# On the Resistive Boundary Conditions for Planar Dielectric Structure

# On the Resistive Boundary Conditions for Planar Dielectric Structure

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We treat about the resistive boundary conditions used for analysis of planar devices with dielectric layered media used in microwave or millimeter-wave region. Especially the relation between the resistive boundary and the actual planar lossy material is discussed both analytically and numerically.

## 1. Introduction

In recent years, many kinds of planar circuits used in microwave or millimeter-wave region have been intensively explored both analytically and numerically, and most of those analytic models are, for example microstrip transmission lines, microstrip antennas, strip gratings, mode filters, and so on. Some of those including metallic planes (strips, patches, or others) often require specific boundary conditions, for instance, for, perfect conductors[1] as well as those of continuity about electromagnetic fields for analyses. It is very effective to use perfect conductor conditions (tangential electric fields on metallic planes are equal to zero) for solving lots of propagation or scattering problems. However, the metallic plane may contain loss due either to the natural losses occurring in materials at microwave or millimeter-wave region or to the addition of resistive materials used to alter the device's property. In such cases, resistive boundary conditions or impedance boundary conditions are often taken into account in analyses, and there has been a lot of papers treating these cases[2)-7)]. But while it is easy to apply resistive boundary condition to some analytic model when the surface resistance of actual material considerd is known, it is not so easy to estimate the thickness or conductivity of actual metallic plane or resistive paint from the resistance used for the analysis. Furthermore, it is difficult to simulate thin lossy material with resistive boundary without knowing the limits in applicability of such boundary condition.

So in this paper, we investigate the relations between the imaginary resistive planes often used for analyses of plane wave scattering by planar devices with dielectric layered media in microwave or millimeter-wave region and the actual planar lossy materials. And numerical examinations are performed about the limits of resistive boundary conditions applicable to actual planar lossy materials.

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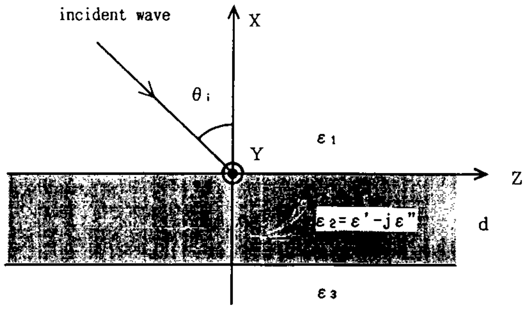
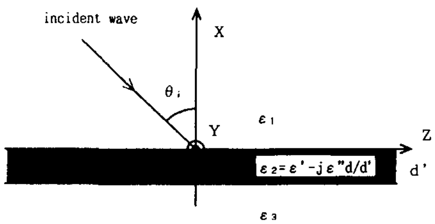
## 2. Analytical Study

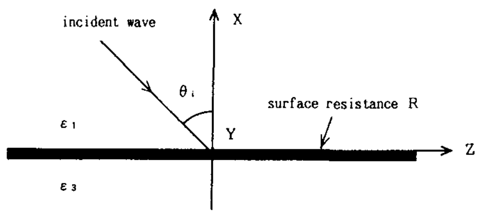
Figure 1 (a) shows a planar lossy material with a thickness $d$ and complex relative permittivity $\varepsilon_2 = \varepsilon' - j\ \varepsilon''$ (conductivity $\sigma$). We assume that the structure is uniform in $y$-direction. For the incidence of a TE or TM wave in the $xz$ plane with an angle of incidence $\theta_i$, all the field components of the incident wave have a form of $\exp[j\ \{\omega_0 t - \sqrt{\varepsilon_1}\ k_0(x\cos\theta_i + z\sin\theta_i)\}]$, where $\omega_0$ is the angular frequency and $k_0 = \omega_0\sqrt{\varepsilon_0\mu_0} = 2\pi/\lambda$ is the wave number in vacuum. Let us consider three lossy materials (Fig.1(a), (b), (c)) whose thickness $d$ tends to zero (from (a) to (c) through (b)) and assume that imaginary part of the relative permittivity varies in such a way that the product of $d$and $\varepsilon''$ keeps a constant value. That is, the planar lossy material (b) has a relative permittivity $\varepsilon'$-$j\varepsilon''d/d'$. Then (a) reduces to the infinitely thin lossy sheet (c). In this stage, $\varepsilon' \gg \varepsilon''$ can be recognized and the product of the thickness and imaginary part of the relative permittivity corresponds to $1/R$, where $R$ [$\Omega$/ square] is surface resistance and the resistive boundary condition has essencial meaning. This condition can be used in actual analysis only when the model Fig.1 (a) can be approximated by model (c).




(a) Planar lossy material with a incident wave.




(b) Limitting process of planar lossy material.




(c) Resistive sheet

Fig 1. Geometry of the problem

We can prepare for the study of the limitation described above analytically by solving Maxwell's equations concerning the fields of model (a). In this paper, we normalize the space variables by $k_0$, putting $k_0x \to x$ and $k_0z \to$z. and accordingly $k_0d \to d$, $R/Z_0 \to R$. Then, Maxwell's equations are expressed as

$$curl\ \sqrt{Y_0}\,\mathbf{E} = -j\mu_0\ \sqrt{Z_0}\,\mathbf{H}, \tag{1}$$

$$curl\ \sqrt{Z_0}\,\mathbf{H}=j\varepsilon_i\ \varepsilon_0\sqrt{Y_0}\,\mathbf{E}, \tag{2}$$

where $Y_0=1/Z_0=\sqrt{\varepsilon_0/\mu_0}.$ (3)

For the incidence of TE wave (the direction of electric field is parallel to y-axis), we have only to use the following elements of electromagnetic fields.

$$\sqrt{Y_0}E_y(x,z)=e_y(x)\exp(-js_o z), \tag{4}$$

$$\sqrt{Z_0}\,H_z(x,y)=e_z(x)\exp(-js_o z) \tag{5}$$

where $s_o$ is related to the incident wave and is given by

$$s_o=\sqrt{\varepsilon_1}\,\sin\theta_i. \tag{6}$$

From Eq. (1), (2) and (4), (5), we get a couple of differential equations concerning the field components $e_y$ and $h_z$ as follows;

$$\begin{bmatrix} de_y(x)/dx \\ dh_z(x)/dx \end{bmatrix} = j\begin{bmatrix} 0 & -1 \\ s_0^2-\varepsilon_i & 0 \end{bmatrix}\begin{bmatrix} e_y(x) \\ h_z(x) \end{bmatrix}. \tag{7}$$

By solving eigenvalue problem concerning the coupling matrix of Eq. (7), the solution of Eq. (7) is given as follws ;

$$\begin{bmatrix} e_y(x) \\ h_z(x) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \xi_i & -\xi_i \end{bmatrix}\begin{bmatrix} \exp[-j\xi_i(x-x_j)] & 0 \\ 0 & \exp[j\xi_i(x-x_j)] \end{bmatrix}\begin{bmatrix} g^+(x_j) \\ g^-(x_j) \end{bmatrix}$$

$(i=1,\ 2,\ 3.\ j=1,\ 2)$ (8)

where $$\xi_i=\sqrt{\varepsilon_i-s_0^2} \tag{9}$$

and indices $i$, $j$ are numbers of region and boundary respectively. $g^+$, $g^-$ correspond to amplitudes of waves propagating along $\pm\ x$-directions. Hereafter we simplify $g^+(x_j)$, $g^-(x_j)$ as $g^+{}_j$, $g^-{}_j$. These electromagnetic fields have to satisfy the conditions of continuity at two boundary ($x=0$ , $x=-d$) , and radiation condition $g^+{}_3=0$. So from Eq. (8) we have

$$\begin{bmatrix} 1 & 1 \\ \xi_1 & -\xi_1 \end{bmatrix}\begin{bmatrix} g^+{}_1 \\ g^-{}_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \xi_2 & -\xi_2 \end{bmatrix}\begin{bmatrix} \exp(-j\xi_2 d) & 0 \\ 0 & \exp(j\xi_2 d) \end{bmatrix}\begin{bmatrix} g^+{}_2 \\ g^-{}_2 \end{bmatrix}\quad (at\ x=0), \tag{10}$$

$$\begin{bmatrix} 1 & 1 \\ \xi_2 & -\xi_2 \end{bmatrix}\begin{bmatrix} g^+{}_2 \\ g^-{}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \xi_3 & -\xi_3 \end{bmatrix}\begin{bmatrix} 0 \\ g^-{}_3 \end{bmatrix}\quad (at\ x=-d). \tag{11}$$

Then eliminating $g^+{}_2$, $g^-{}_2$, and putting $g^-{}_1=1$, we have the following relation about $g^+{}_1$,$g^-{}_3$;

$$\begin{bmatrix} 1 & 1 \\ \xi_1 & -\xi_1 \end{bmatrix} \begin{bmatrix} g^+{}_1 \\ 1 \end{bmatrix} =$$

$$\frac{1}{2\,\xi_2} \begin{bmatrix} 1 & 1 \\ \xi_2 & -\xi_2 \end{bmatrix} \begin{bmatrix} \exp(-j\xi_2 d) & 0 \\ 0 & \exp(j\xi_2 d) \end{bmatrix} \begin{bmatrix} \xi_2 & 1 \\ \xi_2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -\xi_3 \end{bmatrix} g^-{}_3. \tag{12}$$

Solving Eq. (12), the values of unknown amplitudes $g^+{}_1$, $g^-{}_3$ are obtaind. Hereafter we call these amplitudes reflection and transmission coefficients respectively, and use the notation as $r_e=g^+{}_1$, $t_e=g^-{}_3$.

For the incidence of TM wave (the direction of magnetic field is parallel to $y$-axis), we can derive reflection and transmission coefficients through the similar analytical process as TE case. In this case, The following elements of electromagnetic fields have to be used for analysis instead of Eq. (4), (5).

$$\sqrt{Y_0}\, E_z(x,z) = e_z(x)\exp(-js_0 z), \tag{13}$$

$$\sqrt{Z_0}\, H_y(x,\ z) = h_y(x)\exp(-js_0 z). \tag{14}$$

Then,we get a couple of differential equations similar to Eq.(7) as

$$\begin{bmatrix} de_z\ (x)\ /dx \\ dh_y\ (x)\ /dx \end{bmatrix} = j \begin{bmatrix} 0 & -s_0{}^2/\varepsilon_i + 1 \\ \varepsilon_i & 0 \end{bmatrix} \begin{bmatrix} e_z\ (x) \\ h_y\ (x) \end{bmatrix}. \tag{15}$$

By solving eigenvalue problem concerning the coupling matrix of Eq. (15) ,the solution of Eq. (15) is given as follows;

$$\begin{bmatrix} e_z(x) \\ h_y(x) \end{bmatrix} = \begin{bmatrix} \xi_i/\sqrt{\varepsilon_i} & -\xi_i/\sqrt{\varepsilon_i} \\ -\sqrt{\varepsilon_i} & -\sqrt{\varepsilon_i} \end{bmatrix} \begin{bmatrix} \exp[-j\xi_i(x-x_j)] & 0 \\ 0 & \exp[j\xi_i(x-x_j)] \end{bmatrix} \begin{bmatrix} g^+{}_j \\ g^-{}_j \end{bmatrix}.$$

$$(i=1,\ 2,\ 3.\ j=1,2) \tag{16}$$

By taking into accout the conditions of continuity and radiation (as same as in TE case) which the fields of Eq. (16) have to satisfy, we have

$$\begin{bmatrix} \xi_1/\sqrt{\varepsilon_1} & -\xi_1/\sqrt{\varepsilon_1} \\ -\sqrt{\varepsilon_1} & -\sqrt{\varepsilon_1} \end{bmatrix} \begin{bmatrix} g^+{}_1 \\ g^-{}_1 \end{bmatrix}$$

$$= \begin{bmatrix} \xi_2/\sqrt{\varepsilon_2} & -\xi_2/\sqrt{\varepsilon_2} \\ -\sqrt{\varepsilon_2} & -\sqrt{\varepsilon_2} \end{bmatrix} \begin{bmatrix} \exp(-j\xi_2 d) & 0 \\ 0 & \exp(j\xi_2 d) \end{bmatrix} \begin{bmatrix} g^+{}_2 \\ g^-{}_2 \end{bmatrix} \qquad (at\ x=0), \tag{17}$$

$$\begin{bmatrix} \xi_2/\sqrt{\varepsilon_2} & -\xi_2/\sqrt{\varepsilon_2} \\ -\sqrt{\varepsilon_2} & -\sqrt{\varepsilon_2} \end{bmatrix} \begin{bmatrix} g^+{}_2 \\ g^-{}_2 \end{bmatrix} = \begin{bmatrix} \xi_3/\sqrt{\varepsilon_3} & -\xi_3/\sqrt{\varepsilon_3} \\ -\sqrt{\varepsilon_3} & -\sqrt{\varepsilon_3} \end{bmatrix} \begin{bmatrix} 0 \\ g^-{}_3 \end{bmatrix}$$

$$(at\ x=-d). \tag{18}$$

Then eliminating $g^+{}_2$,$g^-{}_2$,and putting $g^-{}_1=1$, we obtain the values of unknown amplitudes $g^+{}_1$, $g^-{}_3$, that is, reflection and transmission coefficients $r_m$, $t_m$ respectively.

We can use the results concerning the coefficients $r_e$, $t_e$, $r_m$ and $t_m$ to obtain electromagnetic fields from Eq. (8) and Eq. (16). Now we examine the approximation about the planar lossy materials (from Fig.1 (a) to (b)) for the incidence of TE wave.

Each scattering coefficient is explicitly given as follows;

$$r_e=\frac{-(\xi_2+\xi_1)(\xi_2-\xi_3)\exp(-j\xi_2 d)+(\xi_2-\xi_1)(\xi_2+\xi_3)\exp(j\xi_2 d)}{-(\xi_2-\xi_1)(\xi_2-\xi_3)\exp(-j\xi_2 d)+(\xi_2+\xi_1)(\xi_2+\xi_3)\exp(j\xi_2 d)}\ , \tag{19}$$

$$t_e=\frac{4\xi_1\xi_2}{-(\xi_2-\xi_1)(\xi_2-\xi_3)\exp(-j\xi_2 d)+(\xi_2+\xi_1)(\xi_2+\xi_3)\exp(j\xi_2 d)}\ . \tag{20}$$

The examination can easily be done about the special case that $\varepsilon_1=\varepsilon_3=1$ and $\theta_i=0°$ (that is $\xi_1=\xi_3=1$). In this case, Eq. (19) and Eq. (20) are reduced to

$$r_e=\frac{-(\xi_2{}^2-1)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\}}{-(\xi_2-1)^2\exp(-j\xi_2 d)+(\xi_2+1)^2\exp(j\xi_2 d),} \tag{21}$$

$$t_e=\frac{4\xi_2}{-(\xi_2-1)^2\exp(-j\xi_2 d)+(\xi_2+1)^2\exp(j\xi_2 d).} \tag{22}$$

For the convenience of approximation, we modify these as

$$r_e=\frac{-(\xi_2{}^2-1)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\}}{(\xi_2{}^2+1)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\}-2\xi_2\ \{\exp(-j\xi_2 d)+\exp(j\xi_2 d)\}}\ , \tag{23}$$

$$t_e=\frac{-4\xi_2}{(\xi_2{}^2+1)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\}-2\xi_2\ \{\exp(-j\xi_2 d)\exp(j\xi_2 d)\}}\ . \tag{24}$$

It is noticed that the model of resistive boundary shown in Fig.1 (c) is represented by two kinds of apporoximation. One of them is that since $d$ is quite small the terms concerning the thickness $d$ can be approximated by taking the first two terms of Taylor series as

$$\exp\ (-j\xi_2 d)\fallingdotseq 1-j\xi_2 d, \tag{25}$$

$$\exp\ (\ j\xi_2 d)\fallingdotseq 1+j\xi_2 d. \tag{26}$$

Another is about the complex relative permittivity $\varepsilon_2=\varepsilon\ '-j\varepsilon\ ''$ whose imaginary part (corresponding to conductivity) is quite large compared with real one. That is,

$$\varepsilon''\gg\varepsilon',\ \varepsilon_1(=\varepsilon_3=1). \tag{27}$$

Taking into account these relation, Eq. (23) and (24) are appoximated as

$$r_e=\frac{-1}{1+2/(\varepsilon''d),} \tag{28}$$

$$t_e=\frac{2/(\varepsilon''d)}{1+2/(\varepsilon''d).} \tag{29}$$

By introducing the notations $R=1/(\varepsilon'' d)$(surface resistance) and considering the directions of coordinate axes and the normalization $R \rightarrow R/Z_0$, we recognize that Eq. (28), (29) are well-known formula of reflection and transmission coefficients for a planar sheet of resistive sheet material[3]. In TM case, similar approximation can be performed. The fomula of scattering coefficients for the special case that $\varepsilon_1=\varepsilon_3=1$ and $\theta_i=0°$ in TM case are given as follows;

$$r_m=\frac{-(\varepsilon_2-\xi_2{}^2/\varepsilon_2)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\}}{-(\xi_2/\sqrt{\varepsilon}-\sqrt{\varepsilon})^2\ \exp(-j\xi_2 d)+(\xi_2/\sqrt{\varepsilon}+\sqrt{\varepsilon})^2 \exp(j\xi_2 d),} \tag{30}$$

$$t_m=\frac{4\xi_2}{-(\xi_2/\sqrt{\varepsilon}-\sqrt{\varepsilon})^2 \exp(-j\xi_2 d)+(\xi_2/\sqrt{\varepsilon}+\sqrt{\varepsilon})^2\ \exp(j\xi_2 d).} \tag{31}$$

For the convenience of approximation, these can be rewritten as

$$\begin{aligned} r_m=&-(\varepsilon_2-\xi_2{}^2/\varepsilon_2)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\} \\ &/\ [-(\varepsilon_2+\xi_2{}^2/\varepsilon_2)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\} \\ &+2\xi_2\ \{\exp(-j\xi_2 d)+\exp(j\xi_2 d)\}]\ , \end{aligned} \tag{32}$$

$$\begin{aligned} t_m=4\xi_2/\ &[-(\varepsilon_2+\xi_2{}^2/\varepsilon_2)\ \{\exp(-j\xi_2 d)-\exp(j\xi_2 d)\} \\ &+2\xi_2\ \{\exp(-j\xi_2 d)+\exp(j\xi_2 d)\}]\ . \end{aligned} \tag{33}$$

Using the same relations (25), (26) and (27), Eq.(32) and (33) are approximated as follows.

$$r_m=\frac{1}{1+2/(\varepsilon'' d)}, \tag{34}$$

$$t_m=\frac{2/(\varepsilon'' d)}{1+2/(\varepsilon'' d)}. \tag{35}$$

By considering the directions of coordinate axes and the normalization $R \rightarrow R/Z_0$, it is clear that Eq. (34), (35) represent scattering coefficients for a planar resistive sheet as same as TE case.

As seen in Eq. (28), (29) and (34), (35), when thickness $d$ is very small and $\varepsilon'' \gg \varepsilon'$each scattering coefficients in both TE and TM case is function of only $R=1/(\varepsilon''\ d)$,that is, each value of $\varepsilon'$, $\varepsilon''$ and $d$ has little meaning individually.This also means that in the case of quite thin and highly conducting dielectric the relations between electric fields and currents (in other words, magnetic fields) in the structure considered expressed by these scattering coefficients are provided almost only by surface resistance $R$, when resistive boundary conditions have essencial meaning. However, both cases treated here are limited to normal incidence and furthermore the limit of thickness or conductivity of lossy material for which resistive boundary condition is available has not been investigated. In the next chapter numerical evaluation is performed about the resistive boundary conditions which are applicable for the analyses of actual models.

## 3. Numerical Investigation

In chaper 2, we derived acattering coefficients of model Fig.1 (a), and studied about the approximation from model (a) to (c) through simple examinations in the case of normal incidence. Those examinations have shown that in the case of very thin and highly conducting planar dielectric, surface resistance is the main factor governing the relation of electromagnetic fields. In this chapter numerical investigation is made about the limit in applicability of resistive boundary conditions. It may be true that we can perform the evaluation by comparing the scattering coefficients of model Fig.1 (a) whose thickness $d$ is nearly zero with those of resistive sheet. However, it would be more suitable for our purpose to directly investigate the accuracy of resistive boundary conditions in the case of TE or TM incidence. In TE and TM case, the boundary conditions on the resistive sheet (at $x=0$) are

$$E_y(0,z)=RJ_y \quad \text{(TE case)}, \tag{36}$$
$$E_z(0,z)=RJ_z \quad \text{(TM case)}, \tag{37}$$

where $R=1/(\varepsilon\,''d)$

and $J_y$ and $J_z$ are elements of conduction current. Evaluation functions about the model Fig.1 (a) we have to check are given as

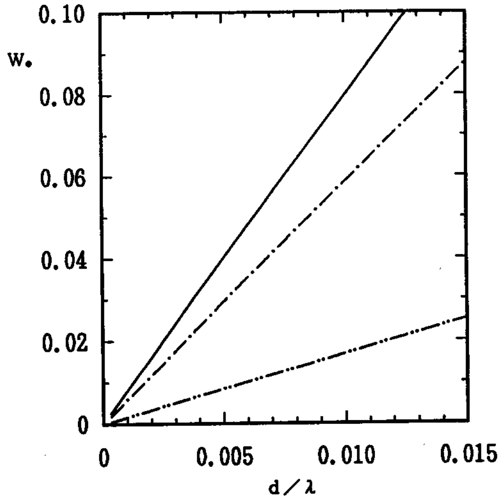
$$W_e=\frac{|E_{My}-R\ \{H_z(-d,0)-H_z(0,0)\}\ |}{|\ E_{My}\ |}, \qquad \text{(TE case)}, \tag{38}$$

$$W_m=\frac{|E_{Mz}-R\ \{H_y(-d,0)-H_y(0,0)\}\ |}{|\ E_{Mz}\ |} \qquad \text{(TM case)} \tag{39}$$

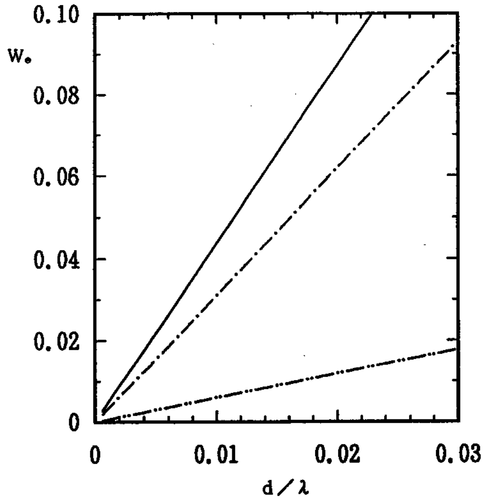
Where $\quad E_M=\{E(0,0)+E(-d,0)\}/2. \tag{40}$

$H(-d,\ 0)-H(0,\ 0)$ is corresponding to conduction curret $J$. Each field is calculated by scattering coefficients and Eq. (4), (5), (8) and Eq.(13), (14), (16). If $W_e$ or $W_m$ (that is, relative errors of the boundary conditions) is nealy zero, it may be true that the resistive boundary conditions like Eq.(36), (37) is almost available to analize the model Fig.1 (a). In the investigation described below, we assume $\varepsilon_2=1-j(\sigma Z_o/k_o)$, that is, the case of ideal conductive material and $\varepsilon_1=\varepsilon_3=1$.

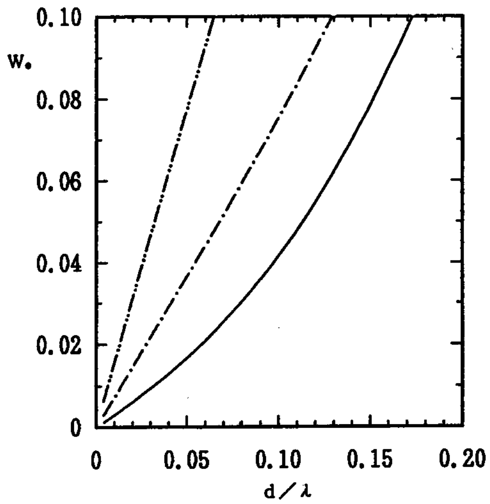
Figure 2 and Fig.3 show the values of $W_e$ and $W_m$ versus the thickness $d$ corresponding to several values of surface resistances with parameters $\theta_i$, (That is, $\varepsilon''$ varies according to $d$). From these results, it can be seen that in the case of TM incidence the values are almost independent of the angles of incidence when the value of $d$ is from 0 to at least 0.03 $\lambda$. On the other hand, the angle $\theta_i$ have apparent effects on the value of TE case. It is thought that these effects are mainly due to the magnetic field of incident wave whose element of $z$-direction has different magnitudes according to the angles of incidence. Concerning the values of $W_e$ and $W_m$ themselves, the values of surface resistances have large effects and so do concerning the limits in applicability of resistive boundary conditions. For example, when $R=100\Omega$/square, the limit of thickness corresponding to the relative error of 3% ($W_e$ or



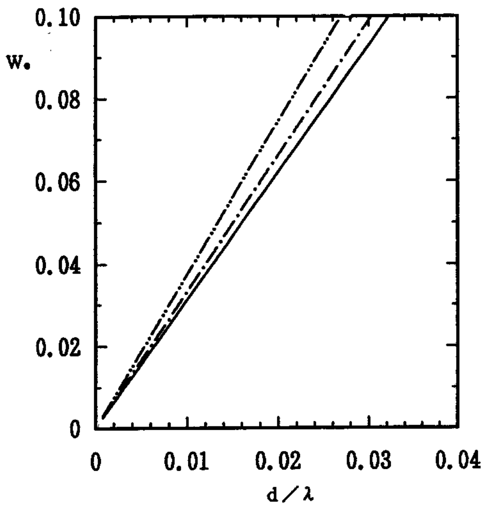

(a) R=500 Ω/square




(b) R=300 Ω/square

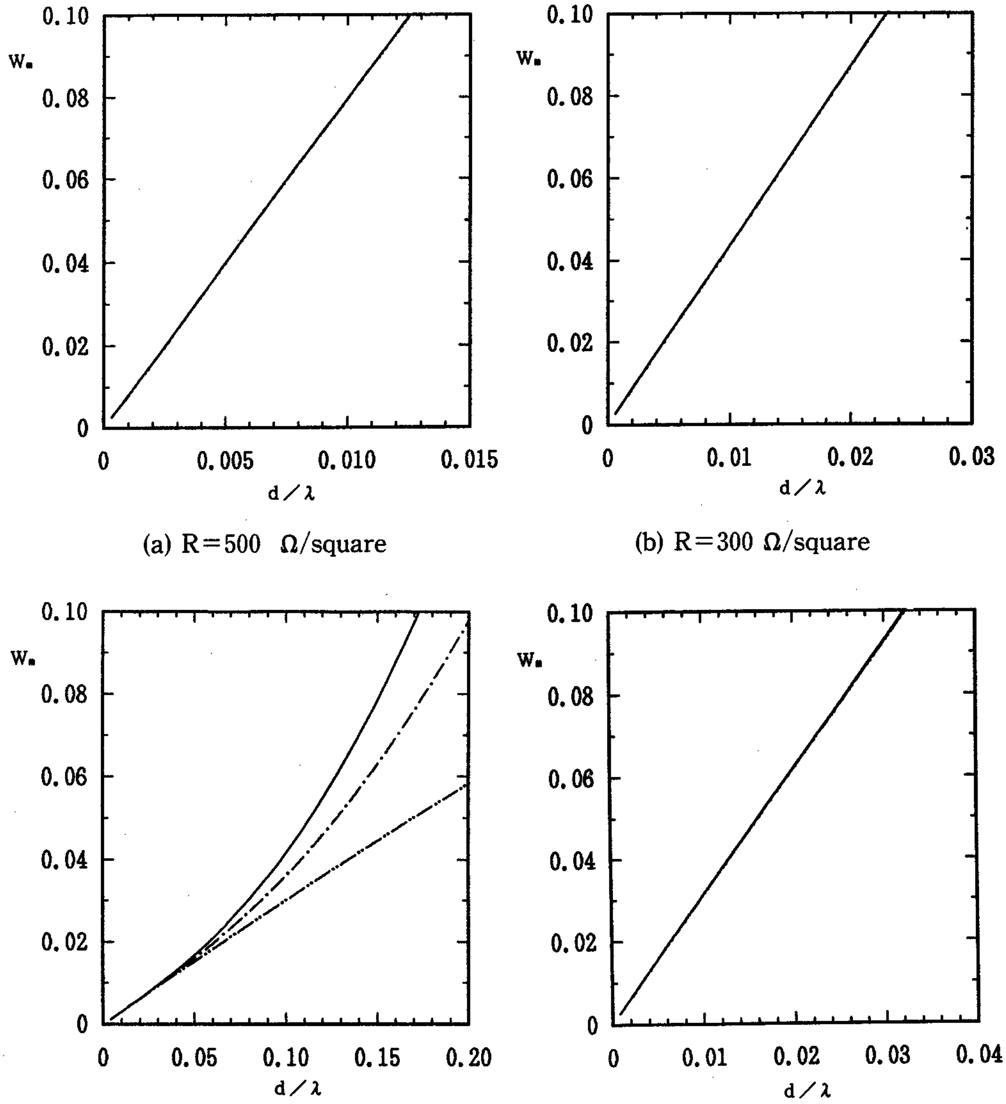



(c) R=100 Ω/square




(d)=50 Ω/square

Fig.2 Relative errors of resistive boundary conditions versus thickness of planar lossy material for the incidence of TE wave.(——— $\theta_i=0°$, —·— $\theta_i=30°$, —··— $\theta_i=60°$)




(a) R=500 Ω/square (b) R=300 Ω/square

(c) R=100 Ω/square (d)=50 Ω/square

Fig.3 Relative errors of resistive boundary conditions versus thickness of planar lossy material for the incidence of TM wave.(——— $\theta_i=0°$, —·— $\theta_i=30°$, —··— $\theta_i=60°$)

$W_m=0.03$) with normal incidence is about $0.08\lambda$, which is the largest value among the results presented here. When the value of $R$ goes away from 100 Ω/square, the limit of thickness reduces according to the value of $R$ (seen among only these results here). When thickness $d$ and conductivity $\sigma$ satisfy the condition indicated by the limit of a certain relative error, the surface resistance $R$ has essencial meaning to a certain extent and resistive boundary condition is almost available for analysis including that error. All the values of $R$ presented here are rather large compared with those for ordinary metallic planes (for instance, copper, silver, gold and so on), but are suitable for several absorbing materials like carbon, or some resistive paint (for example, *Dotaito* sheet) which are used mainly for narrow type (we call *N-type*) microwave or millimeter-wave absorber[8)-10)] with resistive sheets. From the results in which $R$ is set to 500 Ω/squere, it can be seen that for the limitation of error to 3%, it is neccessary to select the thickness $d$ within about 0.04$\lambda$, that is, to select $\varepsilon''$ as 500 (Ω/square)/$d$. As shown thus far, it is, apparently useful to have these relations between the relative error of resistive boundary conditions and the thickness of actual planar lossy materials which provide us with the information about the limit in applicability of such conditions and the estimation of actual thickness or conductivity from the value of surface resistance appeared in analysis.

Figure 4~Fig.7 are results of $W_e(=W_m)$ in the normal incidence calculated from the different point of view. In this calculations, we keep the values of conductivities of planar lossy materials as constant and the values of $W_e$ and $W_m$ are plotted with the variation of thickness $d$. The values of conductivities used in these calculations are same as those of ordinary metals (copper, pure gold, iron of 99.98% pureness, and German silver) in 20°C. From these results, it can be seen that the smaller the conductivity become the larger the limit of thickness $d$ corresponding to a certain relative error (e.g. 3%). Concerning the each result, the thickness allowed for the use of resistive boundary condition is quite small. For example in the case shown in

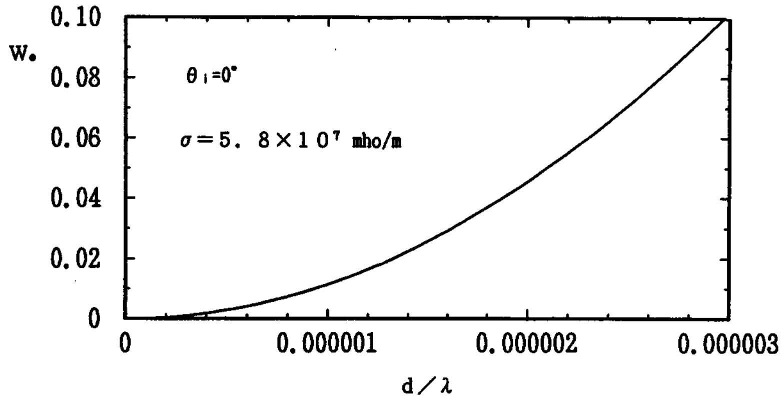



Fig.4 Relative errors of resistive boundary conditions in the case of copper.

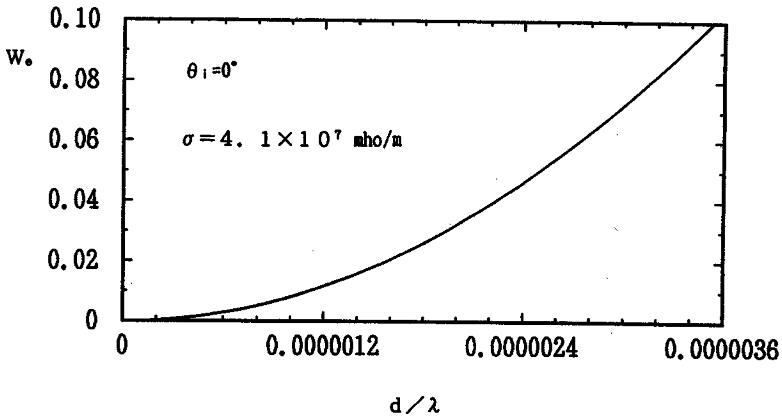



Fig.5 Relative errors of resistive boundary conditions in the case of pure gold.

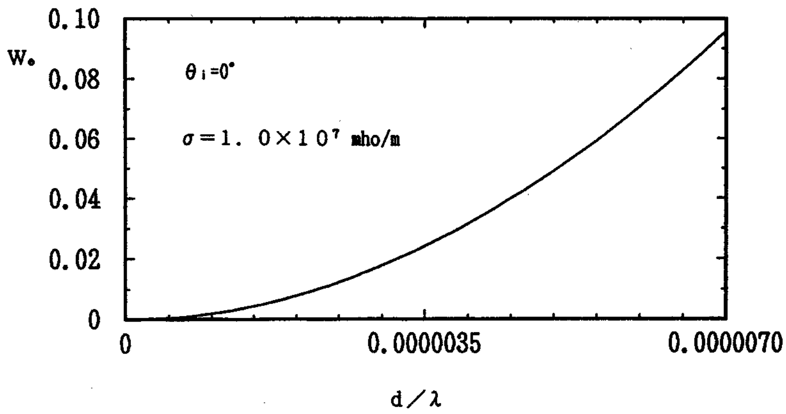



Fig.6 Relative errors of resistive boundary conditions in the case of iron of 99.98% pureness.

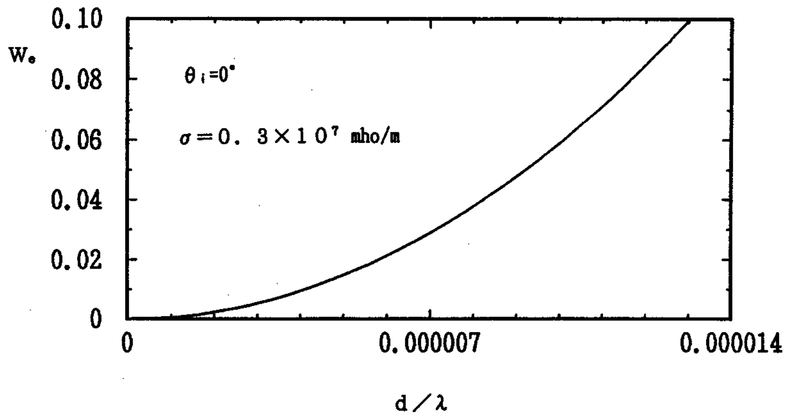



Fig.7 Relative errors of resistive boundary conditions in the case of German silver.

Fig. 4 (copper), the limit of thickness $d$ of 1% error is less than 1/20 of skin depth of copper. This data is very reasonable because resistive boundary condition with rigorous meaning can not easily be assumed in the case that electromagnetic fields attenuate to less than certain extent (e.g. to several times of $1/e$) in conductive material considered here. Similar to the cases of Fig.2 and Fig.3, it is also useful to have these results like Fig.4~Fig.7 from which we obtain the informations about the limit of resistive boundary condition applicable to actual planar lossy material with certain conductivity.

## 4. Conclusion

In this paper, we studied the resistive boundary conditions used for analyses of devices with planar dielectric structure for microwave or millimeter-wave region by regarding the planar boundary as the limit of the material with finite thickness and conductivity. By the analytical study in the case of normal incidence, it was made clear that relation between electromagnetic fields propagating in quitely thin and highly conducting lossy material is provided almost only by surface resistance. And from the numerical investigation, we discussed about the relations between relative errors of resistive boundary conditions and values of thickness of planar lossy materials. These studies shown in this paper will be very significant in the numerical analyses of models including relatively thin and highly conducting sheet materials.

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### References

1 ) F. S. Johansson, IEEE Transactions on Antennas and Propagation, vol. 37, p. 996 (1989).
2 ) T. B. A. Senior, IEEE Transactions on Antennas and Propagation, vol. AP-27, p. 808 (1979).
3 ) R. C. Hall and R. Mittra, IEEE Transactions on Antennas and Propagation, vol. AP-33, p. 1009 (1985).
4 ) T. A. Cwik and R. Mittra, IEEE Transactions on Antennas and Propagation, vol. AP-35, p. 1226 (1987).
5 ) R. C. Hall, R. Mittra and K. M. Mitzner, IEEE Transactions on Antennas and Propagation, vol. 36, p. 511 (1988).
6 ) R. Petit and G.Tayeb, Journal of Optical Society of America, vol. 7, p. 1686 (1990).
7 ) C. M. Krowne, IEEE Transactions on Antennas and Propagation, vol. 37, p. 1207 (1989).
8 ) Y. Naito and K. Suetake, IEICE Journal, vol.48, p. 2152 (1965).
9 ) K. Suetake and Y.Miyazu, IEICE Journal, vol.50, p. 23 (1967).
10) M. Takashima and K.Suetake, IEICE Journal, vol. 50, p.31 (1967).