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Nonlinear Distortion due to Laser Diodes in Fiber-Optic CDMA Systems

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In this paper, we analyze the effects of the nonlinearity in a fiber-optic code division multiple access (CDMA) communication system using a laser diode, assuming that nonlinearity is represented by the power series of the injection current of the laser diode, where we use the current transformed into unipolar one. Furthermore, we use the terms up to the 3 rd-order of the series to express the distorted signal, and propose the calculation formula of the power spectral density of the despread signal in the correlator to compute the S/N of the system. We also justify our formula by comparing the calculated results with the simulated ones for typical coefficients of the series. From the calculation results of the desired and interference components and the signal-to-interferece radio, it is clarified that the level of the components fluctuates considerably with the simultaneous users, that the effects of the 2 nd- and 3 rd- order intermodulation products on the S/N are insignificant, and that the degradation of the S/N by the increase of the users is almost the same as the case of the conventional RF spread spectrum systems.

1. Introduction

Fiber-optic communications using spread spectrum techniques have attracted many investigator's attentions. Various optical code division multiple access (CDMA) systems have been proposed for LAN applications[1-3]. These systems are performed by direct or phase modulator. We concern with asynchronous CDMA systems using a laser diode as the direct modulator. The laser injection current consists of a dc and the CDMA signal components. At a user's receiver, the optical signal through the optical fiber is converted to the electrical CDMA signal using a photodiode. And the user's desired data is detected by correlator. Where we use the individual user's information data $\{-1, 1\}$ and the spread spectrum code $\{-1, 1\}$ sequence used in conventional RF SS systems. And in order to apply these biplolar signals to the direct modulation systems, we transform the signals into the unipolar ones. Also, we assume that the nonlinearity in the system is represented by the power series of the injection current[4]. And we consider the nonlinear effects by using the analytical method of the linear, the 2 nd- and 3 rd- order intermodulation products components in the despread signal[5], which is developed from Bennett[6], and compare the calculated results of the power spectral densities (psds) of the signal with the simulated ones. Furthermore, using the calculated results, we clarify the fluctuations of the desired and interference component levels and the signal-to-interference ratios which are caused by the nonlinearity.

2. System Analysis

In our analysis, we set the following assumptions.

(a) The nonlinear effects in the optical fiber and the photodiode at the receiver are negligible.

(b) The input signal to the laser diode is noise free.

(c) The information data sequence of the individual user, $(b_m^{(k)})$, $b_m^{(k)} \in \{-1, 1\}$, $E[b_m^{(k)}]=0$, and $E[b_n^{(k)} \cdot b_m^{(k)}]=0$ for $n \neq m$, where $E[X]$ denotes the ensemble average of X.

We define the spread spectrum (SS) code sequence of the individual user with a period N_c as $(a_m^{(k)})$, $a_m^{(k)} \in \{-1, 1\}$, and the pulse durations of the transmission data signal $b_k(t)$ and the SS signal $a_k(t)$ as T and T_c respectively. The period of the SS code is $N_c = T/T_c$. The data signal divided by SS code of user k, $d^{(k)}(t)$, is expressed as

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$$d^{(k)}(t)=\sum_{n=-\infty}^{\infty}\sum_{l=0}^{Nc-1} a_l^{(k)}\cdot b_n^{(k)}\cdot g_c[t-(nT+lT_c)] \quad (1)$$

where $g_c(t)=1$ only for $0\leq t<T_c$. Furthermore, we transform the bipolar signal $d^{(k)}(t)$ into the unipolar signal $i^{(k)}(t)$ as follows.

$$i^{(k)}(t)=\frac{1}{2}[d^{(k)}(t)+1] \quad (2)$$

Then, the input signal of the laser diode as shown in Fig. 1, $i(t)$, is expressed as

$$i(t)=I\sum_{k=1}^{K} i^{(k)}(t-\tau_k) \quad (3)$$

where I is the amplitude of the input signal, K the number of the simultaneous users, T_k the transformer, and τ_k the delay representing the asynchronous system.

And we assume that the output signal $r(t)$ of the laser diode can be approximated by the power series up to the 3 rd-order of the input signal[4]. Let $\alpha_0\sim\alpha_3$ be the coefficients of the series, then

$$r(t)=\alpha_0+\alpha_1 i(t)+\alpha_2 i^2(t)+\alpha_3 i^3(t) \quad (4)$$

Now, set $d_t^{(k)}(t)=d^{(k)}(t-\tau_k)$, let $\Sigma_{k=1}^{K}$ and $\Sigma_{i=1}^{K}\Sigma_{j=1}^{K}\cdots$ $(i\neq j\neq\cdots)$ be denoted by Σ_k and $\Sigma'_{i,j,\cdots}$ respectively, then the output signal $r(t)$ is expressed as

$$r(t)=\beta_0+r_c(t) \quad (5)$$

where

$$\beta_0=\alpha_0+2^{-1}\alpha_1 IK+2^{-2}\alpha_2 I^2K(K+1)+2^{-3}\alpha_3 I^3K^2(K+3) \quad (6)$$

and assume that

$$\left.\begin{array}{l}\beta_1=2^{-1}[\alpha_1 I+\alpha_2 I^2K+2^{-2}\alpha_3 I^3(3K^2+3K-2)]\\ \beta_2=2^{-2}[\alpha_2 I^2+2^{-1}3\,\alpha_3 I^3K],\ K\geq 2\\ \beta_3=2^{-3}\alpha_3 I^3,\ K\geq 3\end{array}\right\} \quad (7)$$

then

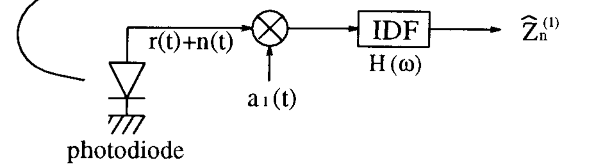

Fig. 1 Fiber-optic CDMA communication system.

$$r_c(t)=\beta_1\sum_k d_t^{(k)}+\beta_2\sum_{j,k}{}' d_t^{(j)} d_t^{(k)}+\beta_3\sum_{i,j,k}{}' d_t^{(i)} d_t^{(j)} d_t^{(k)} \quad (8)$$

And provided that the Gaussian noise $n(t)$ with the psd $W_N(\omega)$ and the mean zero is added on the transmission line, the inputs of the correlators on the receive side are expressed by $r(t)+n(t)$.

The output signal of the multiplier in the correlator of user 1, that is, despread signal, $u(t)$, is expressed as

$$u(t)=[r(t)+n(t)]a_1(t) \quad (9)$$

where $a_1(t)$ is the replica of the code signal for spreading the spectrum of the user 1's data signal on the transmission side.

Let $u_d(t)$ and $u_c(t)$ be the average and the continuous components of despread signal $u(t)$ respectively, then

$$u_d(t)=E[u(t)]=\beta_0 a_1(t) \quad (10)$$

$$u_c(t)=u(t)-E[u(t)]=[r_c(t)+n(t)]a_1(t) \quad (11)$$

The signal-to-noise ratio (S/N) of the system under the consideration is examined by using the psd of the signal $u(t)$. The spectral shaping function $M^{(k)}(\omega)$ shown in Ref. (5) becomes 1 under the assumptions for the data sequence $(b_m^{(k)})$, where $k=1, 2, \cdots, K$.

Since the term $u_d(t)$ is a periodic function with the period T as shown in (10), the line spectral density caused by the term, $W_L^{(1)}(\omega)$, can be expressed by the powers of the harmonic components of the frequency $\omega_d=2\pi/T$ as

$$W_L^{(1)}(\omega)=\frac{\beta_0^2}{N_c}\sum_{n=-\infty}^{\infty} Sa^2\left(\frac{\omega T_c}{2}\right)S^{(1)}(\omega T_c)\cdot\delta(\omega-n\omega_d). \quad (12)$$

The psd of the continuous component $u_c(t)$ can be expressed by those of the desired signal, the interference, and noise components as the followings.

Assume that $d_t^{(1)}$ and $a_1(t)$ are perfectly synchroized, then the psd of the desired signal $\beta_1 d_t^{(1)}a_1(t)$, $W^{(1)}(\omega)$, is similar to that of the data signal $b_1(t)$ as

$$W^{(1)}(\omega)=T\beta_1^2 Sa^2\left(\frac{\omega T}{2}\right) \quad (13)$$

where

$$Sa(x)=\sin x/x$$

The psd of the interference component $\beta_1 d_t^{(k)}a_1(t)$ from other user k, $W_I^{(k)}(\omega)$, is expressed as

$$W_I^{(k)}(\omega)=\frac{T\beta_1^2}{N_c^2}\sum_{l=-\infty}^{\infty}\Phi^{(1)}\left(\frac{2\pi l}{N_c}\right)\cdot\Phi^{(k)}\left(\omega T_c-\frac{2\pi l}{N_c}\right),\quad k\geq 2 \quad (14)$$

Furthermore the psds of the 2 nd- and 3 rd- order intermodulation product components $\beta_2 d_t^{(j)} d_t^{(k)} a_1(t)$ ($j \neq k$) and $\beta_3\, d_t^{(i)} d_t^{(j)} d_t^{(k)} a_1(t)$ ($i \neq j \neq k$), $W_{IM}^{(i,j)}(\omega)$ and $W_{IM}^{(i,j,k)}(\omega)$, are expressed as

$$W_{IM}^{(j,k)}(\omega)=\frac{T\beta_2^2}{2\pi N_c^3}\sum_{l=-\infty}^{\infty}\Phi^{(1)}\left(\frac{2\pi l}{N_c}\right) \cdot\int_{-\infty}^{\infty}\Phi^{(j)}(\omega_1 T_c) \cdot\Phi^{(k)}\left[(\omega-\omega_1)\,T_c-\frac{2\pi l}{N_c}\right]d\omega_1 \qquad (15)$$

$$W_{IM}^{(i,j,k)}(\omega)=\frac{T^3\beta_3^2}{(2\pi)^2 N_c^4}\sum_{l=-\infty}^{\infty}\Phi^{(1)}\left(\frac{2\pi l}{N_c}\right) \cdot\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Phi^{(i)}(\omega_1 T_c)\,\Phi^{(j)}[(\omega_2-\omega_1)T_c] \cdot\Phi^{(k)}\left[(\omega-\omega_2)T_c-\frac{2\pi l}{N_c}\right]d\omega_1 d\omega_2. \qquad (16)$$

respectively, where we set

$$S^{(k)}(x)=1+\frac{2}{N_c}\sum_{m=1}^{Nc-1}\sum_{l=0}^{Nc-m-1}a_l^{(k)}a_{l+m}^{(k)}\cos(mx). \qquad (17)$$

$$\Phi^{(k)}(x)=Sa^2\left(\frac{x}{2}\right)S^{(k)}(x). \qquad (18)$$

The psds of the 2 nd- and 3 rd- order intermodulation products are expressed by $\Sigma'_{j,k}W_{IM}^{(j,k)}(\omega)$ and $\Sigma'_{i,j,k}W_{IM}^{(i,j,k)}(\omega)$ respectively. Therefore, the psd of the interference components, $W_I(\omega)$, becomes

$$W_I(\omega)=\sum_{k(\geq 2)}W_I^{(k)}(\omega)+\sum_{j,k}{}'W_{IM}^{(j,k)}(\omega) +\sum_{i,j,k}{}'W_{IM}^{(i,j,k)}(\omega). \qquad (19)$$

The psd of the noise component $n(t)\,a_1(t)$, $W_n(\omega)$, becomes

$$W_N(\omega)=\frac{1}{N_c}\sum_{l=-\infty}^{\infty}W_n\left(\omega-\frac{2\pi l}{T}\right)\Phi^{(1)}\left(\frac{2\pi l}{N_c}\right). \qquad (20)$$

The psd of $u_c(t)$, $W_c(\omega)$, becomes as

$$W_c(\omega)=W^{(1)}(\omega)+W_I(\omega)+W_N(\omega) \qquad (21)$$

and that of the despread signal $u(t)$, $W(\omega)$, is expressed as

$$W(\omega)=W_L^{(1)}(\omega)+W_c(\omega) \qquad (22)$$

The desired signal component $\hat{Z}_n^{(1)}$ at the output of the integrate-and-dump filter (IDF) in the correlator at time $(n+1)\,T$ becomes

$$\hat{Z}_n^{(1)}=\beta_1\,b_n^{(1)} \qquad (23)$$

The nomalized system function of the IDF, $H(\omega)$, is expressed as $H(\omega)=Sa(\omega T/2)\,e^{-j\omega T/2}$. The desired signal power $(\hat{\sigma}_S^{(1)})^2$, the line spectral component power $(\hat{\sigma}_L^{(1)})^2$, the interference signal power $(\hat{\sigma}_I)^2$, and the noise power σ_N^2 at the output of the IDF $H(\omega)$ are

$$\left.\begin{aligned}(\hat{\sigma}_S^{(1)})^2&=E[(\hat{Z}_n^{(1)})^2]=\beta_1^2\\ (\hat{\sigma}_L^{(1)})^2&=\frac{1}{2\pi}\int_{-\infty}^{\infty}W_L^{(1)}(\omega)\,|H(\omega)|^2 d\omega\\ (\hat{\sigma}_I)^2&=\frac{1}{2\pi}\int_{-\infty}^{\infty}W_I(\omega)\,|H(\omega)|^2\,dw\\ \sigma_N^2&=\frac{1}{2\pi}\int_{-\infty}^{\infty}W_N(\omega)\,|H(\omega)|^2\,d\omega\end{aligned}\right\} \qquad (24)$$

respectively, and the S/N, $\hat{SNR}_1$, is given by

$$\hat{SNR}_1=\frac{(\hat{\sigma}_S^{(1)})^2}{[(\hat{\sigma}_L^{(1)})^2+(\hat{\sigma}_I)^2+\sigma_N^2]}. \qquad (25)$$

3. Calculation Results

In the following calculations, we assume that the Gold code sequences are used as spread spectrum code

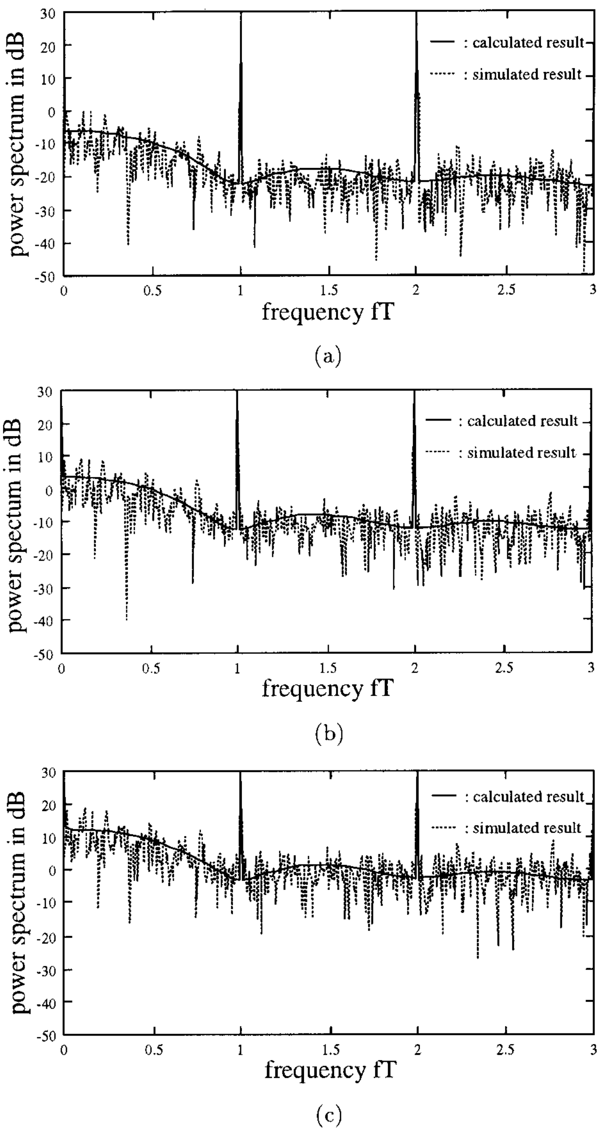

Fig. 2 The power spectrums of despread signals, (a): $i(t)\,a_1(t)$, (b): $i_2(t)\,a_1(t)$, and (c): $i_3(t)\,a_1(t)$; Gold code, $N^c=63$, $K=3$.

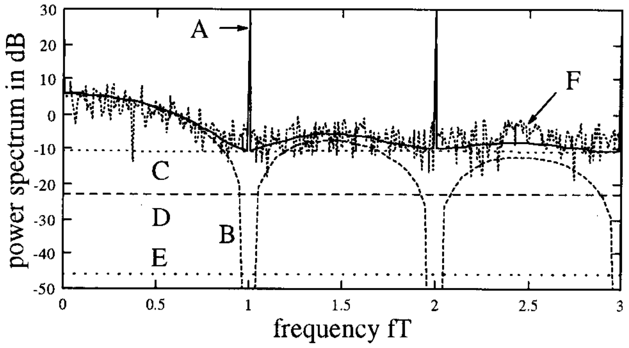

Fig. 3 The power spectrum of despread signal; Gold code, $N_c=63$, $K=3$; A: $2\ [W^{(1)}(\omega)+W_L^{(1)}(\omega)+W_I(\omega)]/T$, B: $2W^{(1)}(\omega)/T$, C: $2\Sigma_{k(\geq 2)}\ W_I^{(k)}(\omega)/T$, D: $2\Sigma'_{j,k}\ W_{IM}^{(i,k)}/T$, E: $2\Sigma'_{i,j,k}\ W_{IM}^{(i,j,k)}/T$, F: simulated results.

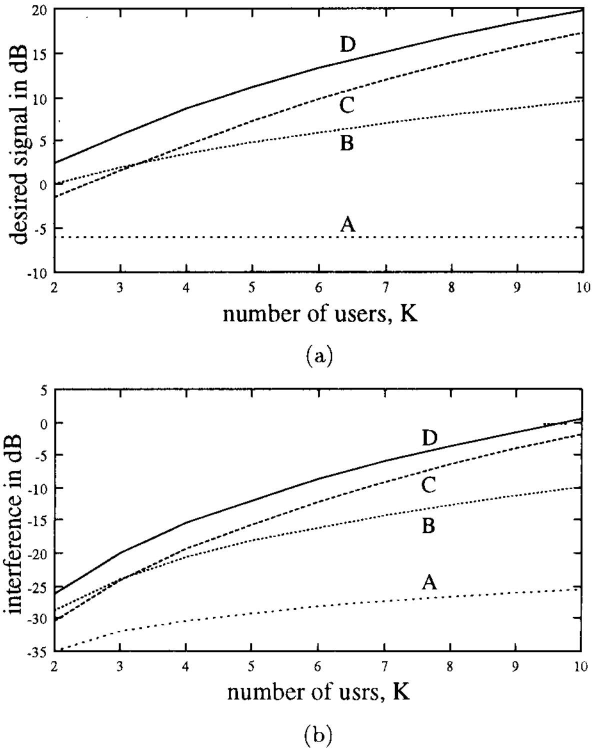

Fig. 4 (a) Desired and (b) interference, $\hat{\sigma}_S^{(1)}$ and $\hat{\sigma}_I$; Gold code, $N_c=511$; A: $\alpha_1=1$, $\alpha_2=\alpha^3=0$; B: $\alpha_1=1$, $\alpha_2=1/2$, $\alpha_3=0$; C: $\alpha_1=1$, $\alpha_2=0$, $\alpha_3=1/6$; D: $\alpha_1=1$, $\alpha_2=1/2$, $\alpha_3=1/6$.

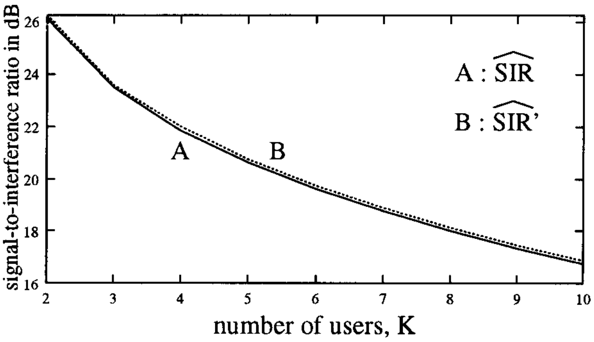

Fig. 5 Signal-to-interference ratio; Gold code, $N_c=511$; A: $\widehat{SIR}_1=(\hat{\sigma}_S^{(1)})^2/[(\hat{\sigma}_L^2)^2+\hat{\sigma}_I^2]$, B: $\widehat{SIR}_1'=(\hat{\sigma}_S^{(1)})^2/[(\hat{\sigma}_L^{(1)})^2+(\hat{\sigma}_I)^2]$.

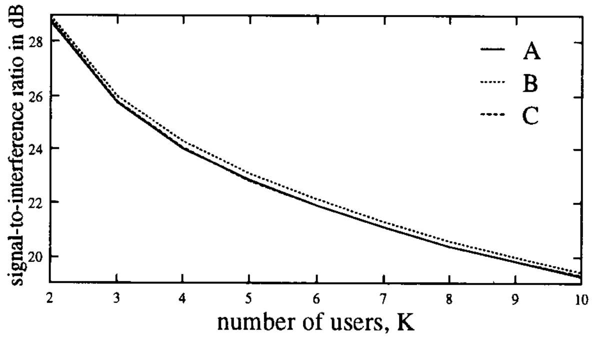

Fig. 6 Signal-to-interference ratio; Gold code, $N_c=511$; A: $\widehat{SIR}_1=(\hat{\sigma}_S^{(1)})^2/\hat{\sigma}_I^2)^2$, B: $\widehat{SIR}'=(\hat{\sigma}_S^{(1)})^2/(\hat{\sigma}'_I)^2$, C: $3\,Nc/2\ (K-1)$.

sequence $(a_m^{(k)})$ of the systems and the amplitude of the injcetion current $I=1$. Futhermore, as an example, we assume that the nonlinearity of the laser diode is represented by the terms up to the 3 rd- order of exponential function expanded in a series, then the coefficients $\alpha_0=0$, $\alpha_1=1$, $\alpha_2=1/2$, and $\alpha_3=1/6$. Figure 2 shows the calculated and simulated results of the psds of the first-, 2 nd-, and 3 rd- order terms, $i\,(t)\ a_1\,(t)$, $i^2\,(t)\ a_1\,(t)$, and $i^3\,(t)\ a_1\,(t)$ in Eq.(4) respectively, which are in good agreement. Also, these results make it clear that the characteristics of the nonlinear distortion due to the laser diode can be given by the presented calculation formulas of the psds and the S/N. Figure 3 shows the calculated results of the psd $[W^{(1)}(\omega)+W_L^{(1)}(\omega)+W_I(\omega)]/\mathrm{T}$ and the simulated results, which are in good agreement. Figure 4 (a) and (b) show that the levels of the desired and the interferene, $\hat{\sigma}_S^{(1)}$ and $\hat{\sigma}_I$, fluctuate considerably with the simultaneous users.

And we set the additive noise is zero, for examing the effects of the 2 nd- and 3 rd- order intermodulation products on the S/N, and calculate the $\widehat{SIR}_1=(\hat{\sigma}_S^{(1)})^2/[(\hat{\sigma}_L^{(1)})^2+\hat{\sigma}_I^2]$ and $\widehat{SIR}_1'=(\hat{\sigma}_S^{(1)})^2/[(\hat{\sigma}_L^{(1)})^2+(\hat{\sigma}'_I)^2]$ for the number of the users, K, where $(\hat{\sigma}'_I)^2=(1/2\,\pi)\ \int_{-\infty}^{\infty}[\Sigma_{k(\geq 2)}\ \mathrm{W}_I^{(k)}(\omega)\mid H\,(\omega)\mid^2]\ dw$, as shown in Fig. 5. Assume that the dc component β_0 of the electrical signal is cut off at the output of the photodiode, then the despread spectrum is represented by the continuous component $W_c\,(\omega)$. The signal-to-interference ratios in this case, $\widehat{SIR}$ and $\widehat{SIR}'$, are shown in Fig. 6, where $\widehat{SIR}=(\hat{\sigma}_S^{(1)})^2/(\hat{\sigma}_I)^2$ and $\widehat{SIR}'=(\hat{\sigma}_S^{(1)})^2/(\hat{\sigma}'_I)^2$. Futhermore, in the distortion free, the relationship $\widehat{SIR}' \approx 3\,Nc/2\,(K-1)$ can be abtained from the approximation formulas of the S/N in Ref. (7). The calculated results are shown in Fig. 6. The results show that the effects of the intermodulation products on the S/N are insignificant.

4. Conclusion

we transform the bipolar CDMA signals into the unipolar signals to use a laser diode in a fiber-optic communication system. And, in order to analyze the effects of nonlinearity of the system due to the laser diode, we derive the calculation formula of the psd of each component in the despread signal in the receiver. We show that the contribution of each component to the S/N is clarified by these formulas. The calculation results show that the effects of the 2 nd- and 3 rd-order intermodulation products on the S/N are insignificant and the line spectrum in the despread one causes some degradations for the S/N. The nonlinearity exists not only in the laser diode but also in the optical fiber and the photodiode in the receiver. It is a subject for further research to clarify the nonlinear effects in the receiver.

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