Adaptive Ensemble Kalman Filter Estimation of Nonlinear Structural Systems with Unknown Noise Covariance

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Adaptive Ensemble Kalman Filter Estimation of Nonlinear Structural Systems with Unknown Noise Covariance

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Abstract

In this paper, an extension of the conventional adaptive Kalman filtering technique to the ensemble Kalman filter is presented. Using the presented method, unknown noise covariances of the filter settings are effectively determined. A simple cantilevered beam vibration problem is provided to verify the effectiveness of the presented adaptive estimation method.

1. INTRODUCTION

Recently, many interests have been attracted for smart structural systems, which are designed to be adjustable via actuators on orbit[1]. One of the main difficulties in constructing effective smart structural systems is to estimate the appropriate current structural states for the actuations because the sensors equipped with spacecraft are usually insufficient to observe whole structural states. To compensate unobserved structural data, numerical simulations based on the finite element (FE) models play an important role.

The FE models usually contains various uncertainties in the systems, such as uncertain structural parameters, and these uncertainties are often identified by using measurement data obtained in validation tests on the ground. The accuracy of the model is limited by how precise the validation test can simulate the orbital spacecraft environment. In future space missions, such as advanced space antennas, the structural design requirements tend to become more and more severe, and the accuracy of the numerical models estimated based on the ground-based validation test could be insufficient.

This research presents an effective model estimation method based on the ensemble Kalman filtering (EnKF) technique[2,3], which can automatically provide optimum estimations of system state variables while assimilating the nonlinear numerical model with the experimental data in sequential manner. Based on the EnKF technique, the structural state estimations can be sequentially obtained on orbit for the actuations in the smart structure system. The accuracies of the EnKF estimations depend on the filter noise settings, that is, the system noise covariance and the measurement noise covariance. These noise settings have to be determined without knowing real state values. In our research, we apply the adaptive estimation technique for the conventional Kalman filter[4,5] to the EnKF, and investigate the effectiveness of the adaptive estimation technique on the EnKF estimations. A simple nonlinear example is provided to verify the effectiveness of the adaptive estimations.

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2. NONLINER STRUCTURAL DYNAMIC SYSTEM EQUATIONS

In the finite element analysis, the governing equations for nonlinear structural dynamics problems are given by

$$\mathbf{M}\ddot{\mathbf{u}}_{t+1} + \mathbf{C}\dot{\mathbf{u}}_{t+1} + \mathbf{Q}(\mathbf{u}_{t+1}) = \mathbf{F}(\mathbf{u}_{t+1}) \tag{1}$$

Here, the vectors $\mathbf{u}_{t+1}, \dot{\mathbf{u}}_{t+1}, \ddot{\mathbf{u}}_{t+1}$ indicate the nodal displacement, velocity, and acceleration vectors, respectively, and the matrices $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, respectively. The internal and external force vectors are denoted by $\mathbf{Q}$ and $\mathbf{F}$, respectively, which are functions of $\mathbf{u}_{t+1}$. The subscript $t+1$ denotes a discretized time step. In this paper, we apply the generalized α method[6] for numerical integration of (1). In the generalized α method, the following governing equations are considered:

$$\mathbf{M}\ddot{\mathbf{u}}_{t+1-\alpha_m} + \mathbf{C}\dot{\mathbf{u}}_{t+1-\alpha_f} + \mathbf{Q}(\mathbf{u}_{t+1-\alpha_f}) = \mathbf{F}(\mathbf{u}_{t+1-\alpha_f}) \tag{2}$$

Here, α_m, α_f are constants that affect numerical damping, and are defined by

$$\alpha_m = \frac{2\rho_\infty - 1}{\rho_\infty + 1}, \alpha_f = \frac{\rho_\infty}{\rho_\infty + 1} \tag{3}$$

where ρ_∞ is the radius of convergence. The discrete time update equations as shown below:

$$\mathbf{\Gamma}\left(\mathbf{u}_t, \mathbf{u}_{t+1}\right) = \mathbf{0} \tag{4}$$

Note that the above equations become nonlinear simultaneous equations and need an iterative method such as the Newton-Raphson method.

3. ENSEMBLE KALMAN FILTER (EnKF)

In this section, we present a basic formulation of the EnKF[2,3], which is a nonlinear extension of the standard Kalman filter. In the EnKF, a priori state estimates are provided by using the Monte Carlo method of nonlinear physical simulations, whereas in the widely used extended Kalman filter (EKF), they are obtained based on linearized systems. Unlike in the EKF, the estimates in the EnKF can inherit many of the nonlinear properties of a priori state estimates.

3.1 Model and observation equations

We define the following model and observation equations:

$$\mathbf{x}_{t+1} = \mathbf{F}_t(\mathbf{x}_t, \mathbf{w}_t) \tag{5}$$

$$\mathbf{y}_{t+1} = \mathbf{H}_{t+1}\mathbf{x}_{t+1} + \mathbf{v}_{t+1} \tag{6}$$

Here, $\mathbf{x}_t$ and $\mathbf{w}_t$ are the state vector and the system noise whose dimensions are n, $\mathbf{F}_t$ is the nonlinear model operator that relates $\mathbf{x}_t$ to $\mathbf{x}_{t+1}$, $\mathbf{H}_{t+1}$ is the transformation matrix from the state vector $\mathbf{x}_{t+1}$ to the measurement $\mathbf{y}_{t+1}$ whose dimension is p, and $\mathbf{v}_t$ is the measurement noise. The covariance matrices of the system noise vector and the measurement noise vector are given by

$$E\left\{\mathbf{w}_t\mathbf{w}_t^T\right\} = \mathbf{Q}_t \tag{7}$$

$$E\left\{\mathbf{v}_t\mathbf{v}_s^T\right\} = \mathbf{R}_t \tag{8}$$

where $E\left\{a\right\}$ denotes the expected value of *a*, the matrices $\mathbf{Q}_t$ and $\mathbf{R}_t$ denote the covariance matrices of the system noise vector and the measurement noise vector, respectively.

In general, the state estimation problem is to find the posterior probability density function (PDF) of the state vector $\mathbf{x}_{t+1}$ given the observation $\mathbf{y}_{t+1}$ with the prior PDF, which is the conditional PDF of $\mathbf{x}_t$ given $\mathbf{y}_{1:t} = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_t]$. The prior PDF and the posterior PDF are described as

$$p(\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t}) = p(\mathbf{x}_t \mid \mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_t) \tag{9}$$

$$p(\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t+1}) = p(\mathbf{x}_t \mid \mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_{t+1}) \tag{10}$$

The state prediction and estimation of $\mathbf{x}_{t+1}$ can be obtained as the expected value of (9) and (10), respectively, as follows:

$$\mathbf{x}_{t+1|t} = E\left\{\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t}\right\} = \int_{-\infty}^{\infty} \mathbf{x}_{t+1} p(\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t}) d\mathbf{x} \tag{11}$$

$$\mathbf{x}_{t+1|t+1} = E\left\{\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t+1}\right\} = \int_{-\infty}^{\infty} \mathbf{x}_{t+1} p(\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t+1}) d\mathbf{x} \tag{12}$$

3.2 Ensemble approximation

The prior and posterior PDFs usually become non-Gaussian when the model equations are nonlinear, and thus, numerical schemes are needed to evaluate (11) and (12). In the EnKF, the Monte Carlo approach is applied, where the PDFs in (11) and (12) are approximated by generating a number of samples, called particles in the EnKF, as follows:

$$p(\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t}) \simeq \frac{1}{M} \sum_{m=1}^{M} \delta\left(\mathbf{x}_{t+1} - \mathbf{x}_{t+1|t}^{(m)}\right) \tag{13}$$

$$p(\mathbf{x}_{t+1} \mid \mathbf{y}_{1:t+1}) \simeq \frac{1}{M} \sum_{m=1}^{M} \delta\left(\mathbf{x}_{t+1} - \mathbf{x}_{t+1|t+1}^{(m)}\right) \tag{14}$$

Here, M is the total number of particles, $\delta(\bullet)$ is Dirac's delta function, the superscript (m) denotes the index number of the particle set, and $\mathbf{x}_{t+1|t}^{(m)}$ and $\mathbf{x}_{t+1|t+1}^{(m)}$ are the prediction and the estimation, respectively, of the state vector of the *m*th particle at time step $t+1$. Substituting (13) and (14) into (11) and (12), respectively, we can obtain the state prediction and estimation of $\mathbf{x}_{t+1}$ as the ensemble mean of particles as shown below:

$$\mathbf{x}_{t+1|t} \simeq \hat{\mathbf{x}}_{t+1|t} = \frac{1}{M} \int_{-\infty}^{\infty} \mathbf{x}_{t+1} \sum_{m=1}^{M} \delta\left(\mathbf{x}_{t+1} - \mathbf{x}_{t+1|t}^{(m)}\right) d\mathbf{x} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{x}_{t+1|t}^{(m)} \tag{15}$$

$$\mathbf{x}_{t+1|t+1} \simeq \hat{\mathbf{x}}_{t+1|t+1} = \frac{1}{M} \int_{-\infty}^{\infty} \mathbf{x}_{t+1} \sum_{m=1}^{M} \delta\left(\mathbf{x}_{t+1} - \mathbf{x}_{t+1|t+1}^{(m)}\right) d\mathbf{x} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{x}_{t+1|t+1}^{(m)} \tag{16}$$

Likewise, the covariance matrices of $\mathbf{x}_{t+1|t}$ and $\mathbf{x}_{t+1|t+1}$ are approximated by the particle set as shown below:

$$\mathbf{P}_{t+1|t} \simeq \hat{\mathbf{P}}_{t+1|t} = \frac{1}{M-1} \sum_{m=1}^{M} \left(\mathbf{x}_{t+1|t}^{(m)} - \hat{\mathbf{x}}_{t+1|t}\right)\left(\mathbf{x}_{t+1|t}^{(m)} - \hat{\mathbf{x}}_{t+1|t}\right)^T \tag{17}$$

$$\mathbf{P}_{t+1|t+1} \simeq \hat{\mathbf{P}}_{t+1|t+1} = \frac{1}{M-1} \sum_{m=1}^{M} \left(\mathbf{x}_{t+1|t+1}^{(m)} - \hat{\mathbf{x}}_{t+1|t+1}\right)\left(\mathbf{x}_{t+1|t+1}^{(m)} - \hat{\mathbf{x}}_{t+1|t+1}\right)^T \tag{18}$$

3.3 State estimation

The state estimation is performed through two steps: a prediction step and an estimation step. At the prediction step, all particles are updated through the following equations:

$$\mathbf{x}_{t+1|t}^{(m)} = \mathbf{F}_t(\mathbf{x}_{t|t}^{(m)}, \mathbf{w}_t^{(m)}) \tag{19}$$

where $\mathbf{w}_t^{(m)}$ is the *m*th realization of system noise vector based on $\mathbf{Q}_t$. At an estimation step, the estimation vector of the *m*th particle is obtained as follows:

$$\mathbf{x}_{t+1|t+1}^{(m)} = \mathbf{x}_{t+1|t}^{(m)} + \mathbf{K}_{t+1}\left(\mathbf{y}_{t+1} + \mathbf{v}_{t+1}^{(m)} - \mathbf{H}_{t+1}\mathbf{x}_{t+1|t}^{(m)}\right) \tag{20}$$

where $\mathbf{v}_t^{(m)}$ is the realization of measurement noise vector for the *m*th particle based on $\mathbf{R}_{t+1}$, and $\mathbf{K}_{t+1}$ is the Kalman gain matrix given by

$$\mathbf{K}_{t+1} = \hat{\mathbf{P}}_{t+1|t}\mathbf{H}_{t+1}^T\left(\mathbf{R}_{t+1} + \mathbf{H}_{t+1}\hat{\mathbf{P}}_{t+1|t}\mathbf{H}_{t+1}^T\right)^{-1} \tag{21}$$

Here, $\hat{\mathbf{P}}_{t+1|t}$ is obtained by the ensemble calculations of the particle set in the EnKF.

4. ESTIMATION PROCEDURE FOR NONLINEAR STRUCTURAL SYSTEM

In the previous section, a standard formulation of the EnKF is presented. In this section, we apply it to nonlinear structural model estimation by combining displacement and structural properties into a state vector. The combined state vector for model estimation is defined by

$$\mathbf{x}_t \doteq \begin{bmatrix} \mathbf{s}_t \\ \mathbf{u}_t \end{bmatrix} \tag{22}$$

where $\mathbf{s}_t$ is a k-dimensional parameter vector whose components are various structural parameters such as damping ratios and densities. Likewise, the system noise vector is expressed as the combined vector as follows:

$$\mathbf{w}_t \doteq \begin{bmatrix} \mathbf{w}_{s,t} \\ \mathbf{w}_{u,t} \end{bmatrix} \tag{23}$$

where $\mathbf{w}_{s,t}$ and $\mathbf{w}_{u,t}$ are the system noise in the structural dynamics equations and the system noise in the parameter time update equations, respectively. In the presented approach, the prediction in (19) is performed through two steps. The first step is the prediction of the parameter vector as follows:

$$\mathbf{s}_{t+1} = \mathbf{s}_t + \mathbf{w}_{s,t} \tag{24}$$

By using the updated parameter vector, in the next step, the time state update of the displacement vector is performed as shown below:

$$\mathbf{\Gamma}\left(\mathbf{u}_t, \mathbf{u}_{t+1}, \mathbf{s}_{t+1}\right) = \mathbf{0} \tag{25}$$

We consider direct observations of displacement vectors, where the observation equations are given by

$$\mathbf{y}_{t+1} = \begin{bmatrix} {}_p\mathbf{0}_q & \tilde{\mathbf{H}}_{t+1} \end{bmatrix} \begin{bmatrix} \mathbf{s}_{t+1} \\ \mathbf{u}_{t+1} \end{bmatrix} + \mathbf{v}_{t+1} = \mathbf{H}_{t+1}\mathbf{x}_{t+1} + \mathbf{v}_{t+1} \tag{26}$$

where $\tilde{\mathbf{H}}_{t+1}$ is a $p \times n$-dimensional Boolean matrix that relates the observation nodes to the FEA nodes.

The estimation procedure for the nonlinear structural model based on the EnKF is shown below.

(Step 1: Generation of an initial particle set)

The initial particle set is generated by adding initial perturbations to the nominal parameter values in each particle as shown below:

$$\mathbf{s}_{0|0}^{(m)} = \mathbf{s}_0 + \mathbf{w}_0^{(m)},\ {}^i w_{s,0} \sim N(0, {}^i\sigma_{s,0}^2) : m = 1, 2, \cdots M \tag{27}$$

(Step 2: Time update of each particle)

In the prediction step, we first produce the realization of $\mathbf{w}_{s,t}^{(m)},\ {}^i w_{s,t} \sim N(0, {}^i\sigma_s^2)$ in each particle. Then, we update the parameter vector as follows:

$$\mathbf{s}_{t+1|t}^{(m)} = \mathbf{s}_{t|t}^{(m)} + \mathbf{w}_t^{(m)}, m = 1, 2, \cdots M \tag{28}$$

Next, the displacement vector of each particle is updated by using (25). Consequently, the state prediction vector is given by

$$\mathbf{x}_{t+1|t}^{(m)} = \begin{bmatrix} \mathbf{s}_{t+1|t}^{(m)} \\ \mathbf{u}_{t+1|t}^{(m)} \end{bmatrix}, m = 1, 2, \cdots M \tag{29}$$

In addition, the covariance matrix $\mathbf{P}_{t+1|t}$ of $\mathbf{x}_{t+1|t}$ is calculated by (17).

(Step 3: Calculation of Kalman gain)

The Kalman gain is calculated by using (21) with $\mathbf{P}_{t+1|t}$ obtained in step 2.

(Step 4: Estimation of each particle)

All particles are updated by using (20) with the Kalman gain given in step 3. Note that the parameter estimations are provided by means of the ensemble mean of the particle set as follows:

$$\hat{\mathbf{s}}_{t+1|t+1} \frac{1}{M} \sum_{m=1}^{M} \mathbf{s}_{t+1|t+1}^{(m)} \tag{30}$$

We return to step 2 until the estimation process finishes. Through these steps, we can sequentially obtain the state estimations of the displacement vector and parameters at each observation time step.

5. ADAPTIVE FILTERING FOR THE ENSEMBLE KALMAN FILTER

The settings of system and measurement noise have strong effects on both the efficiency and the accuracy of estimation results. In a real case, we have to choose the noise settings without knowing the real state values. To find appropriate settings, the adaptive Kalman filtering techniques have been proposed for the conventional Kalman filter[4,5]. In the adaptive Kalman filter, a proper cost function related to estimation residuals is minimized with respect to the noise variances. The widely used cost function in the conventional Kalman filter[5] is given by

$$\begin{aligned} J(\mathbf{a}) &= \sum_{t=t_0}^{t_n} \left(\ln \left| \mathbf{S}(\mathbf{a}) \right| + \mathbf{e}_{t+1}^T \mathbf{S}^{-1}(\mathbf{a}) \mathbf{e}_{t+1} \right) \\ \mathbf{S}(\mathbf{a}) &= \left(\mathbf{H}_{t+1} \mathbf{P}_{t+1|t}(\mathbf{a}) \mathbf{H}_{t+1}^T + \mathbf{R}_{t+1}(\mathbf{a}) \right) \\ \mathbf{e}_{t+1} &= \mathbf{y}_{t+1} - \mathbf{H}_{t+1} \mathbf{x}_{t+1|t} \end{aligned} \tag{31}$$

where $\boldsymbol{a}$ denotes the parameter vector composed of the noise variances as follows:

$$\mathbf{a} = \left[\sigma_{s,1}^2, \cdots, \sigma_{s,n}^2, \sigma_{obs,1}^2, \cdots \sigma_{obs,m}^2 \right] \tag{32}$$

To apply the adaptive Kalman filtering technique to the EnKF, we extend the cost function J to the following equations:

$$\begin{aligned} J(\mathbf{a}) &= \sum_{t=t_0}^{t_n} \left(\ln \left| \hat{\mathbf{S}}(\mathbf{a}) \right| + \hat{\mathbf{e}}_{t+1}^T \hat{\mathbf{S}}^{-1}(\mathbf{a}) \hat{\mathbf{e}}_{t+1} \right) \\ \hat{\mathbf{S}}(\mathbf{a}) &= \left(\mathbf{H}_{t+1} \hat{\mathbf{P}}_{t+1|t}(\mathbf{a}) \mathbf{H}_{t+1}^T + \mathbf{R}_{t+1}(\mathbf{a}) \right) \\ \hat{\mathbf{e}}_{t+1} &= \mathbf{y}_{t+1} - \mathbf{H}_{t+1} \frac{1}{M} \sum_{m=1}^{M} \mathbf{x}_{t+1|t}^{(m)} \end{aligned} \tag{33}$$

In the adaptive filtering, the tuned noise variances are defined by the solution of the following minimization problem:

$$\min_{\mathbf{a}} J(\mathbf{a}) \tag{34}$$

6. A NUMERICAL EXPERIMENT

A simple cantilevered beam vibration problem is considered to verify the effectiveness of the presented adaptive estimation. The beam is divided into five elements and the y tip displacement is set to the measurement point (Figure 1). The length of the beam is set to 0.4 m, and the base excitation frequency is set to 10 Hz, which is near the 1st eigen frequency of the beam. The base excitation amplitude is set to 5mm, which can induces cubic nonlinearities to the system. In this experiment, the 1st mode damping ratio is set as a model parameter.

The numerical experiment is performed through two processes. In the first process, the structural analysis with the real model parameter value is performed to obtain the displacement history of the cantilevered beam vibration. Then, the artificial measurements are produced by adding the random noise to the calculated observations. In the second process, the EnKF estimation is performed with the artificial measurements. The variance of the measurement noise is set to σ^2_{obs} = 1.0×10^{-8}m^2, measurement time step is set as 2ms. The real value of the damping ratio is set to $\zeta = 0.005$, and the initial parameter value in the EnKF is chosen as $\zeta^0 = 0.025$. The total simulation time is 5 s, and the total number of particles is 100.

The solution procedure for the adaptive estimation is summarized in Figure 2. First, we set initial system and measurement noise settings, and perform the EnKF estimation with N_t interval. Then, calculate the cost function *J*. If the convergence check is OK, the adaptive estimation is finished. Otherwise, we update the noise values, and the EnKF is performed with the new noise values. This process continues until the convergence check is OK. In this paper, we utilize MATLAB® optimization toolbox in the update process. In this experiment, the initial system noise is set $\sigma^{2(0)}_{s,1} = 6.3\times10^{-8}$, and the initial measurement noise is set $\sigma^{2(0)}_{obs} = 1.0\times10^{-8}$. The time interval for adaptive estimation is set N_t = 5s.

Figure 3 shows the results of minimization of *J*. The minimization converges at 10 step, where the system noise is $\sigma^{2(Tuned)}_{s,1} = 4.4\times10^{-6}$ and the measurement noise is $\sigma^{2(Tuned)}_{obs} = 6.6\times10^{-9}$. Figure 4 (a) shows the comparisons of estimation errors of ζ, between the initial system noise setting and the tuned filter setting while the comparisons of y displacement estimation errors in the unobserved nodes (node 4) and the observed nodes (node 6) of the cantilevered beam shown in Figure 1. In these figures, the upper plot shows the estimation errors in the initial setting, while the lower plot shows those in the tuned setting. The 3 standard deviation intervals ($\pm3\sigma$) are also depicted in the figures. The standard deviations are evaluated by calculating the square root of variances of corresponding particle set. As can be seen in Figure 4 (a), the convergence rate of model parameter estimation error with the tuned settings is significantly improved when compared to that with the initial settings. Further, the estimation errors in the initial setting lie outside in the area enclosed by $\pm3\sigma$ around the early time step. In the tuned setting, at almost all the time step, the estimation errors lie in the area. In the practical estimation case, the real sate values are unknown, and thus, the accuracies of error boundaries play an important role to evaluate the estimation results. We can see the same trends in Figures 5 (b) and (c). These results indicate the effectiveness of the presented adaptive estimations.

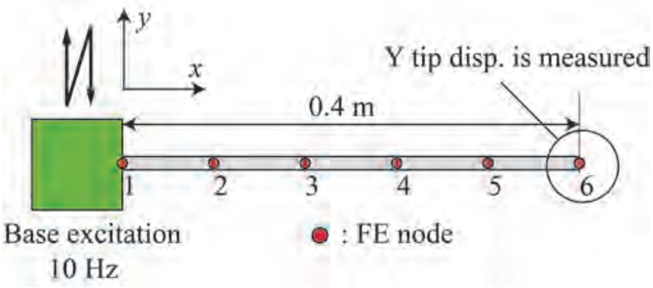


Figure 1. Cantilevered beam model

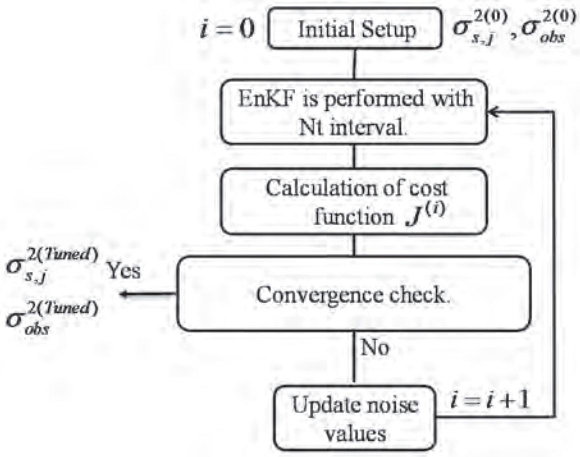


Figure 2. Flow chart in the adaptive estimation

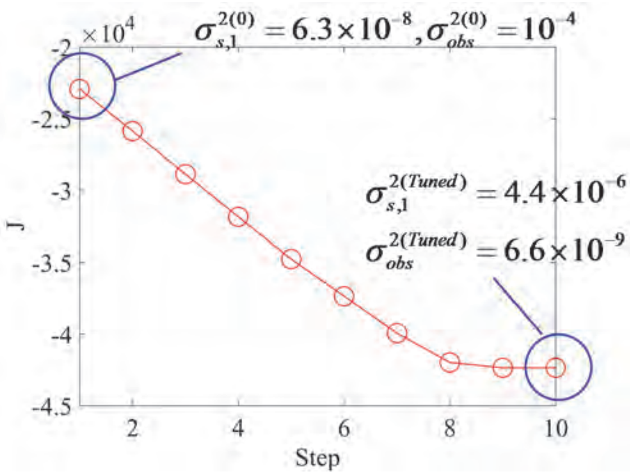
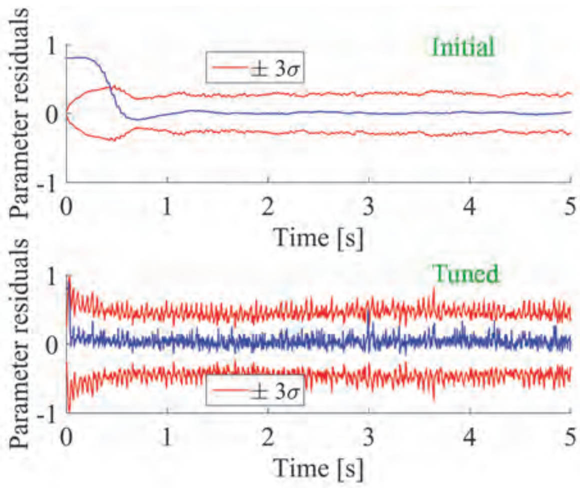
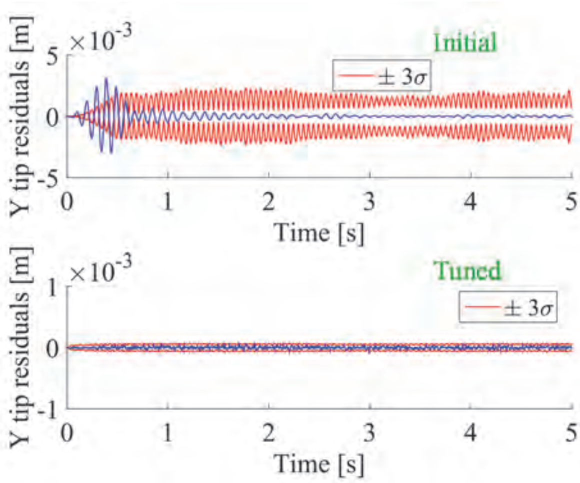


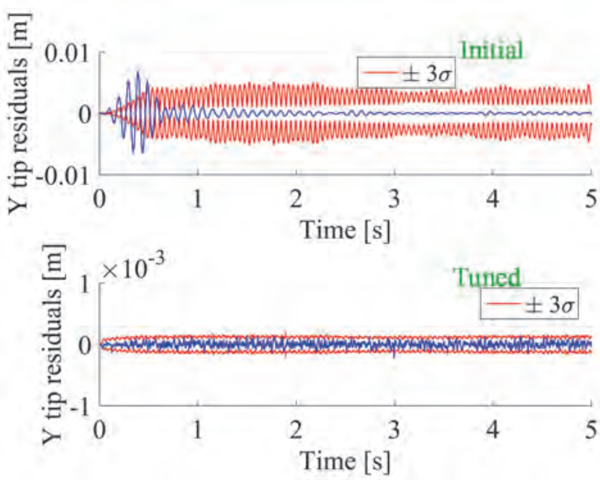
Figure 3. Minimization results



(a) model parameter



(b) y displacement (node 4, unobserved node)



(c) y displacement (node 6, observed node)

Figure 4. Comparisons of estimation errors between the initial noise values ($\sigma^{2(0)}_{s,1} = 6.3\times10^{-8}$, $\sigma^{2(0)}_{obs} = 1.0\times10^{-8}$) and the tuned noise values ($\sigma^{2(Tuned)}_{s,1} = 4.4\times10^{-6}$, $\sigma^{2(Tuned)}_{obs} = 6.6\times10^{-9}$)

6. CONCLUSIONS

An adaptive estimation method of nonlinear structural system with the ensemble Kalman filter was presented. The state space equations of the nonlinear structural dynamics model were derived, and the estimation procedure based on the ensemble Kalman filter was presented. A conventional adaptive filtering technique was applied to the ensemble Kalman filtering. A simple cantilevered beam vibration problem was provided to verify the effectiveness of the presented adaptive estimation method.

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