Robust optimal design of energy supply systems under uncertain energy demands based on a mixed-integer linear model

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Abstract

In designing energy supply systems, designers should consider the robustness in performance criteria against the uncertainty in energy demands. In this paper, a robust optimal design method of energy supply systems under uncertain energy demands is proposed using a mixed-integer linear model so that it can consider discrete characteristics for selection and on/off status of operation and piecewise linear approximations for nonlinear performance characteristics of constituent equipment. First, a robust optimal design problem is formulated as a three-level min-max-min optimization one by expressing uncertain energy demands by intervals based on the interval programming, evaluating the robustness in a performance criterion based on the minimax regret criterion, and considering hierarchical relationships among design variables, uncertain energy demands, and operation variables. Then, a special solution method of the problem is proposed especially in consideration of the existence of

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integer operation variables. In a case study, the proposed method is applied to the robust optimal design of a cogeneration system with a simple configuration. Through the study, the validity and effectiveness of the method is ascertained, and some features of the obtained solutions are clarified.

Keywords: Energy supply, Uncertainty, Robust design, Optimization, Multilevel programming, Mixed-integer linear programming

1. Introduction

In energy supply systems, the values of performance criteria such as annual total cost, primary energy consumption, and CO_2 emission depend not only on design specifications but also on energy demands and corresponding operational strategies. Thus, it is important to determine design specifications optimally in consideration of operational strategies corresponding to seasonal and hourly variations in energy demands. However, many conditions under which energy demands are estimated have some uncertainty at the design stage, and thus the energy demands which occur at the operation stage may differ from those estimated at the design stage. Even if the optimal design is conducted in consideration of the estimated energy demands, the values of performance criteria expected at the design stage may not be attained at the operation stage. Therefore, designers should consider that energy demands have some uncertainty, evaluate the robustness in performance criteria against the uncertainty, and design the systems rationally in consideration of the robustness.

One of the rational approaches to the optimal design is to use mathematical programming methods, and they have been applied increasingly with the development of computation hardware and software. Especially, the mixed-integer linear programming (MILP) method has been utilized widely. This is because it can consider discrete characteristics for selection and on/off status of operation of equipment, and can also treat nonlinear performance characteristics of equipment by piecewise linear approximations. In addition, although the MILP method takes longer computation times than the linear programming method, it can obtain global optimal solutions more easily than the nonlinear programming method. In recent years, since commercial MILP solvers have become more efficient, they have been applied to the optimal design of small-scale commercial and residential energy supply systems in consideration of multi-period operation. However, most of the models used for the optimal design may not be sufficient. For example, Buoro et al., and Wakui and Yokoyama determined only the types of equipment with fixed capacities [1, 2]. Lozano et al. and Carvalho et al. determined the types and numbers of equipment with fixed capacities [3–5]. Buoro et al. and Voll et al. determined the types and capacities of equipment, but treated the capacities as continuous variables [6–8]. Piacentino at al. and Zhou et al. used similar models, but did not take account of the dependence of performance characteristics of equipment on their capacities or part load levels [9, 10]. On the other hand, Yokoyama and Ito, and Yang et al. proposed optimal design methods in consideration of discreteness of equipment capacities to resolve the aforementioned insufficiency of equipment models [11–13]. However, these studies were conducted under certain energy demands.

A simple way to evaluate the robustness in performance criteria under uncertain energy demands is to conduct a sensitivity analysis. Some studies are concerned with sensitivity analyses of performance criteria with respect to changes in energy demands. Ashouri et al. conducted a sensitivity analysis of the optimal design of a building energy system with respect to the changes in conditions related with energy demands and others, and they used deterministic and stochastic optimization approaches [14]. Wang et al. conducted a sensitivity analysis of the optimal design of a building energy system with respect to the changes in energy demands and others, and they used the genetic algorithm to solve the optimization problem [15]. Carvalho et al. conducted a sensitivity analysis to investigate the resilience of the optimal design of an energy system for a hospital with respect to the changes in energy demands and others, and they used an MILP approach for optimization [16]. To conduct such a sensitivity analysis, scenarios for the change in energy demands are inevitable. However, energy demands change with season and time, and there can be innumerable scenarios even if their intervals are given. Thus, it is necessary to limit the number of scenarios, and limited scenarios are not necessarily sufficient for the sensitivity analysis.

On the other hand, many papers on optimization of energy systems planning under uncertainty have been published. Verderame et al. reviewed many papers on planning and scheduling under uncertainty in multiple sectors, and reviewed some papers on energy planning [17]. Zeng et al. also reviewed many papers on optimization of energy systems planning under uncertainty [18]. In these review papers, the approaches adopted for optimization of energy systems planning were categorized into three ones: stochastic, fuzzy, and interval programming. However, it is difficult for designers to specify stochastic distribution and fuzzy membership functions for uncertain parameters in the first and second approaches. From the viewpoint of practical applications, it is much more meaningful for designers to specify fluctuation intervals for uncertain parameters in the third approach. Thus, this paper focuses on the third approach. Lin and Huang introduced an interval-parameter linear programming approach to energy systems planning [19]. Zhu et al. developed an interval-parameter full-infinite linear programming approach to energy systems planning under multiple uncertainties with crisp and functional intervals [20]. They also proposed an interval-parameter full-infinite mixed-integer programming approach to energy systems planning under uncertainties with functional intervals [21]. Dong et al. developed an interval-parameter minimax regret programming method for power management systems planning under uncertainty [22]. However, these methods do not consider the difference between design and operation variables whose values are determined at the design and operation stages, respectively. In addition, most of these methods cannot produce a unique optimal solution but an interval one, which cannot support the decision-making for design. Majewski et al. investigated the trade-off relationship in the objective function between the nominal and worst cases [23]. However, this method produces Pareto optimal solutions depending on the importance given to the nominal and worst cases, which is also unsuitable for design. Yokoyama and Ito proposed a robust optimal design method of energy supply systems in consideration of the economic robustness against the uncertainty in energy demands based on the minimax regret criterion [24]. This method is very natural because the design is determined so that the value of the objective function for the robust optimal design becomes as close as possible to that for the optimal design. In addition, this method considers that values of design and operation variables are determined at the design and operation stages, respectively, and produces a unique optimal solution. Yokoyama et al. revised this robust optimal design method so that it can be applied to energy supply systems with more complex configurations and larger numbers of periods set to consider variations in energy demands [25]. Assavapokee et al. presented a general framework for the robust optimal design based on the minimax regret criterion [26]. Although innumerable scenarios within intervals are considered in these methods, the used models for constituent equipment are not mixed-integer linear but only linear.

Therefore, it is strongly required to develop a robust optimal design method of energy supply systems based on a mixed-integer linear model, so that it can treat not only continuous but also discrete variables. At the first step for this challenge, the authors have proposed a method of comparing performances of two energy supply systems under uncertain energy demands based on a mixed-integer linear model for constituent equipment [27]. In this paper, a robust optimal design method of energy supply systems under uncertain energy demands is proposed using a mixed-integer linear model. A robust optimal design problem is formulated as a three-level min-max-min optimization one by expressing uncertain energy demands by intervals based on the interval programming, evaluating the robustness in a performance criterion based on the minimax regret criterion, and considering hierarchical relationships among design variables, uncertain energy demands, and operation variables. Although this formulation of the robust optimal design problem based on the mixed-integer linear model is similar to that based on the linear model, the solution method have to be changed substantially because of the existence of integer operation variables. In this paper, a special solution method is proposed especially in consideration of the existence of integer operation variables. In a case study, the proposed method is applied to the robust optimal design of a cogeneration system with a simple configuration, and the validity and effectiveness of the method is investigated.

2. Formulation of robust optimal design problem

2.1. Basic concept

In designing an energy supply system under uncertain energy demands, flexibility and robustness have to be taken into account [28]. The former means the feasibility in energy supply for all the possible values of uncertain energy demands, and is related with constraints. The latter means the sensitivity of performance criteria for all the possible values of uncertain energy demands, and is related with objective functions. In this paper, a robust optimal design method is proposed by which the robustness is improved while the flexibility is secured for all the possible values of uncertain energy demands. As a criterion for the robustness, the minimax regret criterion is adopted here [29]. Figure 1 shows a basic concept of the robust optimal design based on the minimax regret criterion. The regret is defined as the difference in an objective function between non-optimal and optimal designs for some values of uncertain energy demands. The minimax regret criterion means that the values of design variables are determined to minimize the maximum regret for all the possible values of uncertain energy demands. Therefore, if this criterion is adopted, the difference in the objective function between the robust optimal and optimal designs can be small for all the possible values of uncertain energy demands.

2.2. Formulation

Following the aforementioned basic concept, a robust optimal design problem for an energy supply system is described as follows: the values of integer and continuous design variables $\boldsymbol{\eta}$ as well as the values of integer operation variables $\boldsymbol{\delta}$ and continuous operation variables $\boldsymbol{z}$ are determined to minimize the maximum regret in the annual total cost f and to satisfy all the constraints for all the possible values of uncertain energy demands $\boldsymbol{y}$. Types, capacities, and numbers of equipment are expressed by integer design variables, while maximum demands of utilities are expressed by continuous design variables. Numbers of equipment at the on status of operation are expressed by integer operation variables, while energy flow rates of equipment are expressed by continuous operation variables. Here, it is assumed that all the objective function and constraints are expressed by linear equations with respect to $\boldsymbol{\eta}$, $\boldsymbol{y}$, $\boldsymbol{\delta}$, and $\boldsymbol{z}$. In addition, it should be noted that although the values of design variables $\boldsymbol{\eta}$ must be determined at the design stage when energy demands are uncertain, the values of operation variables $\boldsymbol{\delta}$ and $\boldsymbol{z}$ can be adjusted for energy demands which become certain at the operation stage. Therefore, there is a hierarchical relationship among the design variables, uncertain energy demands, and operation variables as shown in Fig. 2.

The robust optimal design problem in which the values of design and operation variables are determined to minimize the maximum regret in f under uncertain energy demands $\boldsymbol{y}$ is expressed by

$$\min_{\boldsymbol{\eta}} \max_{\boldsymbol{y}} \left(\min_{\boldsymbol{\delta},\, \boldsymbol{z}} f(\boldsymbol{\eta},\, \boldsymbol{y},\, \boldsymbol{\delta},\, \boldsymbol{z}) - \min_{\boldsymbol{\eta}'} \min_{\boldsymbol{\delta}',\, \boldsymbol{z}'} f(\boldsymbol{\eta}',\, \boldsymbol{y},\, \boldsymbol{\delta}',\, \boldsymbol{z}') \right) \tag{1}$$

where $(\ )'$ denotes a different value of the corresponding variable. Next, the flexibility, or the feasibility in energy supply is incorporated into Eq. (1). To secure the flexibility for all the possible values of uncertain energy demands $\boldsymbol{y}$, an objective function which expresses the infeasibility in energy supply is introduced [30], and the values of design variables $\boldsymbol{\eta}$ are determined to minimize (make zero) the maximum of this objective function for all the possible values of $\boldsymbol{y}$. This idea is applied to the ordinary and robust optimal designs, and the corresponding optimization problems are expressed by

$$\min_{\boldsymbol{\eta}'} \max_{\boldsymbol{y}''} \min_{\boldsymbol{\delta}'',\, \boldsymbol{z}''} p(\boldsymbol{\eta}',\, \boldsymbol{y}'',\, \boldsymbol{\delta}'',\, \boldsymbol{z}'') \tag{2}$$

and

$$\min_{\boldsymbol{\eta}} \max_{\boldsymbol{y}'''} \min_{\boldsymbol{\delta}''',\, \boldsymbol{z}'''} p(\boldsymbol{\eta},\, \boldsymbol{y}''',\, \boldsymbol{\delta}''',\, \boldsymbol{z}''') \tag{3}$$

respectively, where p is the objective function for the infeasibility in energy supply, and $(\ )''$ and $(\ )'''$ denote different values of the corresponding variables. To take account of Eqs. (2) and (3) prior to Eq. (1), they are added to Eq. (1) as penalty terms. As a result, the robust optimal design problem is formulated as

$$\begin{aligned}\min_{\boldsymbol{\eta}} \Bigg[\max_{\boldsymbol{y}} \Bigg\{ & \min_{\boldsymbol{\delta},\, \boldsymbol{z}} f(\boldsymbol{\eta},\, \boldsymbol{y},\, \boldsymbol{\delta},\, \boldsymbol{z}) \\ & - \min_{\boldsymbol{\eta}'} \left(\min_{\boldsymbol{\delta}',\, \boldsymbol{z}'} f(\boldsymbol{\eta}',\, \boldsymbol{y},\, \boldsymbol{\delta}',\, \boldsymbol{z}') + W \max_{\boldsymbol{y}''} \min_{\boldsymbol{\delta}'',\, \boldsymbol{z}''} p(\boldsymbol{\eta}',\, \boldsymbol{y}'',\, \boldsymbol{\delta}'',\, \boldsymbol{z}'') \right) \Bigg\} \\ & + W \max_{\boldsymbol{y}'''} \min_{\boldsymbol{\delta}''',\, \boldsymbol{z}'''} p(\boldsymbol{\eta},\, \boldsymbol{y}''',\, \boldsymbol{\delta}''',\, \boldsymbol{z}''') \Bigg]\end{aligned} \tag{4}$$

where W is the coefficient for penalty terms, and should be given a value large sufficiently. Then, the operation of minimization with respect to $\boldsymbol{\eta}'$ is moved forward and is changed to that of maximization to reformulate Eq. (4) as

$$\min_{\boldsymbol{\eta}}\left[\max_{\boldsymbol{y},\,\boldsymbol{\eta}'}\left\{\min_{\boldsymbol{\delta},\,\boldsymbol{z}} f\left(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z}\right)\right.\right.$$
$$\left.-\left(\min_{\boldsymbol{\delta}',\,\boldsymbol{z}'} f\left(\boldsymbol{\eta}',\,\boldsymbol{y},\,\boldsymbol{\delta}',\,\boldsymbol{z}'\right)+W\max_{\boldsymbol{y}''}\min_{\boldsymbol{\delta}'',\,\boldsymbol{z}''} p(\boldsymbol{\eta}',\,\boldsymbol{y}'',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'')\right)\right\}$$
$$\left.+\,W\max_{\boldsymbol{y}'''}\min_{\boldsymbol{\delta}''',\,\boldsymbol{z}'''} p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''')\right] \tag{5}$$

The optimization problem of Eq. (5) includes the operations of minimization and maximization hierarchically, and is formulated as a kind of multilevel programming problems [31]. Here, a special solution method of this three-level min-max-min optimization problem is proposed especially in consideration of the existence of not only continuous but also integer operation variables. The problem is solved by evaluating upper and lower bounds for the optimal value of the maximum regret iteratively. However, the upper bound has to be evaluated by solving a bilevel max-min optimization problem. Thus, this problem is solved by evaluating lower and upper bounds for the maximum regret iteratively for the optimization with respect to integer operation variables, and adopting the Karush-Kuhn-Tucker conditions at each iteration for the optimization with respect to continuous operation variables. On the other hand, the lower bound is evaluated by solving a single-level optimization problem. A concrete solution procedure is described in the following. A flow chart for an outline of the solution procedure is shown in Fig. 3.

3. Solution of robust optimal design problem

3.1. Evaluation of upper bound

On one hand, appropriate values of $\boldsymbol{\eta}$ and $\boldsymbol{y}''$ are assumed in Eq. (5), and the following optimization problem is considered:

$$\max_{\boldsymbol{y},\,\boldsymbol{\eta}'}\left\{\min_{\boldsymbol{\delta},\,\boldsymbol{z}} f\left(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z}\right) - \left(\min_{\boldsymbol{\delta}',\,\boldsymbol{z}'} f\left(\boldsymbol{\eta}',\,\boldsymbol{y},\,\boldsymbol{\delta}',\,\boldsymbol{z}'\right) + W \min_{\boldsymbol{\delta}'',\,\boldsymbol{z}''} p(\boldsymbol{\eta}',\,\boldsymbol{y}'',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'')\right)\right\} + W \max_{\boldsymbol{y}'''} \min_{\boldsymbol{\delta}''',\,\boldsymbol{z}'''} p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''') \tag{6}$$

The optimal value of Eq. (6) gives an upper bound for that of Eq. (5). The optimal solution of Eq. (6) is obtained by solving two optimization problems corresponding to the first and second lines, and the third line independently. These problems are formulated as bilevel MILP ones which include the operations of maximization and minimization hierarchically. These problems are solved independently as follows. This part is an extention of the robust optimal design method using a linear model [24, 25].

3.1.1. Evaluation of flexibility

The problem corresponding to the third line in Eq. (6)

$$\max_{\boldsymbol{y}'''} \min_{\boldsymbol{\delta}''',\,\boldsymbol{z}'''} p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''') \tag{7}$$

is solved by evaluating lower and upper bounds for the optimal value of this equation repeatedly until both the bounds coincide with each other.

On one hand, a lower bound for the optimal value of the equation is obtained by assuming the value of $\boldsymbol{y}'''$ as follows:

$$\min_{\boldsymbol{\delta}''',\,\boldsymbol{z}'''} p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''') \tag{8}$$

This problem is an MILP one, and can be solved easily.

On the other hand, an upper bound for the optimal value of the equation is obtained by limiting the value of $\boldsymbol{\delta}'''$ as follows:

$$\max_{\boldsymbol{y}'''} \min_{\boldsymbol{\delta}''' \in A} \min_{\boldsymbol{z}'''} p(\boldsymbol{\eta}, \boldsymbol{y}''', \boldsymbol{\delta}''', \boldsymbol{z}''') \tag{9}$$

where the value of $\boldsymbol{\delta}'''$ is selected from its set A which includes the values of $\boldsymbol{\delta}'''$ obtained by solving Eq. (8). The application of the Karush-Kuhn-Tucker conditions to the minimization with respect to $\boldsymbol{z}'''$ transforms Eq. (9) into

$$\max_{\boldsymbol{y}'''} \min_{\boldsymbol{\delta}''' \in A} \max_{\boldsymbol{z}''', \boldsymbol{\mu}''', \boldsymbol{\lambda}''', \boldsymbol{\varepsilon}'''} q(\boldsymbol{\eta}, \boldsymbol{y}''', \boldsymbol{\delta}''', \boldsymbol{z}''', \boldsymbol{\mu}''', \boldsymbol{\lambda}''', \boldsymbol{\varepsilon}''') \tag{10}$$

where $\boldsymbol{\mu}'''$ and $\boldsymbol{\lambda}'''$ are vectors composed of Lagrange multipliers corresponding to equality and inequality constraints, respectively, $\boldsymbol{\varepsilon}'''$ is the vector composed of integer variables which convert the nonlinear complementarity condition generated by the Karush-Kuhn-Tucker conditions into linear equations [32], and q is the function converted from p. Although the operation of maximization with respect to $\boldsymbol{z}'''$, $\boldsymbol{\mu}'''$, $\boldsymbol{\lambda}'''$, and $\boldsymbol{\varepsilon}'''$ is not necessary, it is included for the following procedure. This problem is a three-level MILP one which includes the operations of maximization and minimization hierarchically. However, the operation of minimization is only with respect to $\boldsymbol{\delta}'''$, and is conducted by selecting the value of $\boldsymbol{\delta}'''$ from its finite number of candidates in the set A. Therefore, the introduction of a variable for minimum with respect to $\boldsymbol{\delta}'''$ and inequality constraints changes Eq. (10) into

$$\left.\begin{array}{l} \displaystyle\max_{\boldsymbol{y}''', \boldsymbol{z}''', \boldsymbol{\mu}''', \boldsymbol{\lambda}''', \boldsymbol{\varepsilon}'''} \quad Q \\ \text{sub. to} \quad Q \le q(\boldsymbol{\eta}, \boldsymbol{y}''', \boldsymbol{\delta}''', \boldsymbol{z}''', \boldsymbol{\mu}''', \boldsymbol{\lambda}''', \boldsymbol{\varepsilon}''') \quad (\forall \boldsymbol{\delta}''' \in A) \end{array}\right\} \tag{11}$$

where Q is the minimum of q with respect to $\boldsymbol{\delta}'''$. This problem is also an MILP one, and can be solved easily. Here, the value of $\boldsymbol{y}'''$ to be determined should be independent of the value of $\boldsymbol{\delta}'''$ to be selected. This is because energy demands arise before operational strategies are determined. However, the value of $\boldsymbol{y}'''$ may be dependent on the value of $\boldsymbol{\delta}'''$ to satisfy energy demands. To avoid this dependence, virtual energy supply flows are added to existing ones to satisfy energy demands by virtual ones if and only if existing ones cannot satisfy energy demands. For this purpose, q is modified by considering virtual energy supply flows as penalty terms.

The value of $\boldsymbol{y}'''$ obtained by solving Eq. (9) is used in Eq. (8) to evaluate another lower bound for the optimal value of Eq. (7).

3.1.2. Evaluation of robustness

The problem corresponding to the first and second lines in Eq. (6)

$$\max_{\boldsymbol{y},\, \boldsymbol{\eta}'} \left\{ \min_{\boldsymbol{\delta},\, \boldsymbol{z}} f(\boldsymbol{\eta},\, \boldsymbol{y},\, \boldsymbol{\delta},\, \boldsymbol{z}) - \left(\min_{\boldsymbol{\delta}',\, \boldsymbol{z}'} f(\boldsymbol{\eta}',\, \boldsymbol{y},\, \boldsymbol{\delta}',\, \boldsymbol{z}') + W \min_{\boldsymbol{\delta}'',\, \boldsymbol{z}''} p(\boldsymbol{\eta}',\, \boldsymbol{y}'',\, \boldsymbol{\delta}'',\, \boldsymbol{z}'') \right) \right\} \tag{12}$$

is solved by evaluating lower and upper bounds for the optimal value of this equation repeatedly until both the bounds coincide with each other.

On one hand, a lower bound for the optimal value of the equation is obtained by assuming the values of $\boldsymbol{y}$ and $\boldsymbol{\eta}'$ as follows:

$$\min_{\boldsymbol{\delta},\, \boldsymbol{z}} f(\boldsymbol{\eta},\, \boldsymbol{y},\, \boldsymbol{\delta},\, \boldsymbol{z}) - \min_{\boldsymbol{\delta}',\, \boldsymbol{z}',\, \boldsymbol{\delta}'',\, \boldsymbol{z}''} \left(f(\boldsymbol{\eta}',\, \boldsymbol{y},\, \boldsymbol{\delta}',\, \boldsymbol{z}') + W p(\boldsymbol{\eta}',\, \boldsymbol{y}'',\, \boldsymbol{\delta}'',\, \boldsymbol{z}'') \right) \tag{13}$$

This problem is composed of two MILP ones, and can be solved easily.

On the other hand, an upper bound for the optimal value of the equation is obtained by limiting the value of $\boldsymbol{\delta}$ as follows:

$$\max_{\boldsymbol{y},\,\boldsymbol{\eta}',\,\boldsymbol{\delta}',\,\boldsymbol{z}',\,\boldsymbol{\delta}'',\,\boldsymbol{z}''} \left\{ \min_{\boldsymbol{\delta}\in B} \min_{\boldsymbol{z}} f(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z}) - \left(f(\boldsymbol{\eta}',\,\boldsymbol{y},\,\boldsymbol{\delta}',\,\boldsymbol{z}') + Wp(\boldsymbol{\eta}',\,\boldsymbol{y}'',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'') \right) \right\} \tag{14}$$

where the value of $\boldsymbol{\delta}$ is selected from its set B which includes the values of $\boldsymbol{\delta}$ obtained by solving Eq. (13). The application of the Karush-Kuhn-Tucker conditions to the minimization with respect to $\boldsymbol{z}$ transforms Eq. (14) into

$$\max_{\boldsymbol{y},\,\boldsymbol{\eta}',\,\boldsymbol{\delta}',\,\boldsymbol{z}',\,\boldsymbol{\delta}'',\,\boldsymbol{z}''} \left\{ \min_{\boldsymbol{\delta}\in B} \max_{\boldsymbol{z},\,\boldsymbol{\mu},\,\boldsymbol{\lambda},\,\boldsymbol{\varepsilon}} g(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z},\,\boldsymbol{\mu},\,\boldsymbol{\lambda},\,\boldsymbol{\varepsilon}) - \left(f(\boldsymbol{\eta}',\,\boldsymbol{y},\,\boldsymbol{\delta}',\,\boldsymbol{z}') + Wp(\boldsymbol{\eta}',\,\boldsymbol{y}'',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'') \right) \right\} \tag{15}$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are vectors composed of Lagrange multipliers corresponding to equality and inequality constraints, respectively, $\boldsymbol{\varepsilon}$ is the vector composed of integer variables which convert the nonlinear complementarity condition generated by the Karush-Kuhn-Tucker conditions into linear equations, and g is the function converted from f. Although the operation of maximization with respect to $\boldsymbol{z}$, $\boldsymbol{\mu}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\varepsilon}$ is not necessary, it is included for the following procedure. This problem is a three-level MILP one which includes the operations of maximization and minimization hierarchically. In a similar way to that of coverting Eq. (10) to Eq. (11), the introduction of a variable for minimum with respect to $\boldsymbol{\delta}$ and inequality constraints changes Eq. (15) into

$$\left. \begin{array}{ll} \displaystyle\max_{\boldsymbol{y},\,\boldsymbol{\eta}',\,\boldsymbol{\delta}',\,\boldsymbol{z}',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'',\,\boldsymbol{z},\,\boldsymbol{\mu},\,\boldsymbol{\lambda},\,\boldsymbol{\varepsilon}} & \left\{ G - \left(f(\boldsymbol{\eta}',\,\boldsymbol{y},\,\boldsymbol{\delta}',\,\boldsymbol{z}') + Wp(\boldsymbol{\eta}',\,\boldsymbol{y}'',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'') \right) \right\} \\ \text{sub. to} & G \le g(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z},\,\boldsymbol{\mu},\,\boldsymbol{\lambda},\,\boldsymbol{\varepsilon}) \quad (\forall \boldsymbol{\delta} \in B) \end{array} \right\} \tag{16}$$

where G is the minimum of g with respect to $\boldsymbol{\delta}$. This problem is also an MILP one, and can be solved easily. In a similar way to that in Eq. (11), to avoid the dependence of the value of $\boldsymbol{y}$ on the value of $\boldsymbol{\delta}$, virtual energy supply flows are added to existing ones to satisfy energy demands by virtual ones if and only if existing ones cannot satisfy energy demands. For this purpose, g is modified by considering virtual energy supply flows as penalty terms.

The values of $\boldsymbol{y}$ and $\boldsymbol{\eta}'$ obtained by solving Eq. (14) is used in Eq. (13) to evaluate another lower bound for the optimal value of Eq. (12).

3.2. Evaluation of lower bound

On the other hand, the values of $\boldsymbol{y}$ and $\boldsymbol{\eta}'$ are assumed to be selected only from their combinations obtained by solving Eq. (6), and the following optimization problem is considered in place of Eq. (5):

$$\begin{aligned}\min_{\boldsymbol{\eta}}\Bigg[\max_{(\boldsymbol{y},\,\boldsymbol{\eta}')\in C}\Bigg\{&\min_{\boldsymbol{\delta},\,\boldsymbol{z}} f\,(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z})\\ &-\left(\min_{\boldsymbol{\delta}',\,\boldsymbol{z}'} f\,(\boldsymbol{\eta}',\,\boldsymbol{y},\,\boldsymbol{\delta}',\,\boldsymbol{z}')+W\max_{\boldsymbol{y}''}\min_{\boldsymbol{\delta}'',\,\boldsymbol{z}''} p(\boldsymbol{\eta}',\,\boldsymbol{y}'',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'')\right)\Bigg\}\\ &+W\max_{\boldsymbol{y}'''}\min_{\boldsymbol{\delta}''',\,\boldsymbol{z}'''} p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''')\Bigg]\end{aligned} \tag{17}$$

where C is the set for combinations of values of $\boldsymbol{y}$ and $\boldsymbol{\eta}'$. The optimal value of Eq. (17) gives a lower bound for that of Eq. (5). In Eq. (17), the values of

$$\Phi(\boldsymbol{\eta}',\,\boldsymbol{y})=\min_{\boldsymbol{\delta}',\,\boldsymbol{z}'} f\,(\boldsymbol{\eta}',\,\boldsymbol{y},\,\boldsymbol{\delta}',\,\boldsymbol{z}') \tag{18}$$

and

$$\Psi(\boldsymbol{\eta}')=\max_{\boldsymbol{y}''}\min_{\boldsymbol{\delta}'',\,\boldsymbol{z}''} p(\boldsymbol{\eta}',\,\boldsymbol{y}'',\,\boldsymbol{\delta}'',\,\boldsymbol{z}'') \tag{19}$$

can be evaluated for each candidate of combinations of values of $\boldsymbol{y}$ and $\boldsymbol{\eta}'$ independently. The following procedure is used to solve the problem of Eq. (17).

First, D is defined as the set for values of $\boldsymbol{y}'''$. An appropriate value of $\boldsymbol{y}'''$ is assumed, and is made an element of D. The value of $\boldsymbol{y}'''$ is assumed to be selected only from the elements of D, and the following optimization problem is solved in place of Eq. (17):

$$\min_{\boldsymbol{\eta}}\left[\max_{(\boldsymbol{y},\,\boldsymbol{\eta}')\in C}\left\{\min_{\boldsymbol{\delta},\,\boldsymbol{z}} f(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z})-\Big(\Phi(\boldsymbol{\eta}',\,\boldsymbol{y})+W\Psi(\boldsymbol{\eta}')\Big)\right\} + W\max_{\boldsymbol{y}'''\in D}\min_{\boldsymbol{\delta}''',\,\boldsymbol{z}'''} p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''')\right] \tag{20}$$

This problem is a three-level MILP one which includes the operations of minimization and maximization hierarchically. In a similar way to that of coverting Eq. (10) to Eq. (11), the introduction of variables for maxima with respect to $(\boldsymbol{y},\ \boldsymbol{\eta}')$ and $\boldsymbol{y}'''$, and inequality constraints changes Eq. (12) into

$$\left.\begin{array}{ll}\displaystyle\min_{\boldsymbol{\eta},\,\boldsymbol{\delta},\,\boldsymbol{z},\,\boldsymbol{\delta}''',\,\boldsymbol{z}'''} & (F+WP)\\ \text{sub. to} & F\geq f(\boldsymbol{\eta},\,\boldsymbol{y},\,\boldsymbol{\delta},\,\boldsymbol{z})-\Big(\Phi(\boldsymbol{\eta}',\,\boldsymbol{y})+W\Psi(\boldsymbol{\eta}')\Big)\quad(\forall(\boldsymbol{y},\,\boldsymbol{\eta}')\in C)\\ & P\geq p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''')\quad(\forall\boldsymbol{y}'''\in D)\end{array}\right\} \tag{21}$$

where F and P are the maxima of $f-(\Phi+W\Psi)$ and p with respect to $(\boldsymbol{y},\ \boldsymbol{\eta}')$ and $\boldsymbol{y}'''$, respectively. This problem is also an MILP one, and can be solved easily.

Next, using the value of $\boldsymbol{\eta}$ obtained by solving Eq. (21), the problem

$$\max_{\boldsymbol{y}'''}\min_{\boldsymbol{\delta}''',\,\boldsymbol{z}'''} p(\boldsymbol{\eta},\,\boldsymbol{y}''',\,\boldsymbol{\delta}''',\,\boldsymbol{z}''') \tag{22}$$

is solved by the solution method for Eq. (7) shown previously, and it is tested whether the value of p for the optimal solution is zero or not. If the value of p is zero, it is judged that the optimal solution of Eq. (17) is obtained by solving Eq. (21); otherwise the value of $\boldsymbol{y}'''$ for the optimal solution of Eq. (22) is added to the set D, and the problems of Eqs. (21) and (22) are solved repeatedly until the value of p becomes zero.

The values of $\boldsymbol{\eta}$ and $\boldsymbol{y}''$ obtained by solving Eq. (17) are used in Eq. (6) to evaluate another upper bound for the optimal value of Eq. (5).

4. Case study

4.1. Conditions

The proposed method is applied to a case study on the robust optimal design of a gas engine cogeneration system for electric power and hot water supply. Figure 4 shows the super structure for the system, which has two gas engine cogeneration units with a same capacity and two gas-fired auxiliary boilers with a same capacity. The robust optimal design problem is formulated using the integer and continuous design variables $\boldsymbol{\eta}$, uncertain energy demands $\boldsymbol{y}$, integer operation variables $\boldsymbol{\delta}$, and continuous operation variables $\boldsymbol{z}$ defined as

$$\begin{aligned}\boldsymbol{\eta} = (&\gamma_{\mathrm{GE}1},\ \cdots,\ \gamma_{\mathrm{GE}J_{\mathrm{GE}}},\ \gamma_{\mathrm{GB}1},\ \cdots,\ \gamma_{\mathrm{GB}J_{\mathrm{GB}}},\\ &\eta_{\mathrm{GE}1},\ \cdots,\ \eta_{\mathrm{GE}J_{\mathrm{GE}}},\ \eta_{\mathrm{GB}1},\ \cdots,\ \eta_{\mathrm{GB}J_{\mathrm{GB}}},\ \bar{E}_{\mathrm{buy}},\ \bar{V}_{\mathrm{buy}})^{\mathrm{T}}\end{aligned} \quad (23)$$

$$\left.\begin{aligned}&\boldsymbol{y}_t = (E_{\mathrm{dem}t},\ H_{\mathrm{dem}t}) \quad (t = 1,\ 2,\ \cdots,\ T)\\ &\boldsymbol{y} = (\boldsymbol{y}_1,\ \boldsymbol{y}_2,\ \cdots,\ \boldsymbol{y}_T)^{\mathrm{T}}\end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned}&\boldsymbol{\delta}_t = (\delta_{\mathrm{GE}t},\ \delta_{\mathrm{GB}t}) \quad (t = 1,\ 2,\ \cdots,\ T)\\ &\boldsymbol{\delta} = (\boldsymbol{\delta}_1,\ \boldsymbol{\delta}_2,\ \cdots,\ \boldsymbol{\delta}_T)^{\mathrm{T}}\end{aligned}\right\} \quad (25)$$

and

$$\left.\begin{array}{l} \boldsymbol{z}_t = (E_{\mathrm{GE}t},\ H_{\mathrm{GE}t},\ V_{\mathrm{GE}t},\ H_{\mathrm{GB}t},\ V_{\mathrm{GB}t},\ E_{\mathrm{buy}t},\ V_{\mathrm{buy}t},\ H_{\mathrm{disp}t}) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (t = 1,\ 2,\ \cdots,\ T) \\ \boldsymbol{z} = (\boldsymbol{z}_1,\ \boldsymbol{z}_2,\ \cdots,\ \boldsymbol{z}_T)^{\mathrm{T}} \end{array}\right\} \tag{26}$$

respectively. In Eq. (23), γ and η are binary and integer design variables for the selection and number of equipment, respectively, the subscripts 1 to J denote the 1st to Jth capacities of equipment, the subscripts GE and GB denote gas engine cogeneration unit and gas-fired auxiliary boiler, respectively, $\bar{E}_{\mathrm{buy}}$ and $\bar{V}_{\mathrm{buy}}$ are continuous design variables for the maximum demands of purchased electricity and city gas, respectively, and the superscript T denotes the transposition of a vector. In Eq. (24), E_{dem} and H_{dem} are uncertain electricity and hot water demands, respectively, the subscript t denotes the index for the periods set to consider seasonal and hourly variations in energy demands, and T denotes the number of the periods. In Eq. (25), δ is an integer operation variable for the number of equipment at the on status of operation. In Eq. (26), the components in $\boldsymbol{z}_t$ denote energy flow rates shown in Fig. 4.

Table 1 shows the capacities and performance characteristic values of the gas engine cogeneration units and gas-fired auxiliary boilers to be selected. As shown in this table, if each type of equipment is installed, one of the two candidates for capacities is selected. In addition to the equipment, the maximum demands of electricity and city gas purchased from outside utility companies are also determined. Table 2 shows the capital unit costs of equipment as well as the unit costs for demand and energy charges of utilities. In evaluating the annual capital cost, the capital recovery factor is set at 0.7782 by assuming the interest rate and life of equipment as 0.02 and 15 y, respectively.

A hotel with the total floor area of 3000 m^2 is selected as the building which is supplied with electricity and hot water by the cogeneration system. To take account of seasonal and hourly variations in energy demands, a typical year is divided into three representative days in summer, mid-season, and winter whose numbers of days per year are set at 122, 122, and 121 d/y, respectively, and each day is further divided into 6 sampling time intervals with 4 h/d. Thus, the year is divided into 18 periods correspondingly. Figures 5 (a) and (b) show the hourly variations in average electricity and hot water demands in each season. Electricity and hot water demands for each period are assumed to vary within $\pm\alpha$ times of their averages, and correspondingly their upper and lower limits are given.

Table 3 shows the sizes of the optimization problems, i.e., the numbers of binary/integer variables, continuous variables, and constraints for Eqs. (8), (11), (13), (16), and (21). Since Eqs. (16) and (21) are solved repeatedly by adding variables and constraints, the basic numbers for the first iteration and the incremental numbers for each iteration are shown. All the optimization calculations are conducted using a commercial solver GAMS/CPLEX Ver. 12.6.1 on a MacBook Pro with the Intel Core i5 processor of 2.4 GHz and RAM of 8 GB [33].

4.2. Results and discussion

First, the convergence characteristics of the upper and lower bounds evaluated in the solution process are investigated. As an example, Figs. 6 (a) and (b) show the changes in the upper and lower bounds for the maximum regret in the annual total cost of Eq. (12) and the minimum of the maximum regret in the annual total cost of Eq. (5), respectively, in the case of the uncertainty in energy demands α = 0.25. Figure 6 (b) shows the convergence characteristics of the upper and lower bounds evaluated for the outer loop in the flow chart shown in Fig. 3. As shown in Fig. 6 (b), four iterations I to IV are necessary to attain the coincidence of the upper and lower bounds for the minimum of the maximum regret in the annual total cost. On the other hand, Fig. 6 (a) shows the convergence characteristics of the upper and lower bounds evaluated for the second inner loop in the flow chart shown in Fig. 3. As shown in Fig. 6 (a), three or four iterations are necessary to attain the coincidence of the lower and upper bounds for the maximum regret in the annual total cost for a system in each iteration I to IV shown in Fig. 6 (b). It turns out that the convergence characteristics are preferable in all the cases.

Next, the minimum of the maximum regret in the annual cost is evaluated by changing the value of the uncertainty in energy demands α. Figure 7 shows the minimum of the maximum regret in the annual total cost in relation to α. This figure means that the regret of the robust optimal design for any possible energy demands is smaller than the minimum of the maximum regret. In addition, it also means that there exist some energy demands for which the regret of a design different from the robust optimal one is larger than the minimum of the maximum regret. The increasing rate in the minimum of the maximum regret increases with α in the case study based on a linear model for constituent equipment. However, the minimum of the maximum regret increases while the increasing rate in the minimum of the maximum regret decreases with an increase in α in this case study based on a mixed-integer linear model for constituent equipment. This is because the former model can change the optimal capacities of equipment continuously while the latter model has to change them discretely.

Table 4 shows the optimal values of design variables, or capacities and numbers of equipment as well as maximum demands of utilities, in relation to α. In the overall range of α = 0.0 to 0.25, the capacities and numbers of gas engine cogeneration unit and gas-fired auxiliary boiler do not change, and the maximum demands of electricity and city gas increase with α.

As an example, Figs. 8 (a) and (b) show the hourly variations in electricity and hot water demands, respectively, in winter which give the maximum regret in the annual total cost in the case of α = 0.25. They also include the average energy demands as well as the upper and lower limits for energy demand intervals. In many sampling times, the energy demands which give the maximum regret in the annual total cost coincide with upper or lower limits. However, they do not necessarily coincide with upper or lower limits. Figure 8 shows this feature of the mixed-integer linear model for constituent equipment. As an example, Figs. 9 (a) and (b) show the optimal operational strategies for electricity and hot water supplies, respectively, corresponding to the energy demands in winter which give maximum regret in the annual total cost in the case of α = 0.25. The operational strategies are determined appropriately according to the energy demands. The gas engine cogeneration unit is operated in the thermal following mode. Namely, it is stopped for a low hot water demand, is operated at part load levels for middle hot water demands, and is operated at the rated load level for high hot water demands.

Figure 10 compares the annual total costs of the robust optimal and optimal designs for the energy demands which give the maximum regret in the annual total cost in relation to α. The difference in the annual total cost between the robust optimal and optimal designs coincides with the maximum regret in the annual total cost shown in Fig. 7. This difference ranges only 0.0 to 3.4% of the annual total cost of the optimal design. Therefore, it turns out that the proposed method enables the annual total cost of the robust optimal design to be close to that of the optimal design for all the possible values of uncertain energy demands.

Finally, Fig. 11 shows the overall computation time and its contents, or computation times for Eq. (14) and the other equations, in relation to α. The overall computation time increases drastically with α. The computation time to solve Eq. (14) tends to dominate the overall one with an increase in α. Especially, in the case of α = 0.25, it is extremely hard to solve Eq. (14) directly using the commercial solver.

5. Conclusions

A robust optimal design method of energy supply systems under uncertain energy demands has been proposed using a mixed-integer linear model for constituent equipment. A robust optimal design problem has been formulated as a three-level min-max-min optimization one by adopting the interval programming and minimax regret criterion, and considering hierarchical relationships among design variables, uncertain energy demands, and operation variables. This problem has been solved especially in consideration of the existence of integer operation variables by evaluating upper and lower bounds for the maximum regret and the optimal value of the maximum regret in the performance criterion iteratively. In a case study, the proposed method has been applied to the robust optimal design of a gas engine cogeneration system with a simple configuration. Through the case study, the following main results have been obtained:

- The robust optimal design method based on the linear model proposed previously has been extended successfully to that based on the mixed-integer linear model.
- The proposed method has preferable convergence characteristics in evaluating upper and lower bounds for the maximum regret and the minimum of the maximum regret in the annual total cost repeatedly.
- With an increase in the uncertainty in energy demands, the minimum of the maximum regret increases while its increasing rate decreases. This tendency based on the mixed-integer linear model is different from that based on the linear model.
- The energy demands which give the maximum regret in the annual total cost do not necessarily coincide with upper or lower limits. This tendency based on the mixed-integer linear model is also different from that based on the linear model.
- The difference in the annual total cost between the robust optimal and optimal designs for the energy demands which give the maximum regret in the annual total cost ranges only 0.0 to 3.4 % of the annual total cost of the optimal design.
- It is difficult to obtain these results by conventional optimal design and sensitivity analysis methods where energy demands are treated as certain parameters. The results show the validity and effectiveness of the proposed method.

Through the case study, it has turned out that the computation time increases drastically with the uncertainty in energy demands. Especially, it takes long computation time to evaluate an upper bound for the maximum regret. Therefore, it is inevitable to reduce it so that the proposed robust optimal design method can be applied to practical case studies.

Nomenclature

A : set for candidate values of $\boldsymbol{\delta}'''$

B : set for candidate values of $\boldsymbol{\delta}$

C : set for candidate values of $\boldsymbol{y}$ and $\boldsymbol{\eta}'$

D : set for candidate values of $\boldsymbol{y}'''$

E : electric power, kWh/h

F : maximum of $f-(\Phi+W\Psi)$ with respect to $\boldsymbol{y}$ and $\boldsymbol{\eta}'$, yen/y

f : performance criterion (annual total cost), yen/y

G : minimum of g with respect to $\boldsymbol{\delta}$, yen/y

g : function converted from f, yen/y

H : heat flow rate, kWh/h

J : number of capacity candidates

P : maximum of p with respect to $\boldsymbol{y}'''$, kWh/y

p : infeasibility in energy supply, kWh/y

Q : minimum of q with respect to $\boldsymbol{\delta}'''$, kWh/y

q : function converted from p, kWh/y

T : number of periods

V : city gas flow rate, m^3/h

W : coefficient for penalty terms, yen/kWh

$\boldsymbol{y}$: vector for uncertain parameters (energy demands), kWh/h

$\boldsymbol{z}$: vector for continuous operation variables (energy flow rates), kWh/h, m^3/h

$\overline{(\ )}$: continuous design variable for maximum demand of utility, kW, m^3/h

$(\ )'$, $(\ )''$, $(\ )'''$: different values of variables

Greek symbols

α : uncertainty in energy demands

γ : binary design variable for selection of equipment

δ : integer operation variable for number of equipment at on status of operation

$\boldsymbol{\delta}$: vector for integer operation variables

$\boldsymbol{\varepsilon}$: binary variables for linearizing complementarity constraint

η : integer design variable for number of equipment

$\boldsymbol{\eta}$: vector for integer and continuous design variables, kW, m^3/h

$\boldsymbol{\lambda}$: vector for Lagrange multipliers for inequality constraints

$\boldsymbol{\mu}$: vector for Lagrange multipliers for equality constraints

Φ : function of f with respect to $\boldsymbol{y}$ and $\boldsymbol{\eta}'$, yen/y

Ψ : function of p with respect to $\boldsymbol{\eta}'$, kWh/y

Equipment symbols (subscripts)

GB : gas-fired auxiliary boiler

GE : gas engine cogeneration unit

Subscripts

buy : purchase

dem : demand

disp : disposal

t : index for periods

Superscript

T : transposition of vector

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Captions for tables and figures

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(a) Electricity

(b) Hot water

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(a) Maximum regret of Eq. (12)

(b) Minimum of maximum regret of Eq. (5)

Fig. 7 Minimum of maximum regret in annual total cost in relation to uncertainty in energy demands

Fig. 8 Energy demands which give maximum regret in annual total cost (α = 0.25, winter)

(a) Electricity

(b) Hot water

Fig. 9 Optimal operational strategies corresponding to energy demands which give maximum regret in annual total cost (α = 0.25, winter)

(a) Electricity supply

(b) Hot water supply

Fig. 10 Annual total costs of robust optimal and optimal designs corresponding to energy demands which give maximum regret in annual total cost (α = 0.25)

Fig. 11 Computation time in relation to uncertainty in energy demands

Table 1 Capacities and performance characteristic values of candidates of equipment for selection

Equipment	Capacity/performance*	Candidate	
		#1	#2
Gas engine cogeneration unit	Max. power output kW	25.0	35.0
	Max. hot water output kW	38.4	52.7
	Power generating efficiency	0.335	0.340
	Heat recovery efficiency	0.515	0.511
Gas-fired auxiliary boiler		#1	#2
	Max. hot water output kW	99.0	198.0
	Thermal efficiency	0.886	0.900

*At rated load level

Table 2 Capital unit costs of equipment, and unit costs for demand and energy charges of utilities

Equipment/utility		Unit cost
Gas engine cogeneration unit		225.0×10^3 yen/kW
Gas-fired auxiliary boiler		9.0×10^3 yen/kW
Electricity	Demand charge	1685 yen/(kW·month)
	Energy charge	12.08 yen/kWh
City gas	Demand charge	630 yen/(m^3/h·month) *
	Energy charge	60.0 yen/m^3 *

*At standard state

Table 3 Sizes of optimization problems

Equation	Iteration	Binary/integer variables	Continuous variables	Constraints
Eq. (8)		36	325	847
Eq. (11) for Eq. (9)		180	526	904
Eq. (13) 1st term		36	327	849
Eq. (13) 2nd term		92	677	1987
Eq. (16) for Eq. (14)	Basis	272	1197	2831
	Increment	+180	+486	+865
Eq. (21) for Eq. (17)	Basis	92	705	1967
	Increment	+36	+290	+849

Table 4 Optimal values of design variables

Uncertainty in energy demands α	Capacity and number of equipment		Maximum demand of utility	
	GE	GB	Electricity kW	City gas m^3/h *
0.00	#1×1	#1×1	75.9	9.66
0.05	#1×1	#1×1	81.0	9.79
0.10	#1×1	#1×1	86.0	10.34
0.15	#1×1	#1×1	91.1	10.46
0.20	#1×1	#1×1	96.1	10.55
0.25	#1×1	#1×1	101.5	11.04

*At standard state

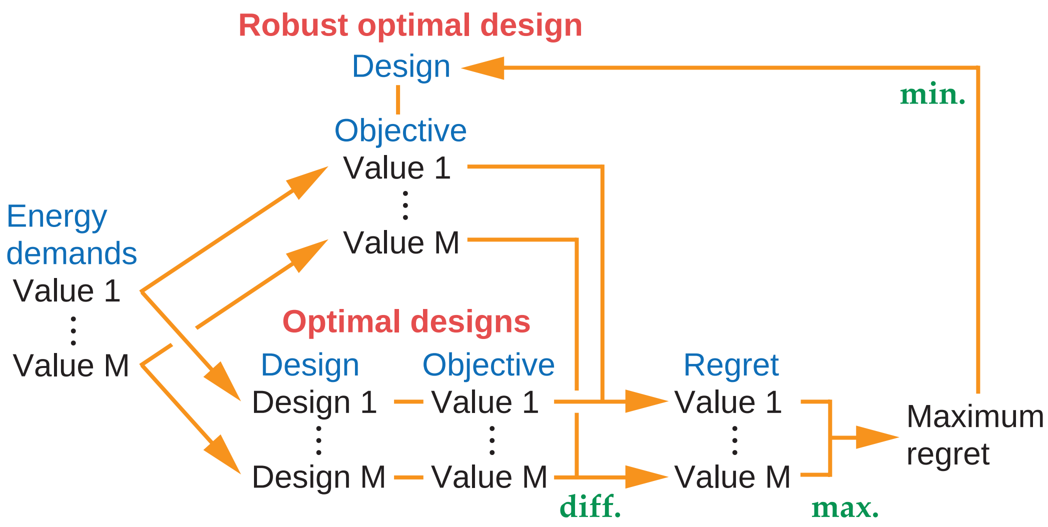


Fig. 1 Concept of robust optimal design based on minimax regret criterion

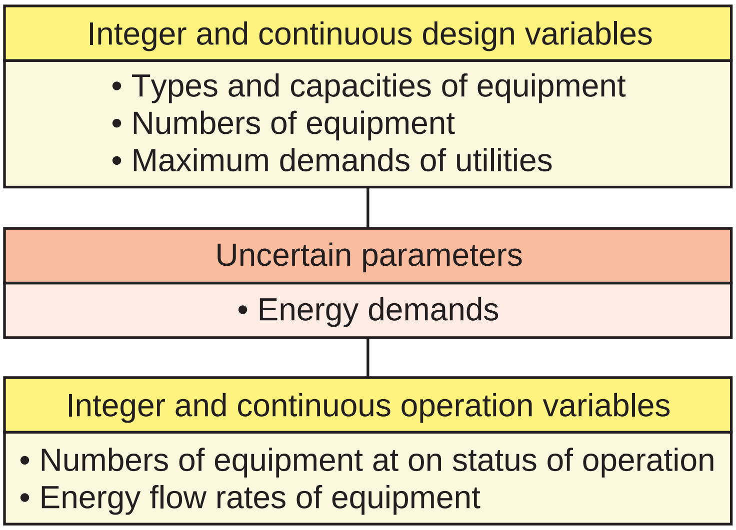


Fig. 2 Hierarchical relationship among design variables, uncertain energy demands, and operation variables

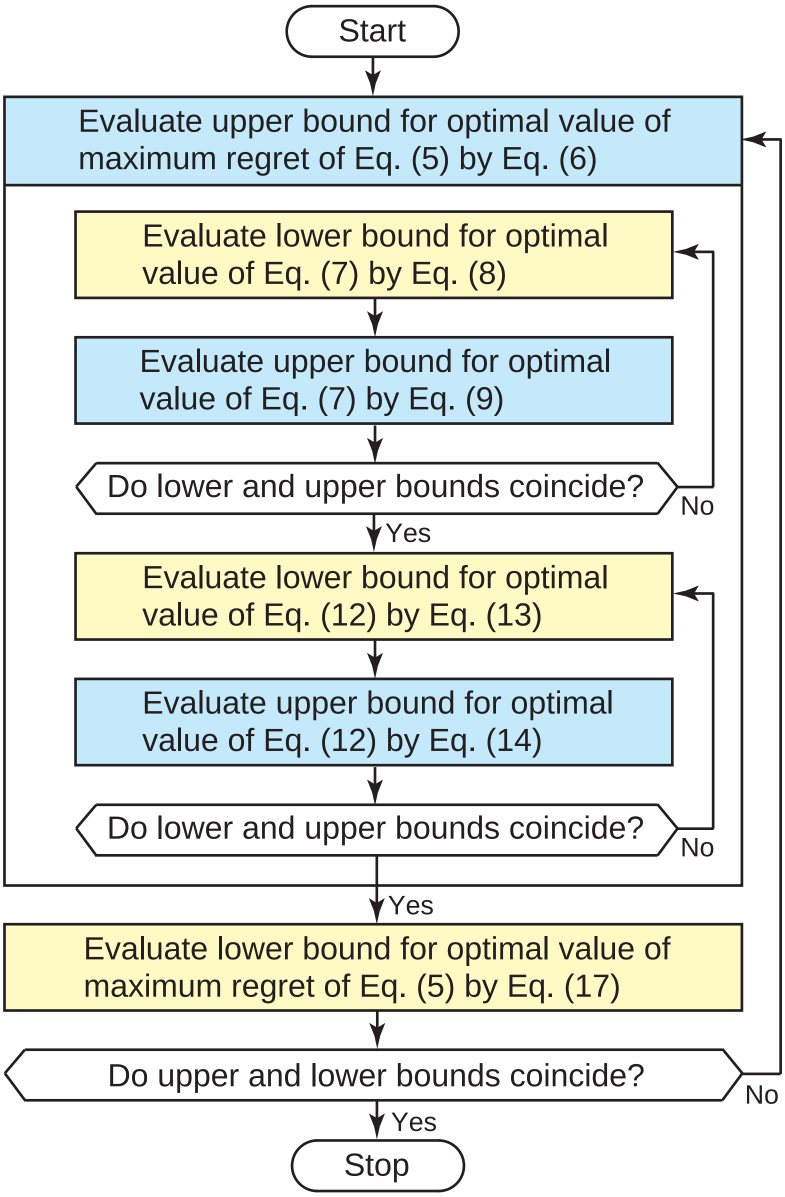


Fig. 3 Flow chart for solution of robust optimal design problem

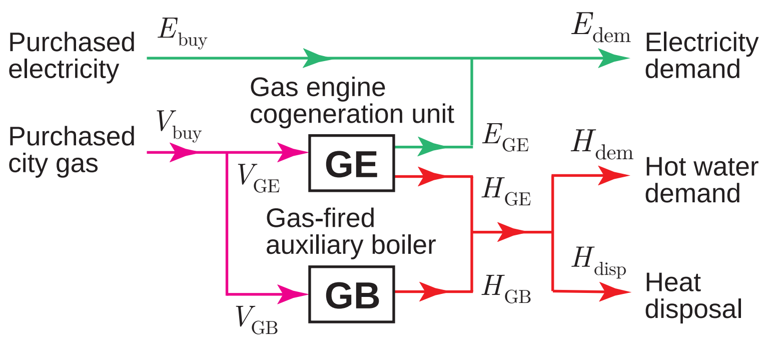
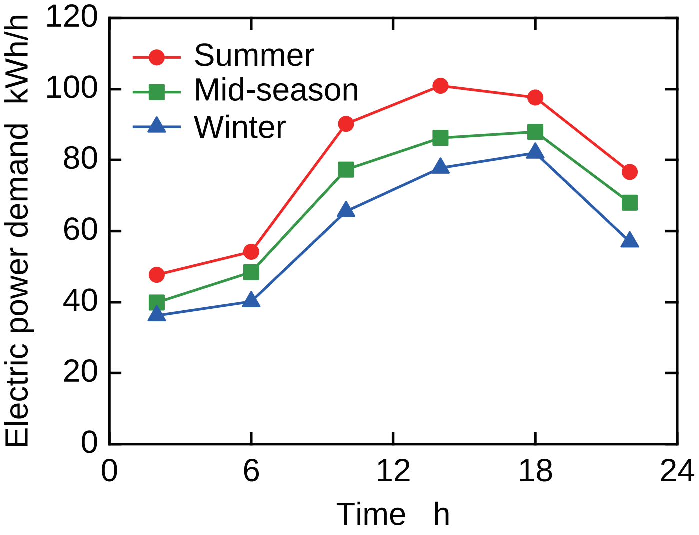
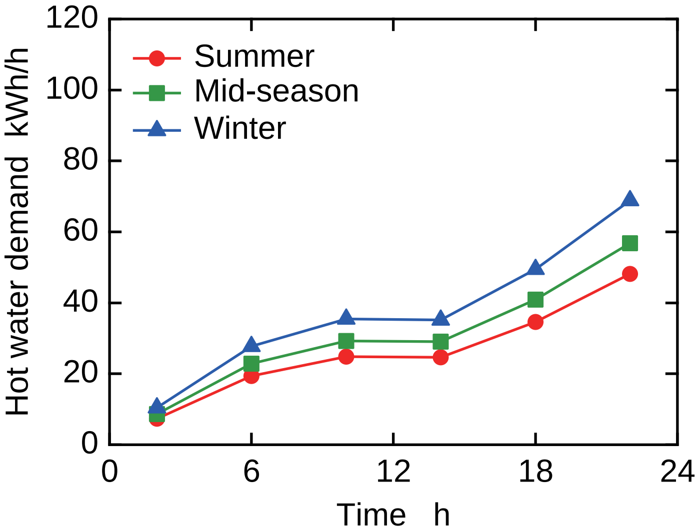


Fig. 4 Configuration of gas engine cogeneration system

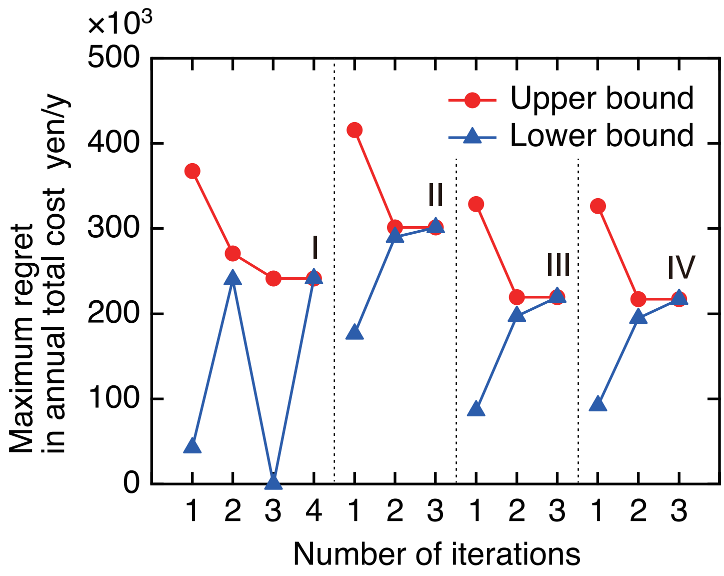


(a) Electricity

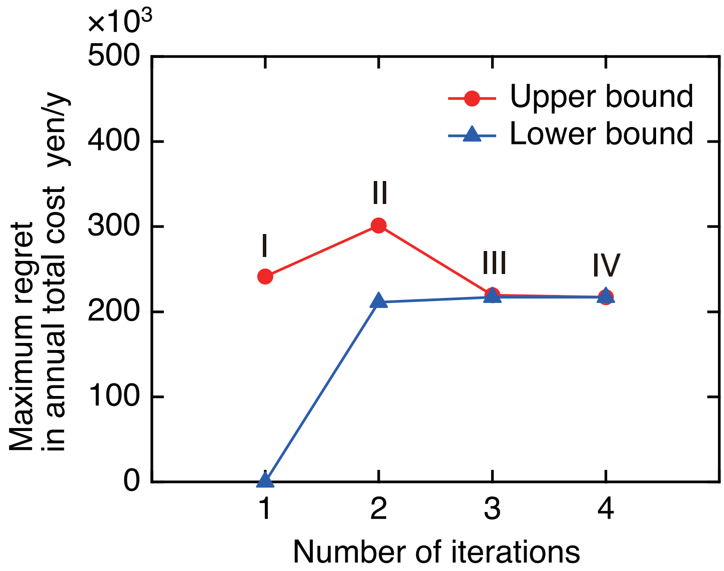


(b) Hot water

Fig. 5 Average energy demands



(a) Maximum regret of Eq. (12)



(b) Minimum of maximum regret of Eq. (5)

Fig. 6 Convergence characteristics of upper and lower bounds (α = 0.25)

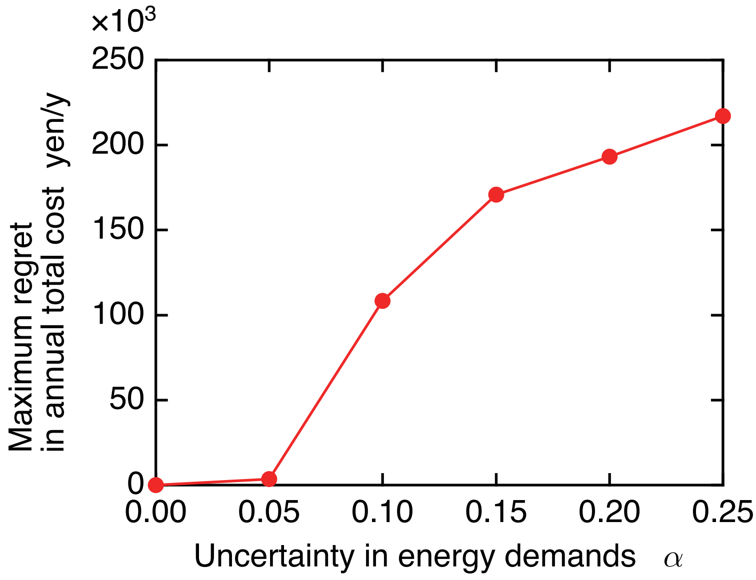
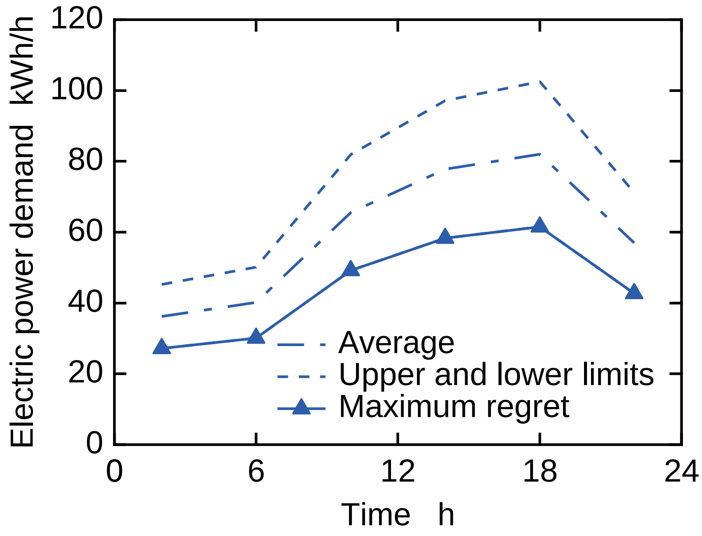
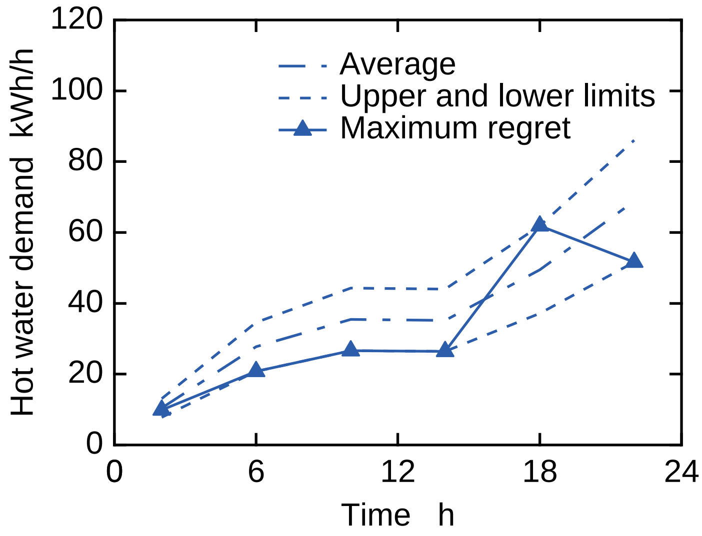


Fig. 7 Minimum of maximum regret in annual total cost in relation to uncertainty in energy demands

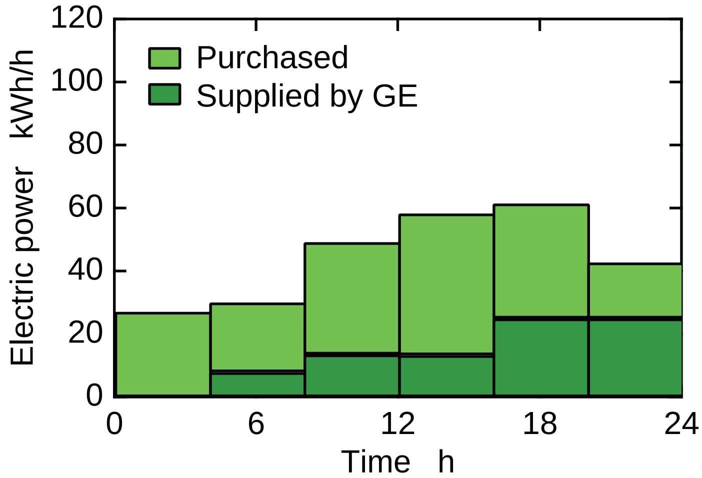


(a) Electricity

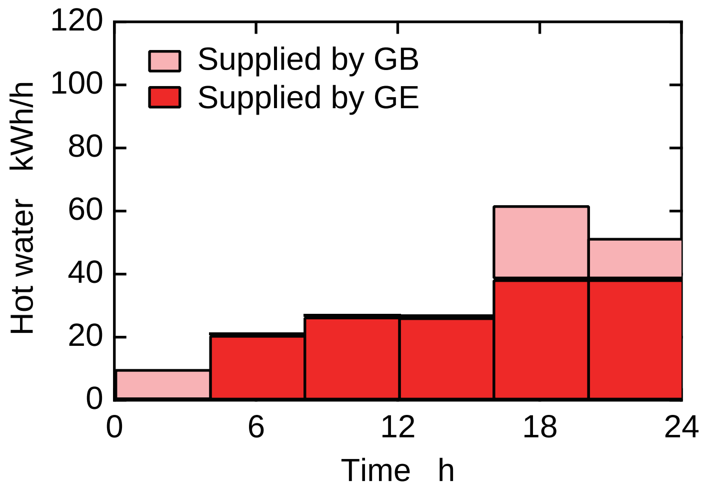


(b) Hot water

Fig. 8 Energy demands which give maximum regret in annual total cost (α = 0.25, winter)



(a) Electricity supply



(b) Hot water supply

Fig. 9 Optimal operational strategies corresponding to energy demands which give maximum regret in annual total cost (α = 0.25, winter)

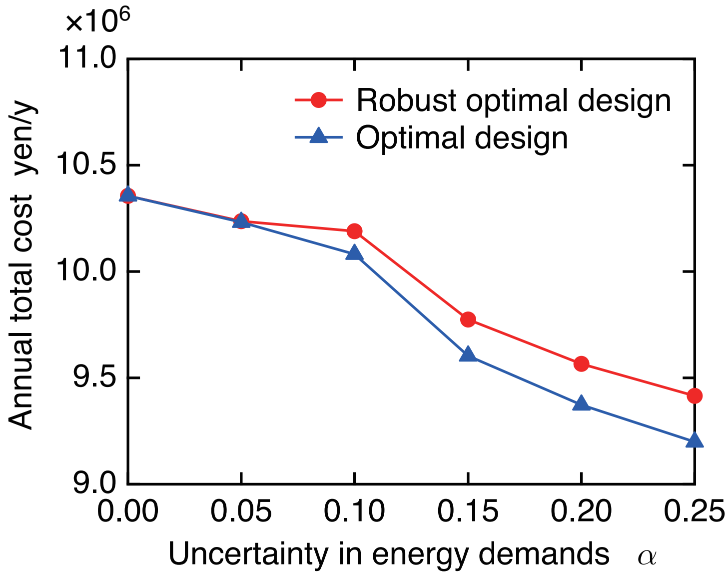


Fig. 10 Annual total costs of robust optimal and optimal designs corresponding to energy demands which give maximum regret in annual total cost (α = 0.25)

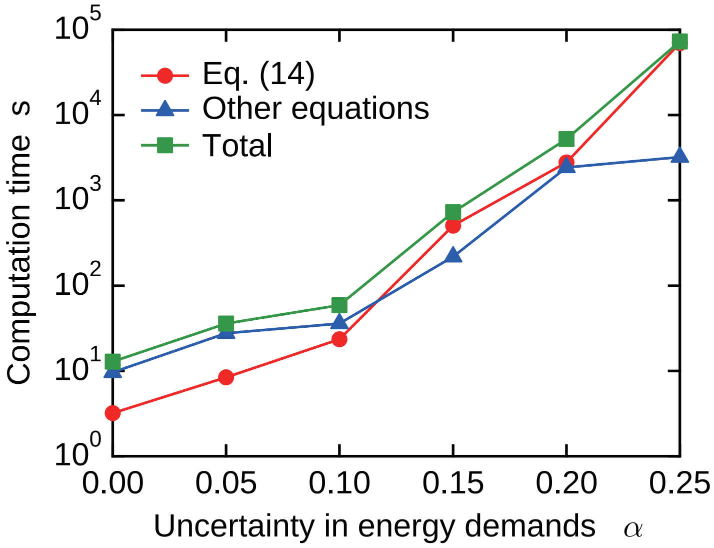


Fig. 11 Computation time in relation to uncertainty in energy demands