Blind Equalizers Using Signal Orthogonalization and Higher Order Statistics

Bulletin of University of Osaka Prefecture
Vol. 44, No. 1, 1995, pp. 7-12

Blind Equalizers Using Signal Orthogonalization and Higher Order Statistics

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(Received June 30, 1995)

This paper presents the studies aimed to improve the convergence characteristics of blind equalization. The blind equalizer which uses gradient algorithm to minimize nonlinear cost function normally has much slower convergence than the conventional adaptive equalizer which is based on MSE criterion. Moreover, it may converge to undesirable equilibrium unless its parameters are appropriately initialized. To cope with ill-conditioned channel situation which is the main reason for slow convergence in gradient algorithm, we apply orthogonalization to the channel and then use the mechanism of blind equalization to update the tap coefficients of the orthogonal signals. Furthermore we derived the condition for solving the lattice tap gain, using higher order cumulant. This result leads to an approximate solution of the blind equalizer and is used to setup the initial parameters. The significant improvement of convergence properties is demonstrated by computer simulation.

1. Introduction

The adaptive equalization techniques based on minimizing mean square errors have been well understood and widely applied to digital communication systems. Normally the system that uses this kind of algorithms requires an initial training sequence during the setup period. After the equalizer achieves converge, the training sequence is replaced by the received data under the assumption that incorrect decisions are negligible. However there are some applications where the mechanism of establishing equalization without training data, or the so called blind equalization, is expected[2].

The pioneering research on blind equalization was carried out by Sato who proposed the first equalizer for PAM data transmission[1]. Sato's work was late generalized in [2] [3] [4] for QAM communication systems. recently, many studies emphasize on analyzing the convergence properties of blind equalization[9,10].

The blind equalizers we mentioned here share the common feature that some modified cost functions related to higher (or lower) order statistics of channel output are employed, because the second order statistics of the channel output does not carry complete phase information if the input is not accessible. Because of the nature of nonlinear optimization, these algorithms are essentially associated with two problems. One is the possibility of being trapped into local minima, and another is the much lower convergence speed than conventional equalizers. There have been some studies on exploring the conditions of undesirable equilibria, but much less has been done on the issue of accelerating the convergence speed, which is very crucial in practice because the blind equalization algorithms might be too slow to be applied. Another category of blind equalization makes use of the technique of higher order statistics (or cumulants) which has received extensive attention in recent years in the fields of system identification, spectrum analysis and channel equalization. Unlike correlation function, higher order cumulants do carry both amplitude and phase information when the system is driven by independent, identical, distributed (IID) non-Gaussian input. Two excellent reviews in this subject can be found in [5] [6]. Cumulants based equalization was investigated in [7] [8] [11], however the increased amount of computation is quite considerable.

In this paper we present an approach aimed to improve the convergence properties of blind equalizers, using signal orthogonalization and higher order statistics. The channel amplitude is equalized by orthogonal conversion, and then the phase characteristics is equalized under the same criterion as normal blind algorithms. To avoid local minima and to accelerate the convergence further, cross-cumulant is used to

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initialize the coefficients of blind equalizers.

2. Blind Equalization

In our studies we use the system model depicted in Fig. 1, where $x(t)$ represents the transmitted data, $H(\omega)$ indicated the channel transfer function to be equalized, $y(t)$ is received signal from the channel and $z(t)$ expresses the output of the equalizer. Thus, the problem of blind equalization is described as to build the inverse channel transfer function $G(\omega)$ up to a linear phase shift, or equivalently to recover the the transmitted data with a fixed delay, where the knowledge of input data $x(t)$ should not be required except to make sure that $x(t)$ is non-Gaussian. (There is no solution to the problem if $x(t)$ is Gaussian, unless $H(\omega)$ is a minimum phase system.) The sufficient condition for constructing the blind equalizer $G(\omega)$ is proved in [3], i.e. the equalizer's output $z(t)$ will recover a delayed version of $x(t)$ if the distribution of $z(t)$ is made consistent with that of $x(t)$. Therefore the cost function of equalization can be defined as

$$J(\mathrm{G})=E\{\psi[z(t,\ G)\}=\int \psi(z)p(z,\ G)dz \tag{1}$$

where $p(z,\ G)$ is is distribution density of $z(t)$. It is expected that the cost function $J(\mathrm{G})$ achieve the global minima when $z(t)$ admits the distribution of $x(t)$. Gradient algorithms are commonly used to minimize the cost function. When the FIR filter is employed as the equalizer, i.e. $z(t)=\boldsymbol{G}^T\boldsymbol{Y}$, where $\boldsymbol{Y}=(y(t),\ y(t-1),\cdots\cdots,\ y(t-N+1)^T$, and $\boldsymbol{G}=(g_0,\ g_1,\cdots\cdots,\ g_{N-1})^T$, the equalizer's coefficients are updated according to

$$\frac{\partial J}{\partial \boldsymbol{G}}=E\left\{\psi'\frac{\partial z(t)}{\partial \boldsymbol{G}}\right\}=E[e\boldsymbol{Y}] \tag{2}$$

$$\boldsymbol{G}(n+1)=\boldsymbol{G}(n)-\mu e\boldsymbol{Y} \tag{3}$$

where $e(t,\ \boldsymbol{G})=\psi'$ is also referred as to pseudo-error signal with respect to that in MSE algorithm. The first blind equalization for PAM systems was studied by Sato who defined the cost function as

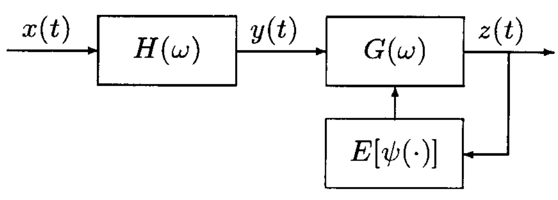

Fig. 1 Blind equalization

$$J(\boldsymbol{G})=E\left\{\frac{1}{2}z^2(t)-\alpha\ |\ z(t)\ |\ \right\} \quad \alpha=\frac{E[\ |\ x(t)\ |^2]}{E|\ x(t)\ |}$$

This cost function only needs the first and second order statistics of $z(t)$ and leads to very simple algorithm:

$$\boldsymbol{G}(n+1)=\boldsymbol{G}(n)-\mu\{z(t)-\alpha\ \mathrm{sign}[z(t)]\}\boldsymbol{Y} \tag{4}$$

It has been proved that Sato cost function has the only global minima at $G(\omega)=H^{-1}(\omega)$ when $x(t)$ is sub-Gaussian[3].

3. Blind Equalization Based on Signal Orthogonalization

The convergence speed of blind equalizers is mainly dominated by correlation matrices of channels. The equalization algorithm converges very slowly when the correlation matrix is ill-conditioned, i.e. the largest eigenvalue is much greater than the smallest one. This may happen if the channel is modeled as a lowpass filter.

To cope with the channel conditions, we apply the principle of blind equalization to a set of orthogonal signals instead of the channel output, as shown in Fig. 2. In the figure, the channel output $y(t)$ is converted into a set of base components $\boldsymbol{V}=(v_1(t),\ v_2(t),\cdots\cdots,\ v_{N-1}(t))^T$ by signal orthogonalization. The base signal $\boldsymbol{V}$ is of the property that its elements are orthogonal between each other, or

$$E[\boldsymbol{V}\boldsymbol{V}^T]=\sigma_x^2\boldsymbol{I} \tag{5}$$

while $\boldsymbol{I}$ is unit matrix and $\sigma_x=E[\ |\ x(t)\ |^2]$. The orthogonalization can be implemented by lattice filter with well-known Levinson-Durbin algorithm[12)]

In this scheme of blind equalizers, the orthogonalization essentially equalizes the amplitude characteristics of the channel. This procedure only required the

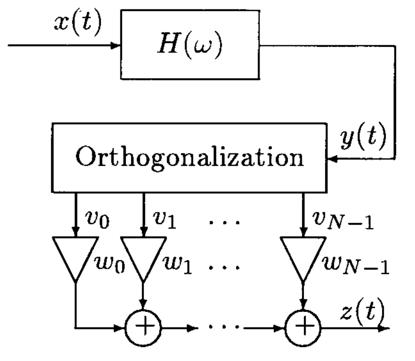

Fig. 2 Channel equalization using signal orthogonalization

second order statistics of $y(t)$ and its convergence is independent on the the channel situation. The overall phase characterizers is equalized by adjusting the tap gain $\boldsymbol{W}$, using nonlinear optimization techniques.

Since the orthogonalization is a linear conversion, the condition for recovering $x(t)$, as we mentioned before, remains valid. The cost function $J(\boldsymbol{W})$ can be adopted. Thus the algorithm of updating is given by

$$J(\boldsymbol{W})=E\{\psi[z(t, \boldsymbol{W})]\}=E[\psi' \boldsymbol{V}] \tag{6}$$

$$\boldsymbol{W}(n+1)=\boldsymbol{W}(n)-\mu\ \psi'\ \boldsymbol{V} \tag{7}$$

For Sato cost function we have

$$J(\boldsymbol{W})=E\left\{\frac{1}{2}z^2(t)-\alpha\ |\ z(t)|\ \right\} \tag{8}$$

$$\boldsymbol{W}(n+1)=\boldsymbol{W}(n)+\mu\{z(t)-\alpha\ \mathrm{sign}[z(t)]\ \boldsymbol{V} \tag{9}$$

For constant modulus (CM) criterion, the algorithm is expressed as

$$J(\boldsymbol{W})=\frac{1}{4}E\{[\ |\ z(t)\ |^2-\beta]^2\} \tag{10}$$

$$\boldsymbol{W}(n+1)=\boldsymbol{W}(n)+\mu y(t)[\ |\ z(t)\ |^2-\beta]\ \boldsymbol{V} \tag{11}$$

where $\beta=E\ |\ x(t)\ |^4/E\ |\ x(t)\ |^2$.

4. Initializing the Blind Equalizer Using Cross-Cumulant

The blind equalization that minimizes a nonlinear cost function always faces the problem of local minima. The analysis on the existence of undesirable equilibria is presented in [9]. To avoid the local optimization, the equalizer must be set initially close to the true solution. Now we will derive an approximate solution for the blind equalizer's coefficients $\boldsymbol{W}$ based on the higher order statistics. The equalizer is first initialized by the approximate result, then switches to the gradient algorithm to obtain more precise solution.

For communication systems the transmitted data are reasonably regarded as symmetric IID sequence. Thus 4th-order cumulant of $y(t)$ is appropriately chosen for blind identification and blind equalization, which is defined as

$$C_y^{(4)}(\tau_1,\tau_2,\tau_3)=m_y^{(4)}(\tau_1,\tau_2,\tau_3)-r_y(\tau_1)r_y(\tau_2-\tau_3)-r_y(\tau_2)r_y(\tau_1-\tau_3)-r_y(\tau_3)r_y(\tau_1-\tau_2) \tag{12}$$

where $m_y^{(4)}(\tau_1,\tau_2,\tau_3)=E[y(t)y(t+\tau_1)y(t+\tau_2)y(t+\tau_3)]$ is the 4th-order moment and $r(\tau)$ is the correlation function. For the details of the properties of 4th-order cumulant and its application, see [5][6].

Similarly, in our studies we define the 4th-order cross-cumulant of orthogonal signal $v_i(t)$ as

$$s_{i,j}=E[v_i^3(t)v_j(t)]-3E[v_j(t)v_j(t)]E[v^2(t)] \tag{13}$$

Under the assumptions that $x(t)$ is IID non-Gaussian, $\boldsymbol{V}$ is the orthogonal signal vector established from $x(t)$, and the system is free from additive noise (Theoretically Gaussian additive noise does not affect the estimates obtained from higher order cumulants[5] the relation between the equalizer's coefficients and 4th-order cross-cumulant, which is given by

$$\begin{pmatrix} s_{0,0} & s_{0,0} & \cdots & s_{0,N-1} \\ s_{0,1} & s_{0,1} & \cdots & s_{1,N-1} \\ \vdots & \vdots & & \vdots \\ s_{N-1} & s_{N-1,0} & \cdots & s_{N-1,N-1} \end{pmatrix} \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_{N-1} \end{pmatrix} = \gamma_x \begin{pmatrix} w_0^3 \\ w_1^3 \\ \vdots \\ w_{N-1}^3 \end{pmatrix} \tag{14}$$

The proof of the equation is given in Appendix. Since solving this nonlinear equation system is a quite delicate task, we will only explore the approximate solution of $\boldsymbol{W}$.

Let us examine (14). If there is an element in $\boldsymbol{W}$, for example w_k, which is dominant over others, i.e. $|\ w_k\ |>|\ w_j\ |$ for all $j\neq k$, then this dominant coefficient will be greatly enhanced in $\boldsymbol{W}_3=(w_0^3, w_1^3,\cdots\cdots, w^3{}_{N-1})^T$. In the other words, it is likely quite possible that $|\ w_k^3\ |\gg|\ w_j^3\ |$ in $\boldsymbol{W}_3$. Thus $\boldsymbol{W}_3$ would be almost determined only by w_k^3, and an approximate solution to (14) could be obtained by setting all w_j zero except $w_k=1$, i.e. $\boldsymbol{W}_3\approx(0,\cdots,1,\cdots,0)^T$. Therefore the coefficients $\boldsymbol{W}$ is calculated approximately from a linear equation system. This approximate result will be used to initialize the coefficients of blind equalizers.

5. Simulation

Computer simulation of the proposed blind equalization algorithms with Sato cost function is carried out for two channels $H_1(\omega)$ and $H_2(\omega)$. The equivalent models of channels are specified by lowpass and bandpass filters depicted in Fig. 3 respectively. Both channels are non-minimum systems, and eye patterns are completely closed. $H_1(\omega)$ is ill-conditioned in the sense of eigenvalue ratio, or $\lambda_{max}/\lambda_{min}=13.4$, when

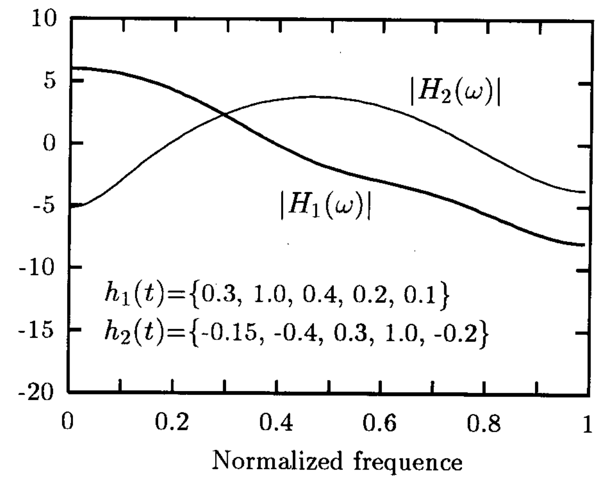

Fig. 3 Channel characteristics of $|H_1(\omega)|$ and $|H_2(\omega)|$.

$N=5$. For $H_2(\omega)$ the ratio is $\lambda_{max}/\lambda_{min}=4.2$. The transmitted data $x(t)$ is 8-level PAM sequence. The signal to noise ratio of channel output is set to $SNR=20dB$. The intersymbol interference of overall system $o(t)$ including channel and equalizer is measured according to

$$ISI=10\cdot\log\left(\frac{\sum_{i\neq m}o^2(i)}{o_m^2}\right)\quad o_m^2=\max_i\{o(i)\}$$

The convergence curves of equalization algorithms are shown in Fig. 4. To achieve the convergence level of $ISI=-20$dB for channel $H_1(\omega)$, the standard Sato algorithm expressed by (4) takes as many as 30,000 symbols, while the orthogonal algorithm reduces the number to 8,000. The approximate solution from 4th-order cumulant has more rapid convergence, but it is unable to suppress ISI to lower than -15dB. However if we use the cumulant solution obtained during

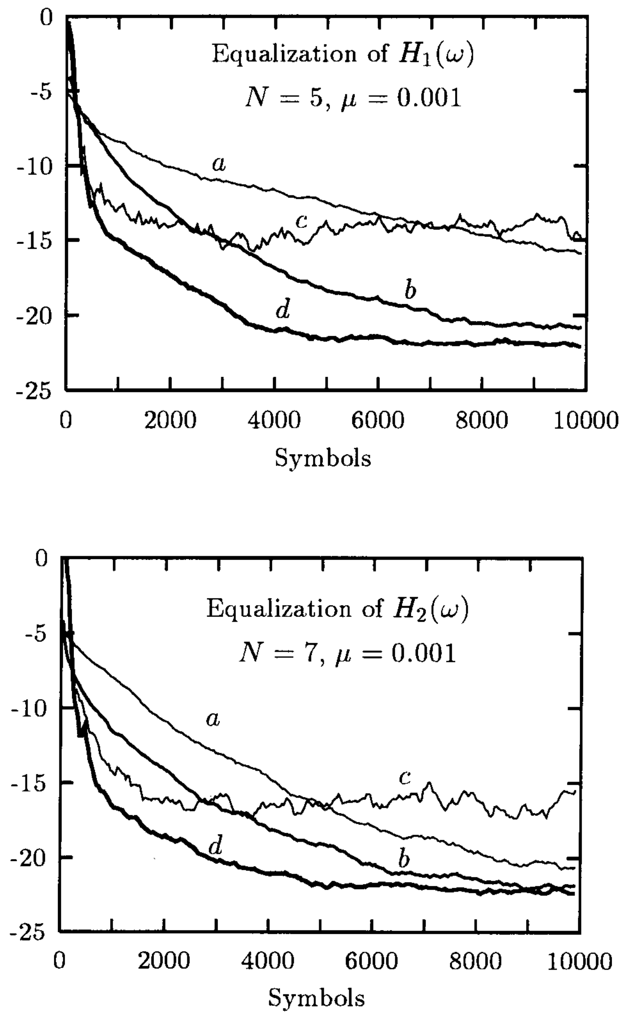

Fig. 4 Average intersymbol interference in dB. a) Sato algorithm; b) Orthogonal algorithm; c) Cumulant solution; d) Orthogonal algorithm initialized by cumulant solution

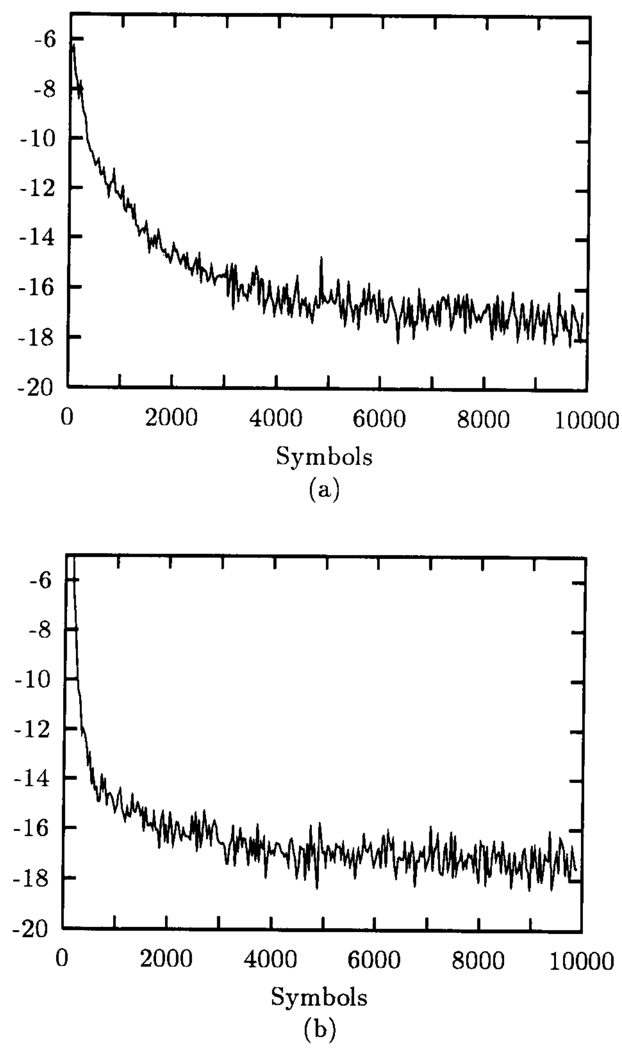

Fig. 5 Mean square error in dB for orthogonal algorithms. a) without initialization; b) with initialization by higher order statistics

the first 500 samples to initialize orthogonal algorithm, the convergence can be achieved within 3,000 symbols. Similar results are obtained for channel $H_2(\omega)$, except Sato algorithm works better than previous case because the channel condition of $H_1(\omega)$ is much worse. Fig. 5 depicts the MSE of equalizer output for orthogonal algorithms with and without initialization by higher order statistic method. The convergence looks faster if it is judged according to MSE rather than *ISI*.

6. Conclusion

We have investigated in this paper an approach of blind equalization. Training sequence is not required in the method. Special efforts were made to improve the the convergence properties of the equalizers by using the techniques of signal orthogonalization and higher order cumulants. Sato cost function was used to demonstrate the improved convergence. The computational complexity of the algorithm is considerable less than that in [11], in which the equalizer is built fully based on 4th-order cumulant. Generally, the estimation based on higher order statistics is of larger deviation. This is also observed in the simulation. Therefore the cumulant solution might be particular suitable for initializing the equalizer rather than solving it. It is not always necessary that the channel be equalized completely by blind algorithms. When the eye patterns become partially opening, the conventional adaptive algorithms are often able to converge faster. In our simulation cases, we can also switch the equalizer directly to MSE algorithm after its parameters are initialized by the higher order statistics method.

Appendix

Since the orthogonal conversion is linear, the impulse response for each orthogonal signal $v_i(t)$ can be written as

$$v_i(t)=\sum_{n=-\infty}^{\infty} q_i(n)x(t-n) \tag{15}$$

The output of equalizer is the combination of $v_i(t)$, i.e.

$$z(t)=\sum_{i=0}^{N-1} w_i v_i(t) \tag{16}$$

The mean square error of the equalizer is expressed as

$$MSE=E[\,|\,z(t)-x(t-d)\,|^2]=$$
$$E\left\{\left|\sum_{i=0}^{N-1} w_i v_i(t)-x(t-d)\right|^2\right\} \tag{17}$$

where d is a fixed, arbitrary delay. Although MSE is not measurable due to unknown $x(t)$ in blind equalization, it must be actually minimized by the equalizer. It is easy to find that minimizing (17) leads to the solution of

$$\sum_{i=0}^{N-1} w_i E[v_i(t)v_j(t)]-E[x(t-d)v_j(t)],$$
$$j=0,\cdots,N-1 \tag{18}$$

Using (5) and (15), we can obtain

$$w_j=q_j(d),\ j=0,\cdots,N-1 \tag{19}$$

Furthermore, if perfect equalization is achieved, the impulse response of overall system will admit a δ-function. That means

$$\sum_{i=0}^{N-1} w_i q_i(t)=\delta(t-d) \tag{20}$$

The 4th-order cross-cumulant of the orthogonal in the studies is defined in (13). It is not difficult to deduce that

$$s_{i,j}=\gamma_x \sum_{n=-\infty}^{\infty} q_i^3(t)q_j(t) \tag{21}$$

where $\gamma_x=E[x^4(t)]$. Multiply (21) by w_j and take summary with respect to index j, we have

$$\sum_{i=0}^{N-1} s_{i,j}w_j=\gamma_x \sum_{n=-\infty}^{\infty} q_i^3(t) \sum_{i=0}^{N-1} w_i q_i(t) \tag{22}$$

Substituting (19) and (20) into (22), this relation can be rewritten as

$$\sum_{i=0}^{N-1} s_{i,j}w_j=\gamma_x \sum_{n=-\infty}^{\infty} q_i^3(t)\delta(t-d)=\gamma_x q_i^3(d)=\gamma_x w_i^3 \tag{23}$$

Rewrite (23) in matrix form, we obtain the equation (14).

References

1) Y. Sato, "A Method of Self-Recovering Equalization for Multilevel Amplitude-Modulation Systems", IEEE Trans. Com. Vol. COM-23, pp. 679–682, June 1975.
2) D. N. Godard, "Self-Recovering Equalization and Carrier Tracking in Two-dimensional Data Communication Systems", IEEE Trans. Comm., Vol. COM-28, pp. 1867-1875, Nov. 1980.
3) A. Benvenist, M. Goursat and G. Ruget, "Robust Identifi-

cation of a Nonminimum System: Blind Adjustment of a Linear Equalizer in Data Communications", IEEE Trans. Automatic Control, Vol. AC-25, pp. 385-399, June 1980.

4) A. Benveniste and M. Goursat, "Blind Equalizers", IEEE Trans. Comm., Vol COM-32, pp. 871-883, Aug. 1984.

5) J. M. Mendel, "Tutorial on Higher-Order Statistics (Spectra) in Signal Processing and System Theory: Theoretical results and some applications", Proc. of IEEE, Vol-79, 278-305, Mar. 1991.

6) C. L. Nikias, "Bispectrum Estimation: A Digital Signal Processing Framework", Proc. of IEEE, Vol-75, pp. 869-891, Jul. 1987.

7) G. Picchi and G. Parti, "Blind Equalization and Carrier Recovery Using a 'Stop-and-Go' Decision-Directed Algorithm", IEEE Trans. Comm. , Vol. COM-35, pp. 877-887, Sept. 1987.

8) B. Porat and B. Friedlander, "Blind Equalization of Digital Communication Channels Using High-Order Moments", IEEE Trans. Signal Processing, Vol. SP-39, pp. 522-527, Feb. 1991.

9) Z. Ding, R. A. Kennedy, B. D. O. Anderson and C. R. Johnsŏn, "Ill-Convergence of Godard Blind Equalizers in Data Communication Systems", IEEE Trans. , Comm. Vol. COM-39, pp. 1313-1327, Sept. 1991.

10) V. weerackody, S. A. Kassam and K. R. Laker, "Convergence Analysis of an Algorithm for Blind Equalization", IEEE Trans., Comm. , Vol. COM-39, pp. 857-865, June 1991.

11) D. Hatzinakos and C. L. Nikias, "Blind Equalization Using a Triceptrum-Based Algorithm", IEEE Trans. Comm., Vol. COM-39, pp. 669-681, May 1991.

12) J. Proakis, *Digital Communications*, Second Edition, McGraw-Hill, 1989.