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Unsteady Forced Convection around a Sphere Immersed in a Porous Medium at Large Peclet Number

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The unsteady forced convection around a sphere immersed in a fluid-saturated porous medium is investigated at large Peclet number. It is assumed that the sphere is suddenly heated and, subsequently, maintains a constant temperature over the surface and that the fluid in a porous medium flows according to Darcy's law.

1. Introduction

Convection heat transfer around a body embedded in a fluid-saturated porous medium is a very important subject area because of the wide applications in geophysics and engineering[1,2]. The present paper is concerned with forced convection around a sphere immersed in a porous medium. Previous studies on forced convection around a sphere in a porous medium have been made by Sano[3,4] and Cheng[5]. In references [3] and [5], the steady-state convection around an isothermal sphere is considered and asymptotic solutions for small[3] and large[5] Peclet numbers are obtained, assuming a Darcy flow for the velocity field. In reference [4], on the other hand, asymptotic solution for small Peclet number is obtained for unsteady forced convection around a sphere which is suddenly heated and, subsequently, maintains a constant temperature over the surface. No solutions for large Peclet number have been obtained for unsteady forced convection around a sphere.

The purpose of the present paper is to give asymptotic solution for large Peclet number for the unsteasy forced convection problem around a sphere immersed in a Darcy flow. As in reference [4], the sphere is assumed to be suddenly heated and, subsequently, to maintain a constant temperature over the surface.

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2. Governing Equations

We assume that the superficial velocity of the flow far upstream is uniform ($=U_\infty$) and that initially, the surface of the sphere and the surroundings are at the same temperature T_∞, whereupon at time $\tau'=0$ the surface temperature is suddenly changed to a constant value T_w. The energy equation governing the temperature field around the sphere can be written in non-dimensional form as

$$Pe\left(\frac{\partial t}{\partial \tau}+u\frac{\partial t}{\partial r}+\frac{v}{r}\frac{\partial t}{\partial \theta}\right)=\nabla^2 t, \tag{1}$$

$$u=(1-r^{-3})\cos\theta,\ v=-\frac{1}{2}(2+r^{-3})\sin\theta, \tag{2}$$

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right), \tag{3}$$

where non-dimensional quantities are defined as

$$\left.\begin{array}{c} r=\dfrac{r'}{r_0},\ u=u\dfrac{u'}{U_\infty},\ v=\dfrac{v'}{U_\infty},\ \tau=\dfrac{U_\infty\rho_f c_f}{r_0\rho_c c_c}\tau' \\ t=\dfrac{t'-T_\infty}{T_w-T_\infty},\ \alpha=\dfrac{\lambda_c}{\rho_f c_f},\ Pe=\dfrac{U_\infty r_0}{\alpha}. \end{array}\right\} \tag{4}$$

In the above equations, t' is the locally averaged temperature, τ' the time, (r', θ, φ) spherical coordinate with $r'=0$ at the center of the sphere and $\theta=0$ in the direction of uniform flow, u' and v' the superficial velocity in r'- and θ- directions, respectively, r_0 the radius of the sphere, ρ_f and c_f the density and specific heat of the fluid, ρ_c, c_c and λ_c the density, specific heat and effective thermal conductivity of the saturated porous medium.

The non-dimensional initial and boundary conditions are

$$\left.\begin{array}{ll}\tau<0 & t=0, \\ \tau\geq 0 & t=1 \ at \ r=1 \\ & t\to 0 \ as \ r\to\infty.\end{array}\right) \tag{5}$$

3. Asymptotic Solution for Large Peclet Number

We shall now proceed to obtain asymptotic solution of the energy equation (1) for large Peclet number. By introducing the following variable

$$Y=Pe^{1/2}(r-1), \tag{6}$$

we can write Eq. (1) as

$$\frac{\partial t}{\partial \tau}+(3Y+O(Pe^{-\frac{1}{2}}))\cos\theta\frac{\partial t}{\partial Y} -\frac{3}{2}(1+O(Pe^{-\frac{1}{2}}))\sin\theta\frac{\partial t}{\partial\theta}=\frac{\partial^2 t}{\partial Y^2}+O(Pe^{-\frac{1}{2}}). \tag{7}$$

In the limit $Pe\to\infty$, Eq. (7) becomes

$$\frac{\partial t}{\partial \tau}+3Y\cos\theta\frac{\partial t}{\partial Y}-\frac{3}{2}\sin\theta\frac{\partial t}{\partial\theta}=\frac{\partial^2 t}{\partial Y^2}, \tag{8}$$

with

$$\left.\begin{array}{l} t=1 \ at \ Y=0 \\ t\to 0 \ as \ Y\to\infty.\end{array}\right) \tag{9}$$

We now assume that

$$t=t(\eta) \tag{10}$$

and

$$\eta=\frac{Y}{\delta(\theta,\tau)}, \tag{11}$$

where δ is an unknown function of θ and τ and may be considered to be proportional to the thickness of the thermal boundary layer. In terms of Eqs. (10) and (11), Eq. (9) may be written as

$$\frac{d^2t}{d\eta^2}+\eta\frac{dt}{d\eta}\left(\frac{1}{2}\frac{\partial\delta^2}{\partial\tau}-3\delta^2\cos\theta-\frac{3}{4}\sin\theta\frac{\partial\delta^2}{\partial\theta}\right)=0. \tag{12}$$

In order that t is a function only of η, we have to require that

$$\frac{1}{2}\frac{\partial\delta^2}{\partial\tau}-3\delta^2\cos\theta-\frac{3}{4}\sin\theta\frac{\partial\delta^2}{\partial\theta}=\beta \ (constant). \tag{13}$$

This equation determines the unknown function $\delta(\theta,\tau)$. The solution of Eq. (12) satisfying the boundary condition

$$\left.\begin{array}{l} t=1 \ at \ \eta=0 \\ t\to 0 \ as \ \eta\to\infty\end{array}\right) \tag{14}$$

is

$$t=erfc\left(\sqrt{\frac{\beta}{2}}\eta\right). \tag{15}$$

Equation (13) can be solved using the method of characteristics. It is equivalent to a system of total differential equations as

$$\frac{d\tau}{2}=\frac{d\theta}{-3\sin\theta}=\frac{d\delta^2}{\beta+3\delta^2\cos\theta}. \tag{16}$$

From this equation, we can easily obtain

$$\tau=-\frac{2}{3}\ln\left(\tan\frac{\theta}{2}\right)+C_1 \tag{17}$$

and

$$\delta^2=\frac{4\beta}{9}\frac{1}{\sin^4\theta}(3\cos\theta-\cos^3\theta+C_2), \tag{18}$$

where C_1 and C_2 are integral constants. From Eqs. (17) and (18), we have the following general solution of Eq. (13).

$$\delta^2=\frac{4\beta}{9}\frac{1}{\sin^4\theta}\left\{3\cos\theta-\cos^3\theta+\phi\left(\frac{3}{2}\tau+\ln\left(\tan\frac{\theta}{2}\right)\right)\right\}, \tag{19}$$

where ϕ denotes an arbitrary function. The requirement that $\delta^2=0$ when $\tau=0$ gives,

$$\phi\left(\ln\left(\tan\frac{\theta}{2}\right)\right)=\cos^3\theta-3\cos\theta. \tag{20}$$

Using the relation

$$\cos\theta=\frac{1-\exp\{2\ln\tan(\theta/2)\}}{1+\exp\{2\ln\tan(\theta/2)\}}, \tag{21}$$

we can determine the function $\phi(x)$ from Eq. (20) as

$$\phi(x)=\frac{1-\exp(2x)}{1+\exp(2x)}\left\{\left(\frac{1-\exp(2x)}{1+\exp(2x)}\right)^2-3\right\}. \tag{22}$$

Thus, δ has been determined completely and we have

$$t=erfc\,\frac{3Pe^{1/2}(r-1)\sin^2\theta}{2\sqrt{2\{3\cos\theta-\cos^3\theta+\phi(\frac{3}{2}\tau+\ln(\tan\frac{\theta}{2}))\}}}. \tag{23}$$

It is seen that an arbitrary constant β disappears in Eq. (23). For $\tau\to\infty$, Eq. (23) becomes

$$t=erfc\,\frac{3Pe^{1/2}(r-1)\sin^2\theta}{2\sqrt{2\{3\cos\theta-\cos^3\theta+2\}}}, \tag{24}$$

which agrees with the soluton given by Cheng[5] for the steady state.

4. Discussion

Figure 1 shows the isothermal lines for $Pe=1000$ calculated from the solutions obtained in the preceding section. The isotherms are drawn for $t=0.8$, 0.5, 0.3

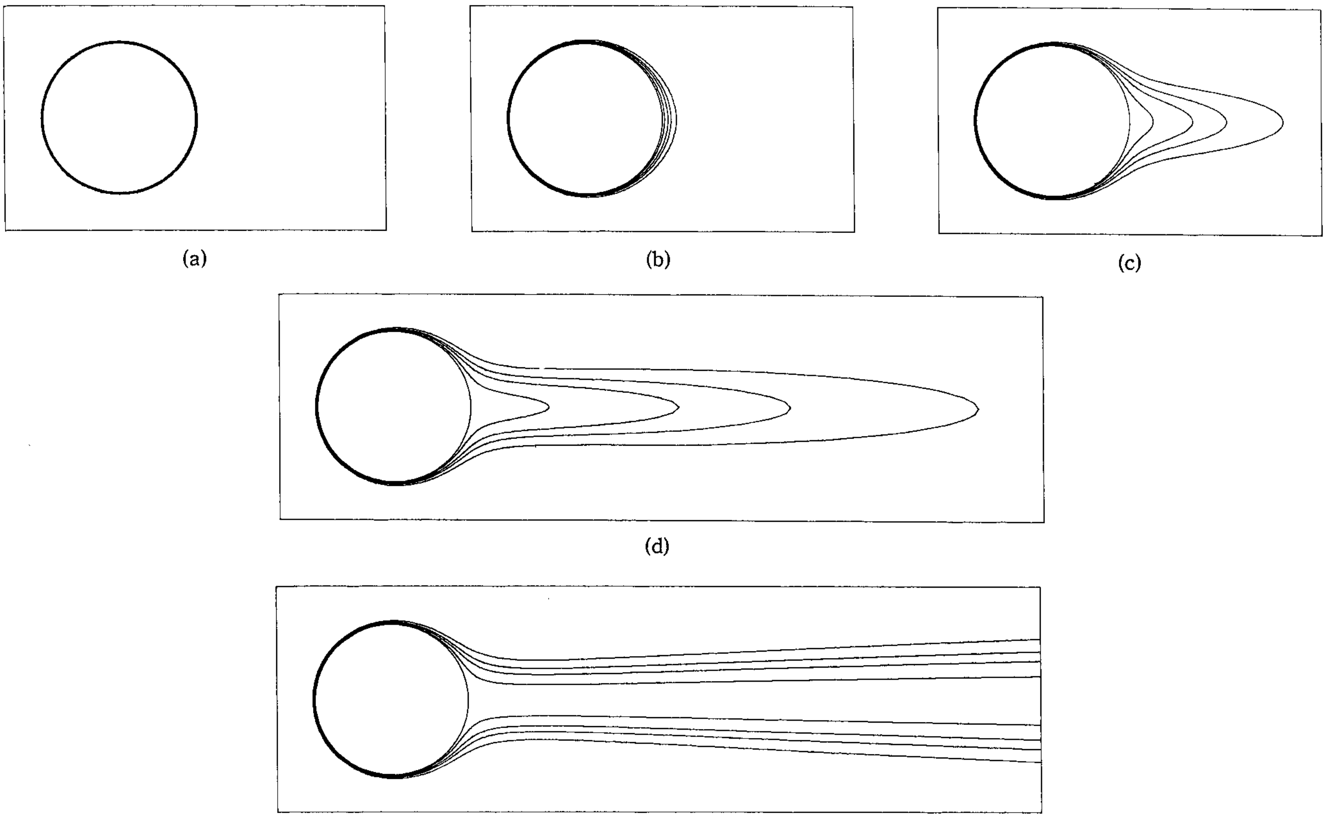

Fig. 1 Isothermal lines for Pe=1000; (a)$\tau=0.1$, (b)$\tau=0.6$, (c)$\tau=1.4$, (d)$\tau=1.8$, (e)$\tau\to\infty$

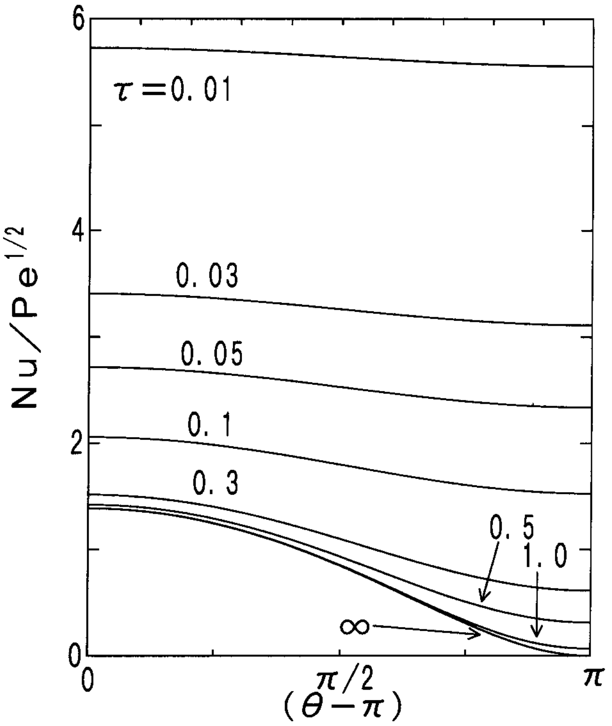

Fig. 2 Distribution of the local Nusselt number

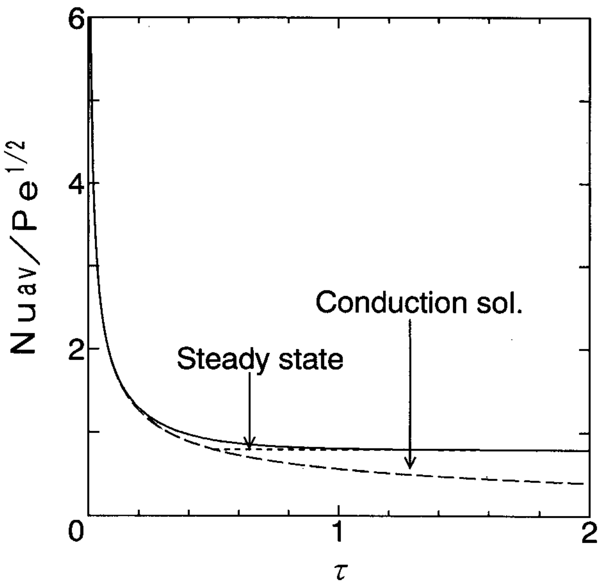

Fig. 3 Timewise variation in the mean Nusselt number

and 0.1. For $\tau=0.1$, isothermals are nearly concentric, suggesting that the heat transfer process is dominated mainly by conduction. As τ increases, isothermals begins to grow in the downstream direction owing to the effect of convection.

The local Nusselt number defined by $Nu=hr_0/\lambda_c$, where $h=-(\lambda_c/(T_w-T_\infty))(\partial T'/\partial r')_{r'=r_0}$ is the heat transfer coeficient, may be calculated from Eq. (23) as

$$Nu=\frac{Pe^{1/2}\sin^2\theta}{\sqrt{2\pi\{3\cos\theta-\cos^3\theta+\phi(\frac{3}{2}\tau+\ln(\tan\frac{\theta}{2})\}}}. \quad (25)$$

For $\tau\to\infty$ Eq. (25) becomes

$$Nu=\frac{Pe^{1/2}\sin^2\theta}{\sqrt{2\pi\{3\cos\theta-\cos^3\theta+2\}}}, \quad (26)$$

which agrees with the steady-state result by Cheng[5].

Figure 2 shows the distribution of $Nu/Pe^{1/2}$ for several value of τ. It is seen that steady-state distribution is almost achieved at $\tau=1$, especially at the front side of the sphere.

Figure 3 shows the relation between Nu_{av} and τ, where Nu_{av} is the mean Nusselt number averaged over the surface and is calculated numerically from the relation

$$Nu_{av}=\frac{1}{2}\int_0^\pi Nu\,\sin\theta d\theta. \quad (27)$$

For comparison, the conduction solution is also shown in the figure. It is seen that Nu_{av} decreases monotonously to its steady value as τ increases: no oscillatory behavior can be found in the transient process. Furthermore, it is seen that, for $\tau\leq 0.2$, the present result almost agrees with the conduction solution and, as τ increases, the difference between them becomes larger. This result is consistant with that obtained previously from Fig. 1.

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