Effects of the Basset Term on the Motions of a Spheric Solid Particle Under Oscillatory Motion of Waves

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Abstract

For investigating the mechanics of sediment suspension, the present paper deals with the behavior of a solid particle under standing waves based on the equation of motion of the solid particle.

Namely, the value of the ratio of the each term of the equation of motion to the gravitational force term is calculated numerically by using the experimental results and as a consequence, the degree of the Basset term in the equation of motion is estimated. It is shown that the value of the ratio of the Basset term to the gravitational force term is about 30% at most.

I. Introduction

The establishment of countermeasures for preventing from serious disasters caused by phenomina of sediment movement on beaches, such as beach erosion, filling-up of a harbour and closure of a river mouth, is one of the most important problems in coastal engineering. So far, in order to solve such problems, attempts have been made to examine the mechanics of the sediment suspension and transport quantitatively by numerous investigators. However, because of complications of sediment movement due to wave action, satisfactory results have not been obtained yet.

The author, so far, has studied the mechanics of the sediment suspension and transport from the point of view that the behaviors of a solid particle under waves should be examined in detail in order to estimate the sediment suspension and transport quantitatively.

In this paper, as the first step of the investigation of the behavior of the solid particle based on the equation of motion of a solid particle, the degree of the effect of the Basset term on the equation is estimated by using our experimental results obtained previously[1)~3)].

Namely, under the assumption that the velocity field is described by the small amplitude wave theory, the values of the ratio of the each term in the equation of motion to the gravitational force term are calculated numerically and as a consequence, the degree of the effect of the Basset term on the equation of motion is estimated.

II. Basic Equation of Motion for a Spheric Solid Particle in a Variable Velocity Field

(1) In the Case of Law Reynolds Numbers (Re$\leqq$1, Re : Reynolds numbers)

(a) Tchen's equation[4)]: Tchen proposed the model of the equation of motion for a

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spheric solid particle in fluid moving with a variable velocity v. It is based on an equation of motion for a particle moving under the influence of gravity in a fluid at rest, which is frequently called B.B.O. equation, and is derived under the following assumption: i) the fluid velocity field is homogeneous and infinite extent, ii) there is no mutual action between particles, iii) there is also no particle rotation, iv) the Stokes linear resistance can be employed. The resulting equation is given by:

$$M\frac{dv_p}{dt} = m\frac{dv_f}{dt} - \frac{1}{2}m\left(\frac{dv_p}{dt} - \frac{dv_f}{dt}\right) - 3\pi\mu d(v_p - v_f)$$
$$-\frac{3}{2}d^2\sqrt{\pi\rho_f\mu}\cdot\int_{t_1}^{t}\frac{\frac{dv_p}{dt_1} - \frac{dv_f}{dt_1}}{\sqrt{t-t_1}}dt_1 - g(M-m) \tag{1}$$

where the index f and p refers to the fluid and the solid particle, respectively, $M=\pi d\rho_p/6$, μ = the coefficient of viscosity (=1.0295×10^{-5}g·sec/cm), ρ_p = the density of the solid particle, ρ_f = the density of the fluid and t_0 = the starting time.

(b) Corrsin and Lumley's equation[5]: They proposed the model of the equation of motion for a spheric solid particle in the velocity field which is inhomogeneous in space-time as follows:

$$M\frac{d(v_p)_i}{dt} = m\left\{\frac{D(v_f)_i}{Dt} - \nu\nabla^2(v_f)_i\right\} - \frac{1}{2}m\left\{\frac{d(v_p)_i}{dt} - \frac{\partial(v_f)_i}{\partial t} - (v_p)_j\frac{\partial(v_f)_i}{\partial x_j}\right\}$$
$$-3\pi\mu d\{(v_p)_i - (v_f)_i\} - \frac{3}{2}d^2\sqrt{\pi\rho_f\mu}\int_{t_0}^{t}\frac{\frac{dv_p}{dt} - \frac{\partial(v_f)_i}{\partial t} - (v_p)_k\frac{\partial(v_f)_i}{\partial x_k}}{\sqrt{t-t_1}}dt_1$$
$$-g(M-m) \tag{2}$$

where the subscript introduces conventional Cartesian tensor notation, and $D/Dt=\partial/\partial t+(v_p)_k\ \partial/\partial x_k$.

(2) In the Case of High Reynolds Numbers (Re>1)

Graf[6] gived the equation of motion for a spheric particle moving under the influence of gravity in a variable velocity fluid at large Reynolds numbers as follows:

$$M\frac{dv_p}{dt} = m\frac{dv_f}{dt} - km\left(\frac{dv_p}{dt} - \frac{dv_f}{dt}\right) - C_D\frac{\pi d^2}{4}\cdot\frac{\rho_f(v_p - v_f)^2}{2}$$
$$-\frac{3}{2}d^2\sqrt{\pi\rho_f\mu}\int_{t_0}^{t}\frac{\frac{dv_p}{dt_1} - \frac{dv_f}{dt_1}}{\sqrt{t-t_1}}dt_1 - g(M-m) \tag{3}$$

where k = the virtual mass coefficient, which depends on an acceleration parameter $(dv_p/dt - dv_f/dt)\cdot d/(v_p-v_f)^2$ for a spheric particle k=0.5 and C_D = the steady-state drug coefficient and in functional relationship with the particle Reynolds number $(v_p - v_f)\cdot d/\nu$.

Although the Basset term in this Eq. (3) is defined as well as that of Eq. (1), in general, it is impossible to define this term for high Reynolds numbers and any important it might have is probably carried over to the virtual mass term, but more commonly it is altogether omitted.

III. Analysis of Basic Equation of Motion for a Spheric Solid Particle

(1) Determination of the Basic Equation of Motion for Analysis

So far as already shown in II., some models of the equation of motion for a spheric particle were proposed. In the field of the oscillatory wave motion, however, the velocities are changeable in space-time. So the equation of motion for a solid particle should employ the Eq. (2) where the linear resistance and the virtual mass term were corrected as those in Eq. (3), because the particle Reynolds numbers are larger than 1. However, since the purpose of this tudy is to estimate the approximate values of the ratio of each term to the gravitational force term in the equation of motion, herein, Eq. (1) which is in the simplest form is employed and furthermore, only the vertical velocity component is analyzed under the following assumption.

(2) Assumptions

i) The fluid velocity field of oscillatory wave motion can be fully described by a small amplitude wave theory (linear theory).

ii) The Stokes' resistance, which is the linear resistance, holds true, because Eq. (1) is employed as the basic equation of motion for a solid particle.

iii) The time t used in the Basset term is given the values from the starting time (t_0) to the wave period (T). The values of t_0 is given that of the beginning in reading of the position of the solid particle, herein zero.

(3) Some Characteristics of the Spheric Solid Particle

i) The particle diameter (d) is 0.2cm.

ii) The fall velocity of the solid particle (w_0) is 2.15cm/sec.

iii) The particle Reynolds numbers due to the fall velocity ($Re=w_0d/\nu$) are 43 (ν: coefficient of kinematic viscosity = 0.01cm^2/sec.)

iv) The specific gravity of the solid particle is 1.032.

(4) Methods of Numerical Calculation

(a) The values of the ratio of each term to the gravitational force term:

Herein, for the numerical calculation by finite difference method are used the values of each term in Eq.(1) divided by the gravitational force term $g(M-m)$, which are given as follows:

i) The values of the ratio of Basset term to the gravitational force term (BT1)

$$BT1=\frac{\frac{3}{2}d^2\sqrt{\pi\rho_f\mu}}{g(M-m)}\cdot\int_0^t\frac{\frac{dw_p}{dt_1}-\frac{dw_f}{dt_1}}{\sqrt{t-t_1}}dt_1=0.0813\int_0^t\frac{\frac{dw_p}{dt_1}-\frac{dw_f}{dt_1}}{\sqrt{t-t_1}}dt_1 \tag{4}$$

ii) The values of the ratio of the virtual mass term to the gravitational force term (VM)

$$VM=\frac{\frac{1}{2}m\left(\frac{dw_p}{dt}-\frac{dw_f}{dt}\right)}{g(M-m)}=0.0159\times\left(\frac{dw_p}{dt}-\frac{dw_f}{dt}\right) \tag{5}$$

iii) The values of the ratio of Stokes' resistance term to the gravitational force term (RE)

$$RE=\frac{3\pi\mu d(w_p-w_f)}{g(M-m)}=0.1447(w_p-w_f) \tag{6}$$

iv) The values of the ratio of the resultant term of the pressures to the gravitational force term (PR)

$$PR=\frac{m\frac{dw_f}{dt}}{g(M-m)}=0.03189\frac{dw_f}{dt} \tag{7}$$

v) The values of the ratio of the term of the force required to accelerate the particle to the gravitational force term (FP)

$$FP=\frac{M\frac{dw_p}{dt}}{g(M-m)}=0.03291\frac{dw_p}{dt} \tag{8}$$

From Eq. (4)~(8) shown above, the degree of each term in the equation of motion were estimated by the finite difference method.

(b) Methods of calculating the Basset term numerically:

In this paper, the following two methods were employed in calculating the Basset term;

i) Method (I): This method is that of calculating the Basset term directly by finite difference method; this calculation is conducted every Δt_1 (=0.075 sec.) the time interval in reading the experimental values, which is equivalent to the flash interval of the stroboscope. The obtained difference equation is as follows:

$$\int_0^t \frac{\frac{dw_p}{dt_1} - \frac{dw_f}{dt_1}}{\sqrt{t-t_1}} dt_1 = \sum_{i=1}^{n} \frac{\left(\frac{\Delta w_p}{\Delta t_1} - \frac{\Delta w_f}{\Delta t_1}\right)_{t_{i-1}, x_{i-1}, z_{i-1}}}{\sqrt{n\Delta t_1 - (i-1)\Delta t_1}} \Delta t_1 \tag{9}$$

$$\left(\frac{\Delta w_p}{\Delta t_1}\right)_i = \left\{\frac{x_{i+1} - x_i}{\Delta t_1} - \frac{x_i - x_{i-1}}{\Delta t_1}\right\} / \Delta t_1 = \frac{x_{i+1} - 2x_i + x_{i-1}}{(\Delta t_1)^2} \tag{10}$$

$$\left(\frac{\Delta w_f}{\Delta t_1}\right)_i = \left\{\frac{(w_f)_{i+1} + (w_f)_i}{2} - \frac{(w_f)_i + (w_f)_{i-1}}{2}\right\} / \Delta t_1 = \frac{(w_f)_{i+1} - (w_f)_{i-1}}{2\Delta t_1} \tag{11}$$

where Δt_1=0.075 sec, T=1.15 sec, $n=T/\Delta t_1=15$

ii) Method (II): This method is that of estimating the Basset term by calculating all the other terms except the Basset term in Eq. (1) by finite difference method. Namely, if the result is denoted by BT2, it is given from Eq. (1) and Eq. (5)~(8) as follows:

$$BT2 = -FP + PR - VM - RE - 1.0 \tag{12}$$

(5) Results of Analysis

Some examples calculated by the two methods shown above are shown in **Table 1** (1)~(4), where h = water depth, z = the vertical distance from the still water surface, k = wave number, x = the horizontal distance from the wave reflection wall, H =wave height, T = wave period, t = the lapse time from the starting time, w_p = the vertical velocity of the solid particle, w_f = the vertical velocity of the water particle calculated by using the wave theory, and *BT1, VM, RE, PR, FP* and *BT2* in this table correspond to the results calculated by using Eq. (4)~(8) and Eq. (12), respectively.

In addition, only the values of w_f, w_f, $w_p - w_f$, *BT1* and *BT2* shown in this table are plotted in **Figs. 1** (1)~(4).

IV. Considerations

As mentioned above, **Figs. 1** (1)~(4) represented the values of the Basset term calculated by the two methods, the experimental values of the velocities of the solid particle and the theoretical ones of the fluid particle. From these figures, it is found that when a solid particle moves in the fluid field described by the small applitude wave theory, the value of the ratio of the Basset term to the gravitational force term by Method (I) is about 30% at most and changeable in time-space, and on the other hand, the values of Basset term estimated by Method (II) are several times those of Method (I). As the causes of the differences between these two methods are shown the following ones; namely, the error resulting from the assumption in III. (2) shown above, and imperfection of Eq. (1), the coarse of divisions of the time in numerical calculation and assuming the starting time, etc. are considered. However, the fact that an outstanding difference exists between Method (I) and (II) like this suggests that Eq. (1) can't describe the motion of the solid particle under waves perfectly, and there exists another term besides each term in Eq. (1). Namely, it seems that these differences may be due to something similar to the turbulence existing in the oscillatory motion of waves, such as already shown in the previous paper[3].

V. Conclusions

As mentioned above, in this paper, the behavior of a solid particle under standing waves has been treated using the experimental results based on the equation of motion concerning the solid particle and some considerations have been made on the Basset term in the equation of motion.

As a result, it has been found that the values of the ratio of the Basset term to the gravitational force term are about 30% at most and changeable in time and space. In addition, the disagreement of these results between Method (I) and (II) seems to suggest that there may exist some dispersive effect similar to turbulence between the solid particle and waves. Owing to the lack of the data, however, it is necessary to make more detailed experiment and comprehensive study.

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Table 1 Results of numerical calculation

(1) h = 30cm, z = −15cm, kx = $3\pi/4$, H = 13.75cm, T = 1.15 sec (Run. 22−16)

$BT1$	VM	RE	PR	FP	$BT2$	$W_p\,(\frac{cm}{sec})$	$W_f\,(\frac{cm}{sec})$	$BT2$-$BT1$	t
−0.002	−0.005	−0.547	1.801	1.855	−0.502	0.632	4.412	−0.500	Δt_1
0.020	0.057	−0.529	1.607	1.782	−0.703	4.865	8.519	−0.723	$2\Delta t_1$
0.081	0.157	−0.454	1.182	1.547	−1.068	8.741	11.879	−1.149	$3\Delta t_1$
0.180	0.242	−0.315	0.393	0.907	−1.441	11.598	13.777	−1.621	$4\Delta t_1$
0.227	0.110	−0.193	−0.790	−0.591	−1.116	11.966	13.298	−1.343	$5\Delta t_1$
0.266	0.087	−0.125	−2.106	−2.001	−1.067	8.948	9.809	−1.333	$6\Delta t_1$
0.265	−0.002	−0.095	−3.009	−3.118	−0.793	2.989	3.647	−1.058	$7\Delta t_1$
0.247	−0.036	−0.108	−3.028	−3.207	−0.676	−4.375	−3.626	−0.923	$8\Delta t_1$
0.234	−0.023	−0.129	−2.137	−2.260	−0.725	−10.740	−9.849	−0.959	$9\Delta t_1$
0.182	−0.090	−0.168	−0.775	−0.988	−0.528	−14.521	−13.357	−0.710	$10\Delta t_1$
0.088	−0.149	−0.252	0.492	0.202	−0.310	−15.436	−13.697	−0.398	$11\Delta t_1$
−0.051	−0.197	−0.372	1.302	0.940	−0.069	−14.106	−11.536	−0.018	$12\Delta t_1$
−0.148	−0.119	−0.482	1.630	1.442	−0.210	−11.334	−8.003	−0.062	$13\Delta t_1$
−0.224	−0.076	−0.550	1.670	1.571	−0.275	−7.826	−4.026	−0.051	$14\Delta t_1$
−0.279	−0.039	−0.590	1.603	−1.579	−0.347	−4.158	−0.082	−0.068	$15\Delta t_1$

(2) h = 30cm, z = −15cm, kx = $3\pi/4$, H = 8.19cm, T = 1.15 sec (Run. 22−29)

$BT1$	VM	RE	PR	FP	$BT2$	$W_p\,(\frac{cm}{sec})$	$W_f\,(\frac{cm}{sec})$	$BT2$-$BT1$	t
−0.004	−0.011	−0.651	1.555	1.587	−0.371	−0.603	3.893	−0.367	Δt_1
0.019	0.059	−0.634	1.228	1.393	−0.591	2.866	7.246	−0.610	$2\Delta t_1$
0.056	0.097	−0.579	0.714	0.940	−0.743	5.582	9.586	−0.799	$3\Delta t_1$
0.190	0.328	−0.431	0.026	0.705	−1.576	7.496	10.477	−1.766	$4\Delta t_1$
0.354	0.384	−0.184	−0.773	−0.008	−1.965	8.307	9.577	−2.319	$5\Delta t_1$
0.483	0.289	0.050	−1.539	−0.996	−1.882	7.138	6.791	−2.365	$6\Delta t_1$
0.619	0.290	0.251	−2.049	−1.523	−2.067	4.205	2.468	−2.686	$7\Delta t_1$
0.479	−0.281	0.255	−2.093	−2.746	−0.322	−0.764	−2.523	−0.801	$8\Delta t_1$
0.447	−0.060	0.136	−1.626	−1.806	−0.896	−6.063	−7.004	−1.343	$9\Delta t_1$
0.217	−0.399	−0.024	−0.800	−1.652	0.275	−10.089	−9.927	0.058	$10\Delta t_1$
0.081	−0.216	−0.237	0.102	−0.340	−0.105	−12.409	−10.768	−0.185	$11\Delta t_1$
−0.054	−0.190	−0.379	0.833	0.470	−0.069	−12.258	−9.642	−0.015	$12\Delta t_1$
−0.183	−0.158	−0.500	1.294	1.012	−0.060	−10.532	−7.079	0.122	$13\Delta t_1$
−0.238	−0.055	−0.574	1.503	1.442	−0.310	−7.675	−3.709	−0.072	$14\Delta t_1$
−0.332	−0.067	−0.616	1.511	1.425	−0.232	−4.337	−0.079	0.100	$15\Delta t_1$

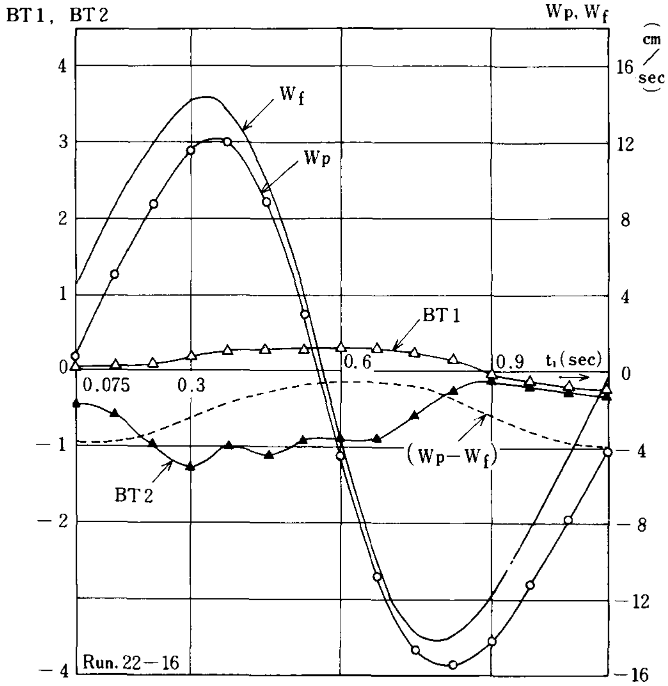
Table 1 Results of numerical calculation

(3) h = 30cm, z = −25cm, kx = $3\pi/4$, H = 7.04cm, T = 1.15 sec (Run. 23–11)

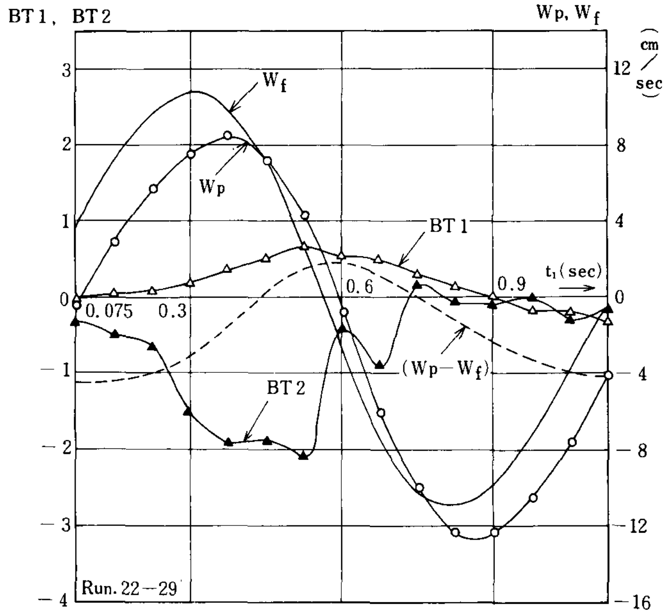
$BT1$	VM	RE	PR	FP	$BT2$	$W_p(\frac{cm}{sec})$	$W_f(\frac{cm}{sec})$	$BT2$-$BT1$	t
0.015	0.042	−0.515	0.503	0.607	−0.631	−2.288	1.268	−0.647	Δt_1
0.036	0.056	−0.481	0.381	0.510	−0.704	−0.989	2.333	−0.740	$2\Delta t_1$
0.065	0.073	−0.436	0.206	0.364	−0.796	0.028	3.039	−0.861	$3\Delta t_1$
0.091	0.064	−0.388	−0.018	0.113	−0.807	0.584	3.265	−0.898	$4\Delta t_1$
0.149	0.135	−0.319	−0.262	0.008	−1.086	0.725	2.928	−1.235	$5\Delta t_1$
0.201	0.117	−0.231	−0.476	−0.251	−1.112	0.443	2.039	−1.313	$6\Delta t_1$
0.189	−0.026	−0.199	−0.605	−0.679	−0.700	−0.640	0.736	−0.889	$7\Delta t_1$
0.186	−0.005	−0.210	−0.599	−0.631	−0.754	−2.166	−0.715	−0.940	$8\Delta t_1$
0.143	−0.081	−0.240	−0.456	−0.639	−0.496	−3.644	−1.986	−0.640	$9\Delta t_1$
0.143	−0.001	−0.268	−0.225	−0.235	−0.721	−4.661	−2.806	−0.863	$10\Delta t_1$
0.077	−0.103	−0.305	0.027	−0.186	−0.379	−5.150	−3.045	−0.456	$11\Delta t_1$
0.016	−0.087	−0.371	0.237	0.065	−0.369	−5.292	−2.727	−0.385	$12\Delta t_1$
0.041	0.031	−0.391	0.369	0.445	−0.716	−4.698	−1.998	−0.757	$13\Delta t_1$
0.004	−0.036	−0.393	0.424	0.364	−0.511	−3.757	−1.043	−0.515	$14\Delta t_1$
−0.082	−0.061	−0.426	0.418	0.307	−0.401	−2.975	−0.028	−0.320	$15\Delta t_1$

(4) h = 30cm, z = −25cm, k_x = $3\pi/4$, H = 12.28cm, T = 1.15 sec (Run. 23–34)

$BT1$	VM	RE	PR	FP	$BT2$	$W_p(\frac{cm}{sec})$	$W_f(\frac{cm}{sec})$	$BT2$-$BT1$	t
−0.014	−0.040	−0.472	0.829	0.776	−0.436	−1.224	2.039	−0.421	Δt_1
0.025	0.104	−0.450	0.723	0.962	−0.893	0.800	3.909	−0.918	$2\Delta t_1$
0.038	0.034	−0.402	0.503	0.590	−0.719	2.608	5.387	−0.756	$3\Delta t_1$
0.044	0.016	−0.385	0.117	0.154	−0.668	3.474	6.134	−0.713	$4\Delta t_1$
0.019	−0.060	−0.400	−0.411	−0.550	−0.401	3.013	5.779	−0.420	$5\Delta t_1$
0.009	−0.021	−0.428	−0.935	−1.011	−0.475	1.196	4.156	−0.484	$6\Delta t_1$
−0.016	−0.053	−0.454	−1.246	−1.399	−0.340	−1.610	1.528	−0.324	$7\Delta t_1$
0.009	0.048	−0.456	−1.206	−1.148	−0.651	−4.576	−1.427	−0.659	$8\Delta t_1$
−0.012	−0.038	−0.452	−0.839	−0.946	−0.403	−7.015	−3.891	−0.391	$9\Delta t_1$
−0.001	0.018	−0.459	−0.310	−0.283	−0.587	−8.446	−5.275	−0.586	$10\Delta t_1$
−0.003	−0.004	−0.454	0.179	0.178	−0.542	−8.568	−5.433	−0.538	$11\Delta t_1$
−0.018	−0.021	−0.462	0.503	0.477	−0.492	−7.806	−4.611	−0.474	$12\Delta t_1$
0.021	0.048	−0.453	0.646	0.768	−0.718	−6.356	−3.227	−0.739	$13\Delta t_1$
0.016	−0.006	−0.438	0.675	0.687	−0.569	−4.661	−1.635	−0.585	$14\Delta t_1$
0.001	−0.011	−0.443	0.654	0.655	−0.547	−3.098	−0.033	−0.548	$15\Delta t_1$

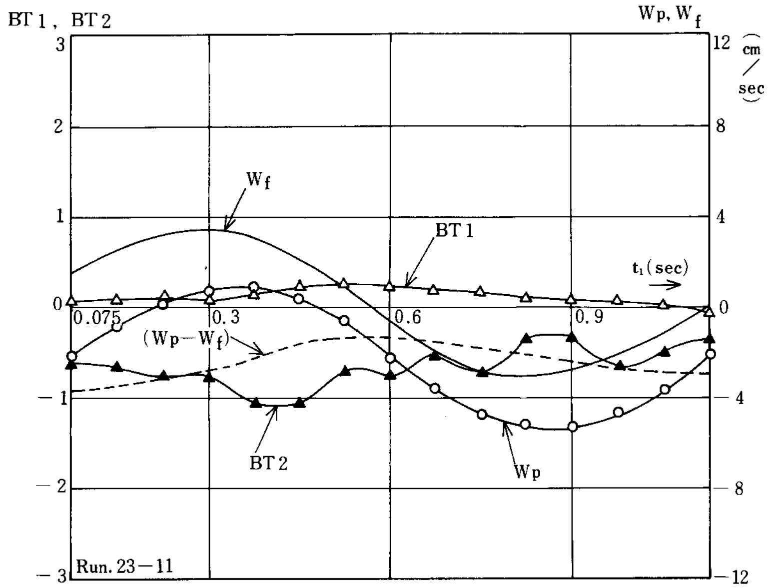


(1) h = 30cm, z = −15cm, kx = $3\pi/4$, H = 13.75cm, T = 1.15 sec.

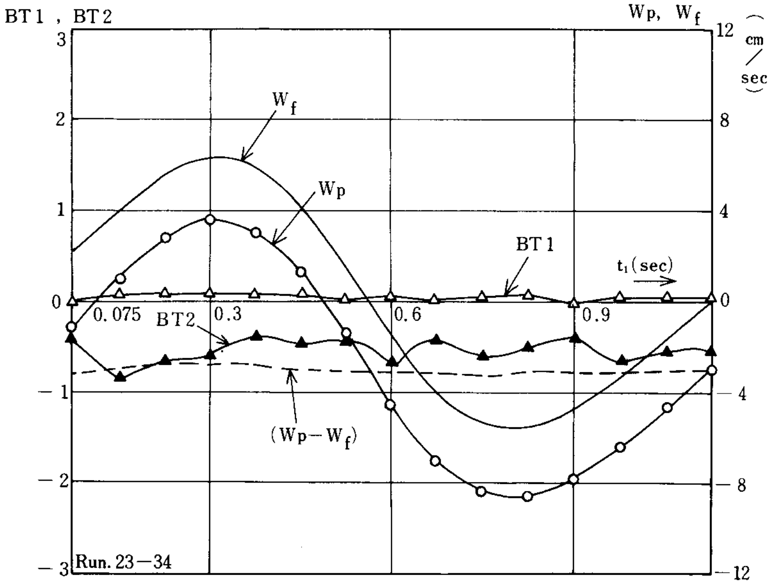


(2) h = 30cm, z = −15cm, kx = $3\pi/4$, H = 8.19cm, T = 1.15 sec.

Fig. 1 Comapriosn of the values of the Basset term calculated by the two methods



(3) $h = 30\text{cm}, z = -25\text{cm}, kx = 3\pi/4, H = 7.04\text{cm}, T = 1.15$ sec.



(4) $h = 30\text{cm}, z = -25\text{cm}, kx = 3\pi/4, H = 12.28\text{cm}, T = 1.15$ sec.

Fig. 1 Comparison of the values of the Basset term calculated by the two methods