \cdots, n$) in this stage are given by

$$\underbrace{n(1-\alpha)^2 P_X \tilde{x}_0 \frac{\partial \tilde{x}}{\partial G}}_{\text{pre-commitment effect of } g_0\ (-)} + P_G(\alpha\tilde{X} + (1-\alpha)\tilde{x}_0) = D_0', \tag{7}$$

$$\underbrace{P_X \tilde{x}\{(n-1)\frac{\partial \tilde{x}}{\partial G} + \frac{\partial \tilde{x}_0}{\partial G}\}}_{\text{pre-commitment effect of } g_i\ (-)} + P_G\tilde{x} = D_i'. \tag{8}$$

Taking into account the effect that their investment will also increase their competitors' output, each firm chooses their investment levels in common property resources. This "pre-commitment" effect of investment reduces the incentive to invest in common property resources. As the degree of nationalization of the semi-public firm increases, the effect of semi-public firm become weakened.

From (7) and (8), we obtain the following reaction functions:

$$RF_0 : g_0 = \frac{(2-\alpha+\alpha n(1-\alpha)(\varphi+2-\alpha))(a-c+\sum_{i=1}^{n} g_i)}{bd\varphi^2-\alpha n(1-\alpha)(\varphi+2-\alpha)-(2-\alpha)}, \tag{9}$$

$$RF_i : g_i = \frac{2(1-\alpha)^2(a-c+g_0+\sum_{i\neq j} g_i)}{bd\varphi^2-2(1-\alpha)^2}. \tag{10}$$

(9) and (10) indicate that investment choices are strategic complements.[6] An increase in the investment of other firms raises the marginal benefit for the investment of firm i. From (9) and (10), we derive the equilibrium output and investment in subgame perfect Nash equilibrium in the second stage as follows:

$$\hat{x}_0 = \frac{d(a-c)\varphi}{\theta}, \quad \hat{x} = (1-\alpha)\hat{x}_0 \tag{11}$$

$$\hat{g}_0 = \frac{(a-c)[2-\alpha+\alpha n(1-\alpha)(\varphi+2-\alpha)]}{\theta} \tag{12}$$

$$\hat{g} = \hat{g}_i = \hat{g}_j = \frac{2(a-c)(1-\alpha)^2}{\theta}, \tag{13}$$

where we assume that $\theta \equiv bd\varphi^2-\alpha n(1-\alpha)\{2+(1-\alpha)(2+n)\}-(2-\alpha)>0$.

[6] From (7) and (8), we have these second-order conditions: $bd > \frac{2-\alpha+\alpha n(1-\alpha)(\varphi+2-\alpha)}{\varphi^2} > 1$ and $bd > (\frac{1-\alpha}{\varphi})^2 > 1$

Then, we find that

$$\frac{\partial \hat{x}_0}{\partial n} = \frac{\partial \tilde{x}_0}{\partial n} + \frac{\partial \tilde{x}_0}{\partial G}\frac{\partial \hat{G}}{\partial n} < 0 \tag{14}$$

$$\frac{\partial \hat{x}}{\partial n} = (1-\alpha)\frac{\partial \hat{x}_0}{\partial n} < 0 \tag{15}$$

$$\begin{aligned}\frac{\partial \hat{X}}{\partial n} &= \frac{\partial \tilde{X}}{\partial n} + \frac{\partial \tilde{X}}{\partial G}\frac{\partial \hat{G}}{\partial n} \gtreqless 0 \\ &\Leftrightarrow \quad bd \gtreqless \frac{(2-\alpha)(1-2\alpha) + n\{2(2-\alpha) + n(1-\alpha)(2+\alpha)\}}{\varphi^2},\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial \hat{g}_0}{\partial n} &= -\frac{2(1-\alpha)^2(A-c)\{bd(2-\alpha)(1-\alpha)\varphi + \alpha n^2(1-\alpha)^2 - (2-\alpha)\}}{\theta^2} \gtreqless 0, \\ &\Leftrightarrow \quad bd \lesseqgtr \frac{2-\alpha-\alpha n^2(1-\alpha)^2}{\varphi(2-\alpha)(1-\alpha)},\end{aligned} \tag{17}$$

$$\frac{\partial \hat{g}}{\partial n} = -\frac{4(a-c)(1-\alpha)^3\{bd\varphi - 1 - \alpha(1+n)(1-\alpha)\}}{\theta^2} < 0, \tag{18}$$

$$\begin{aligned}\frac{\partial \hat{G}}{\partial n} &= -\frac{2bd\varphi(a-c)(1-\alpha)^2\{n(1-\alpha) - \alpha(2-\alpha)\}}{\theta^2} \gtreqless 0, \\ &\Leftrightarrow \quad n \lesseqgtr \frac{\alpha(2-\alpha)}{1-\alpha},\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial \hat{G}_{-0}}{\partial n} &= n\frac{\partial \hat{g}}{\partial n} + \hat{g} \gtreqless 0 \\ &\Leftrightarrow \quad bd \gtreqless \frac{(2-\alpha) - \alpha n^2(1-\alpha)^2}{\varphi\left(2-\alpha-n(1-\alpha)\right)},\end{aligned} \tag{20}$$

$$\begin{aligned}\frac{\partial \hat{G}_{-1}}{\partial n} &= \frac{\partial \hat{g}_0}{\partial n} + (n-1)\frac{\partial \hat{g}}{\partial n} + \hat{g} \gtreqless 0 \\ &\Leftrightarrow \quad bd \gtreqless \frac{2(1-\alpha)\left(1+\alpha-\alpha^2+\alpha(1-\alpha)n\right)}{\varphi\left(2-\alpha^2-n(1-\alpha)\right)}\end{aligned} \tag{21}$$

where $\hat{G}_{-0} \equiv n\hat{g}$ and $\hat{G}_{-1} \equiv \hat{g}_0 + (n-1)\hat{g}$. $\partial \hat{g}/\partial n < 0$ holds for any n, indicating that an increase in the number of private firms actually decreases individual investment of private firms. However, whether each signs of $\partial \hat{g}_0/\partial n$ and $\partial \hat{G}/\partial n$ are positive or negative depends on the value of b, d, α, and N. When α is small, firm 0 prioritize their own profit to social welfare in decisions. In this case, an increased number of new entrant private firms causes reduced investment amount of the semi-public firm, resulting in reduced total amount of investment and reduced total output.

Furthermore, we have

$$\lim_{n\to\infty} \hat{x}_0 = \lim_{n\to\infty} \hat{x} = \lim_{n\to\infty} \hat{g} = 0, \quad \lim_{n\to\infty} \hat{g}_0 = \lim_{n\to\infty} \hat{G} = \lim_{n\to\infty} \hat{X} = \frac{(a-c)d}{bd-\alpha},$$

which implies that as the number of entrants approaches infinity, individual output and private firm's investment close to zero, while the total output and investment come close to positive and finite value.

In the first stage, private firms will continuously entry into the market until profit of entrant firms is driven to zero (we ignore the integer constraint for the number of entrants). Therefore, the number of entrants under free entry is $\hat{n}_f$ such that

$$\begin{aligned}\hat{\pi}(\hat{n}_f) &= P(\hat{X}(\hat{n}_f), \hat{G}(\hat{n}_f)) \cdot \hat{x}(\hat{n}_f) - C(\hat{x}(\hat{n}_f)) - D(\hat{g}(\hat{n}_f)) - F = 0, \\ &\Leftrightarrow \quad \frac{d(a-c)^2(1-\alpha)^2\{bd\varphi^2 - 2(1-\alpha)^2\}}{\theta^2} - F = 0.\end{aligned}$$

Because $\hat{\pi}(n)$ is strictly decreasing in n and $\lim_{n\to\infty} \hat{\pi} = -F$, we confirm that $\lim_{F\to 0} \hat{n}_f = \infty$. Therefore, the number of entrants under free entry goes to infinity when there are no fixed costs.

Then, we consider the second-best problem for a social planner who can control the number of firms entering the market. Let $\hat{W}(n)$ denote the total surplus as follows:

$$\hat{W} = \int_0^{\hat{X}} P(s, \hat{G})ds - C(\hat{x}_0) - nC(\hat{x}) - D(\hat{g}_0) - nD(\hat{g}) - (n+1)F.$$

Then, we have

$$\begin{aligned}\hat{W}' &= \hat{\pi} - P_X \hat{x}\left[\frac{\partial \hat{x}_0}{\partial n} + n\frac{\partial \hat{x}}{\partial n} + n\left[(1-\alpha)\frac{\partial \tilde{x}_0}{\partial G}\frac{\partial \hat{g}_0}{\partial n} + \frac{\partial \hat{g}}{\partial n}\left((n-1)\frac{\partial \tilde{x}}{\partial G} + \frac{\partial \tilde{x}_0}{\partial G}\right)\right]\right] \\ &\quad + P_G\left[\hat{q}_0\frac{\partial \hat{G}_{-0}}{\partial n} + n\hat{q}\left(\frac{\partial \hat{G}_{-1}}{\partial n} - \alpha\frac{\partial \hat{g}_0}{\partial n}\right)\right],\end{aligned}$$

where we assume the second-order condition $\hat{W}'' < 0$ to be satisfied. The social planner chooses $n = \hat{n}_{sb}$, which maximizes $\hat{W}(n)$, implying

$$\hat{W}'(n)|_{n=\hat{n}_{sb}} = 0, \text{if } \hat{n}_{sb} > 0.$$

Considering $\hat{\pi}(\hat{n}_f) = 0$, we have

$$\begin{aligned}\hat{W}'(\hat{n}_f) &= \overbrace{-P_X\hat{x}\left[\overbrace{\frac{\partial \hat{x}_0}{\partial n} + n\frac{\partial \hat{x}}{\partial n}}^{\text{direct effect of entry } (-)} + \overbrace{n\left[(1-\alpha)\frac{\partial \tilde{x}}{\partial G}\frac{\partial \hat{g}_0}{\partial n} + \frac{\partial \hat{g}}{\partial n}\left((n-1)\frac{\partial \tilde{x}}{\partial G} + \frac{\partial \tilde{x}_0}{\partial G}\right)\right]}^{\text{indirect effect of entry}}\right]}^{\text{business stealing effect of entry } (-)} \\ &\quad \underbrace{+P_G\left[\hat{q}_0\frac{\partial \hat{G}_{-0}}{\partial n} + n\hat{q}\left(\frac{\partial \hat{G}_{-1}}{\partial n} - \alpha\frac{\partial \hat{g}_0}{\partial n}\right)\right]}_{\text{common property effect of entry}} \gtreqless 0. \qquad (22)\end{aligned}$$

The first term of Eq.(22) is the business stealing effect. Private firms take into account neither the negative direct impact of the entries on their competitors nor the indirect impact through the changes

in their competitors' investment amount. The sign of the business stealing effect is always negative, because the direct effect dominates the indirect effect even when the indirect effect is positive value. For instance, when α is large, the indirect effect become positive value. The second term of (22) is the common property effect of entry. Private firms take no account of the positive external effect of their investments in common property resources on other firms. Whether the sign of common property effect is positive or negative depends on the value of b, d, α, and n. When α is small, the sign of this effect is negative because $\partial \hat{G}_{-0}/\partial n$, $\partial \hat{G}_{-1}/\partial n$, and $\partial \hat{g}_0/\partial n$ are negative. Therefore, when α is small, the sign of both effects (the common property effect and the business stealing effect) are negative, and the number of entrants under free entry is socially excessive. When α is large, the sign of common property effect is positive. If common property effect dominates the business stealing effect, the sign of $\hat{W}'(\hat{n}_f)$ is positive, and the number of private firms under free entry is socially insufficient. Meanwhile, when the business stealing effect dominates the common property effect, the sign of $\hat{W}'(\hat{n}_f)$ is negative, and the number of entrants under free entry is socially insufficient. Furthermore, we have n^* that satisfies the following condition,

$$\begin{aligned}\hat{W}'(n)\mid_{F=0}= \quad & \frac{(1-\alpha)^2 d(a-c)^2}{\theta^3}\left[\begin{array}{l}(1-\alpha)b^2d^2\varphi^3 \\ -2\left(\alpha^3+\alpha+2\left(\alpha^5-5\alpha^4+10\alpha^3-9\alpha^2+2\alpha+1\right)n+2\right)\end{array}\right] \\ & +\frac{bd\varphi(1-\alpha)^2 d(a-c)^2}{\theta^3}\left[\begin{array}{l}2+\alpha(9-\alpha(7-4\alpha))+2\alpha^2(1-\alpha)^4n^4 \\ +2n(1-\alpha)(\alpha(3-\alpha)(4-\alpha(5-2\alpha)+4)-1)\end{array}\right] \\ & +\frac{n^2bd\varphi(1-\alpha)^4 d(a-c)^2}{\theta^3}\left[\begin{array}{l}-4\alpha^3+5\alpha^2+5\alpha-8 \\ +2n\left(2\alpha^4-6\alpha^3+4\alpha^2+\alpha-1\right)\end{array}\right]=0,\end{aligned}$$

and $\hat{W}'' < 0$. Therefore, we find that $\lim_{F\to 0}\hat{n}_{sb} = n^*$. In other words, the second-best number of firms is positive and finite even when there are no fixed (set-up) costs. The number of free entry is socially excessive when there are no fixed (set-up) costs. This is because when $F = 0$, the optimal number of entrants becomes positive finite, while the number of entrant firms becomes infinity under free entry.

We now show an example of mixed oligopoly using appropriate parameter values to highlight the impact on welfare. Setting $a = 10, b = 2, c = 1, d = 1$,and $F = 3$, we compare $\hat{n}_f$ and $\hat{n}_{sb}$. Table 1 displays numerical results, showing that the number of entrants ($\hat{n}_f$) is (but is not always) larger under free entry than that under the second-best problem ($\hat{n}_{sb}$). There are some cases that a marginal entry increases the total investment level, and consequently, the sign of the common property effect of entry

Table 1: Numerical Example (the case with fixed cost)

α	$\hat{n}_f$	$\hat{n}_{sb}$	the business stealing effect	the common property effect
0	2.35	0.69	-3.46	-0.39
0.1	2.45	0.8	-3.4	-0.29
0.2	2.54	0.91	-3.34	-0.18
0.3	2.62	1.02	-3.27	-0.04
0.4	2.66	1.13	-3.18	0.13
0.5	2.62	1.21	-3.07	0.38
0.6	2.43	1.24	-2.91	0.74
0.7	1.87	1.11	-2.65	1.33
0.8	0.28	0.48	-2.1	2.43
0.9	0	∞	-0.87	1.02

Note: This data is rounded off to the second decimal place.

is positive. When the semi-public firm is highly nationalized as cases such as $\alpha = 0.8$ and $\alpha = 0.9$, the common property effect of entry dominates the business stealing effect and the number of entry firms is socially insufficient under free entry. Meanwhile, when the firm is privatized to some degree as cases such as $\alpha = 0.4$ to 0.7, the business stealing effect dominates the common property effect and the number of entry firms will be socially excessive under free entry. On the other hand, when the firm is highly privatized as cases such as $\alpha = 0$ to 0.3, a new entry of a private firm decreases the total investment level. Therefore, the common property effect and the business stealing effect work in the same direction and the number of entrants under free entry will be socially excessive.

5 Conclusion

This paper has presented a new perspective on social welfare of free entry into a co-opetitive mixed oligopoly market. Applying the demand boosting strategy to the standard mixed oligopoly model, we analyzed a model of a co-opetitive mixed oligopoly. We found that the magnitude between the business stealing effect and the common property effect is the determinant factor for the number of entrants under free entry to be socially insufficient or excessive. In our model, the firms play the following three-stage game: in the first stage, each firm chooses whether to enter the market: in the second stage, they independently and simultaneously choose the levels of their investment which increases market size: in the third stage, in which total investment is observed, each firm compete on output quantity as in the

standard Cournot model. We solve this game by backward induction and the equilibrium concept is the subgame perfect Nash equilibrium.

The major contribution of this study is finding that the magnitude relation between the business stealing effect and the common property effect depends on the degree of privatization and the value of fixed cost. By analyzing the both cases with fixed cost and without it, we clarified that the degree of privatization and the value of fixed cost are able to use as key criterions for determining whether the government would be better off regulating or deregulating on private firms' entry into the market. In the cases with fixed cost, if the semi-public firm is highly nationalized, a marginal entry of a private firm increases the total investment for demand boosting. The number of entrants in free entry equilibrium is socially insufficient because the common property effect of entry dominates the business stealing effect. Meanwhile, if the semi- public firm is highly privatized, a marginal entry of a private firm decreases the level of total investment. Thus, the number of entrants under free entry will be socially excessive because both the common property effect and the business stealing effect exert negative effects on social welfare. Further, we examined the effect of free entry on social welfare in the cases without fixed (set-up) cost. Although the number of entrant firms under free entry is infinity, the socially optimal number of firms is positive finite. This indicates that the number of entrants at free entry equilibrium is socially excessive under the circumstances that all the firms in the market are not required to pay fixed (set-up) cost.

Our analysis can be extended further in several ways. One conceivable extension would be to consider product differentiation and the positive externality of investment on other products' market size. Employing the extension, we can investigate price competition instead of quantity competition. Moreover, we can examine the welfare evaluation of entry in the model of price competition with common property resources by adopting, for example, a circular-market model of Salop (1979). Likewise, the government subsidy on co-opetitive investment serves to remedy the problems of welfare deterioration that are caused by socially excessive or insufficient entry would be improved. In addition to the above, in some situations, a leader firm strategically chooses their preliminary investment before an entry stage, as seen in Etro (2006) and Czarnitzki et al. (2014). In this case, the welfare evaluation of free entry may be affected by the strategic investment of the leader firm and their objectives. These issues await further research in the future.

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