Analysis of Oblique Propagating Leaky Waves by Anisotropic Dielectric Waveguides with Index-Modulated Gratings

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A rigorous formulation is presented for the guidance of light waves by the anisotropic dielectric waveguides with index-modulated gratings, under the most general condition of oblique propagation. An numerical method to determine the propagation constants is formulated for the hybrid leaky waves in an exact fashion. The accurate solutions of arbitrary level can be calculated with increasing the matrix size necessary for the computatinos. Numerical examples of wave guiding properties are given for the index-modulated waveguides consisting of lithium niobate whose optical axis makes a slanted angle with respect to the periodic variation of the grating.

1. Introduction

The guidance of light waves in periodic structures exhibits many interesting and useful phenomena, and these phenomena have been employed in devices with dielectric gratings in the fields of optics for many years. On the other side anisotropic materials like lithium niobate are widely applied to the waveguides in integrated optics. The anisotropic dielectric gratings are needed in the periodic interdigitated electrode devices[1] such as swiches, modulators, mode converters and optical computing applications. Numerous methods of analyzing these periodic structures have been reported by using many perturbation approaches[2)-4)]. Recently, the rigorous methods of analyzing anisotropic gratings are reported on a field of the scattering problems[5)-9)], However, very little rigorous method[10)-12)]has been applied on the wave guidance for the case of the oblique propagating waves or the waveguides containing anisotropic materials. At least for the leaky waves obliquely propagating in the anisotropic grating waveguides the rigorous formulation have never been reported so far to the authors knowledge. In this paper a method of analyzing the anisotropic waveguides with index-modulated gratings is presented for the case of the oblique propagating leaky waves. The materials consisting of the dielectric waveguides are arbitrary anisotropic. The analysis is the

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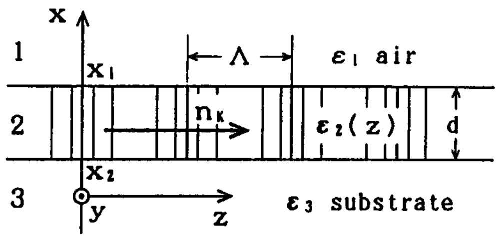
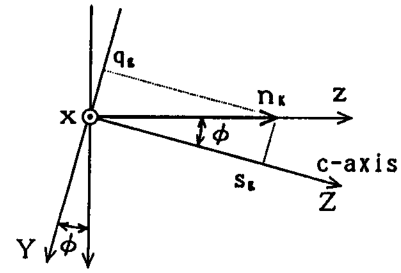
rigorous approach formulated in a unified matrix form in which the electromagnetic fields in grating and external regions are expressed as a summation over all possible space harmonics. Thus it is possible to obtain the accurate solutions of arbitrary level by increasing the size of the matrix necessary for the calculation. Numerical examples of wave guiding properties are given for the case of index-modulated waveguides caused by acoustic shear waves. The film and substrate regions of waveguides are assumed to be uniaxial crystals consisting of lithium niobate. The peculiar properties of the leaky wave due to the anisotropy of the materials are shown.

2. Formulation of Problem

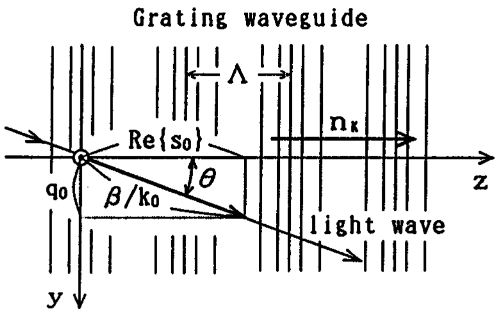
We now consider the anisotropic dielectic slab waveguide whose film layer is modulated by an acoustic shear wave propagating along the z-axis as shown in Fig.1. The propagating direction of the light wave makes an angle θ to the z-axis on the y-z plane. The acoustic waves causes a periodic perturbation in the relative permittivity, and the film layer becomes the index-modulated grating with a period equal to that of the acoustic wavelength Λ. The semi-infinite region above the grating film layer is designated as the air with relative permittivity ε_1, and the substrate is an anisotropic dielectric medium with tensor permittivity ε_3. The index-modulated anisotropic grating in the film is a function of both space and time



(a) Geometry of a grating waveguide.



(b) Coordinate system of waveguides and uniaxial crystals.



(c) Oblique propagating waves with respect to the grating.

Fig.1 An anisotropic slab waveguide with index-modulated grating.

in a strict sense. However, the grating caused by the traveling acoustic wave is essentially stationary, since the velocity of sound is some five orders of magnitude smaller than that of light. In this paper we use the coordinate variables normalized by $k_0=2\pi/\lambda$, and put $k_0x\rightarrow x$. $k_0y\rightarrow y$ and $k_0z\rightarrow z$ for simplicity.
Therefore a relative permittivity tensor in the film region can be expressed as

$$\varepsilon_2(z)=\bar{\varepsilon}_2+\delta\Delta\varepsilon_2\cos(n_K z) \tag{1}$$

where, n_K is the magnitude of the grating vector normalized by k_0, and is expressed as $n_K=\lambda/\Lambda$ in terms of the wavelength λ of light wave and grating period Λ. The modulation factor δ is given as the particle displacement of the acoustic wave, and the tensor $\Delta\varepsilon_2$ is the perturbation of the relative permittivity tensor caused by the acoustic wave. The tensors ε_2 and ε_3 of the film and the substrate are arbitrary anisotropic.

We assume that the acoustic shear wave is polarized along the x-axis. Then we can express the particle displacement of the acoustic wave as

$$\xi_x=\delta\sin(n_K z),\quad \xi_y=\xi_z=0 \tag{2}$$

where δ is the modulation factor. While we consider that the film region consists of the uniaxial crystals, and the relative permittivity tensor is described by using the coordinate systems (x,Y,Z) of the crystal as

$$\varepsilon_c=\mathrm{diag}[\varepsilon_0\ \varepsilon_0\ \varepsilon_e],\quad \text{on } (x,Y,Z) \tag{3}$$

where ε_0 and ε_0 are the ordinary and the extraordinary relative permittivities, respectively. The direction of the crystal axis is chosen to be an angle ϕ to the z-axis on the y-z plane as shown in Fig.1(b). Therefore the representation of the particle displacement in eq.(3) can be rewritten in terms of the coordinate systems (x,Y,Z) as

$$\xi_x=\delta\sin(q_g Y+s_g Z) \tag{4}$$

with

$$q_g=-n_K\sin(\phi),\quad s_g=n_K\cos(\phi). \tag{5}$$

The strain tenser S_k (i=1~6) caused by the acoustic wave is described by

$$S_1=S_2=S_3=S_4=0, \tag{6}$$
$$S_5=\partial\xi_z/\partial x+\partial\xi_x/\partial Z=\delta\ s_g\cos(q_g Y+s_g Z), \tag{7}$$
$$S_6=\partial\xi_x/\partial Y+\partial\xi_y/\partial x=\delta\ q_g\cos(q_g Y+s_g Z). \tag{8}$$

Consequently the change in the optical impermeability is given by

$$\Delta\eta_i=\sum_{j=1}^{6}p_{ij}\ S_j \tag{9}$$

where $\Delta\eta_i$ (i=1~6) are the change in the element of optical impermeability tensor and P_{ij} is the strain-optic coefficients[13]. In the end the change in the relative permittivity tensor on the coordinate systems of the crystal is expressed as

$$\delta\Delta\varepsilon_c\cos(q_g Y+s_g Z)=-\varepsilon_c\begin{bmatrix}\Delta\eta_1 & \Delta\eta_6 & \Delta\eta_5\\ \Delta\eta_6 & \Delta\eta_2 & \Delta\eta_4\\ \Delta\eta_5 & \Delta\eta_4 & \Delta\eta_3\end{bmatrix}\varepsilon_c\ . \tag{10}$$

Finally, we obtain the relative permittivity tensor in film with help of the coordinate transformation between the (x, Y, Z) in the crystal and the (x,y,z) in the waveguide as

$$\bar{\varepsilon}=R\varepsilon_c R^{-1},\quad \Delta\varepsilon=R\Delta\varepsilon_c R^{-1} \tag{11}$$

with

$$R=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} . \tag{12}$$

The rotator R is related to the waveguide and the crystal coordinate system by a coordinate rotation about the x axis.

3. Fields in grating and uniform regions

We consider that a surface wave is incident from a uniform dielectric waveguide upon a grating layer at an angle θ with respect to the direction of the grating variation as shown in Fig.1(c). The incident surface wave has a normalized propagation constant q_0 in the y direction. In the case of wave guidance by the grating waveguide, q_0 is taken as a known parameter which has a pure real value. A normalized propagation constant in the z direction is s_0 which is the fundamental space harmonic for the grating waveguide. Therefore s_0 is the single unknown to be determined by the characteristic equation of the grating waveguide. The phase constant β of the leaky wave along the direction of the propagation is concerned with q_0 and s_0 as

$$\beta/k_0=\sqrt{q_0{}^2+\ \{R_e(s_0)\}^2}\ . \tag{13}$$

While the imaginary part $Im(s_0)$ of the normalized propagation constant in the z direction means the attenuation constant with respect to the direction of the grating vector. Therefore, if an angle θ instead of q_0 is given as a prior parameter, we should execute the trivial calculations to determine q_0 as

$$q_0=Re\{s_0\}\tan\theta\ . \tag{14}$$

Maxwell's equations can be rewritten in terms of the coordinate variables normalized by k_0 as follows

$$\mathrm{curl}\sqrt{Y_0}E=-j\sqrt{Z_0}H, \tag{15}$$

$$\mathrm{curl}\sqrt{Z_0}H=j\varepsilon(z)\sqrt{Y_0}E \tag{16}$$

with

$$Y_0=1/Z_0=\sqrt{\mu_0/\varepsilon_0} \tag{17}$$

where, the time dependence exp $(j\omega t)$ is assumed throughout this paper. The relative permittivity tensor $\varepsilon(z)$ is a periodic function of z, so that each element ε_{ij} of the tensor ε can be expressed in terms of the Fourier expansion as

$$\varepsilon_{ij}(z)=\sum_m b_{ij,m}\exp(jmn_K z). \tag{18}$$

The components of electromagnetic fields E_i and H_i $(i=x,y,z)$ can be expressed from the periodicity of the film region as following field expansions in terms of the space-harmonics;

$$\sqrt{Y_0}E_i(x,y,z)=\sum_{m} e_{im}(x)\exp\{-j(q_0y+s_mz)\}, \tag{19}$$

$$\sqrt{Z_0}H_i(x,y,z)=\sum_{m} h_{im}(x)\exp\{-j(q_0y+s_mz)\} \tag{20}$$

with

$$s_m=s_0+mn_K \tag{21}$$

Now let the truncation size of the expansions be (2M+1). We introduce the column matrices whose elemets are coefficients of the expansions as

$$\mathbf{e}_i=[e_{i(-M)}\cdots e_{io}\cdots e_{iM}]^t, \tag{22}$$

$$\mathbf{h}_i=[h_{i(-M)}\cdots h_{io}\cdots h_{iM}]^t. \tag{23}$$

Substitution of eqs. (19) and (20) into Maxwell's equations (15) and (16) yields the following first order differential equations in matrix form;

$$d\mathbf{f}/dx=j\mathbf{C}\mathbf{f},\quad \mathbf{f}=\begin{pmatrix}\mathbf{e}_y\\ \mathbf{h}_z\\ \mathbf{e}_z\\ \mathbf{h}_y\end{pmatrix},\qquad \mathbf{C}=\begin{pmatrix}\mathbf{C}_{11} & \mathbf{C}_{12}\\ \mathbf{C}_{21} & \mathbf{C}_{22}\end{pmatrix}, \tag{24}$$

$$C_{11}=\begin{pmatrix}[q][\varepsilon_{xx}]^{-1}[\varepsilon_{xy}] & [q][\varepsilon_{xx}]^{-1}[q]-[1]\\ [s]^2+[\varepsilon_{yx}][\varepsilon_{xx}]^{-1}[\varepsilon_{xy}]-[\varepsilon_{yy}] & [\varepsilon_{yx}][\varepsilon_{xx}]^{-1}[q]\end{pmatrix}, \tag{25}$$

$$C_{12}=\begin{pmatrix}[q][\varepsilon_{xx}]^{-1}[\varepsilon_{xz}] & -[q][\varepsilon_{xx}]^{-1}[s]\\ [\varepsilon_{yx}][\varepsilon_{xx}]^{-1}[\varepsilon_{xz}]-[\varepsilon_{yz}]-[s][q] & -[\varepsilon_{yx}][\varepsilon_{xx}]^{-1}[s]\end{pmatrix}, \tag{26}$$

$$C_{21}=\begin{pmatrix}[s][\varepsilon_{xx}]^{-1}[\varepsilon_{xy}] & [s][\varepsilon_{xx}]^{-1}[q]\\ [q][s]+[\varepsilon_{zy}]-[\varepsilon_{zx}][\varepsilon_{xx}]^{-1}[\varepsilon_{xy}] & -[\varepsilon_{zx}][\varepsilon_{xx}]^{-1}[q]\end{pmatrix}, \tag{27}$$

$$C_{22}=\begin{pmatrix}[s][\varepsilon_{xx}]^{-1}[\varepsilon_{xz}] & -[s][\varepsilon_{xx}]^{-1}[s]+[1]\\ -[q]^2+[\varepsilon_{zz}]-[\varepsilon_{zx}][\varepsilon_{xx}]^{-1}[\varepsilon_{xz}] & [\varepsilon_{zx}][\varepsilon_{xx}]^{-1}[s]\end{pmatrix} \tag{28}$$

where, $\mathbf{f}(x)$ is a $4(2M+1)\times 1$ column matrix with the elements described by eqs. (22) and (23), and the matrix $\mathbf{C}$ indicates a $4(2M+1)\times 4(2M+1)$ coupling matrix with $2(2M+1)\times 2(2M+1)$ matrices $C_{ln}(l,n=1,2)$. The sub-matrices $[\varepsilon_{ij}]$ $(i,j=x,y,z)$, $[q]$ and $[s]$ are $(2M+1)\times(2M+1)$ matrices, and they are written by using the Kronecker delta δ_{mn} as

$$[\varepsilon_{ij}]=[b_{ij,(n-m)}], \tag{29}$$

$$[q]=q_0[\delta_{mn}], \tag{30}$$

$$[s]=[s_m\delta_{mn}]. \tag{31}$$

Moreover, $[1]$ and $[0]$ indicate the unit and the zero matrices, respectively, and $[\varepsilon_{xx}]^{-1}$ is the inverse matrix of $[\varepsilon_{xx}]$. It is worth noting that the matrices $[\varepsilon_{ij}]$ are composed of the Fourier coefficients of $\varepsilon_{ij}(z)$. For instance, $[\varepsilon_{xy}]$ implies the matrix whose (m,n) element is the (n-m)th Fourier component $b_{xy,(n-m)}$ of $\varepsilon_{xy}(z)$.

For this grating, the coupling matrix $\mathbf{C}$ is independent of x. In this case the solutions for the first order coupled-wave equation (24) are reduce to the eigenvalue problem of the matrix $\mathbf{C}$. Then, let $\{\kappa_m\}$ be eigenvalues of the matrix $\mathbf{C}$, and $\mathbf{T}$ be a diagonalizer composed of the corresponding eigenvectors, and so we can transform $\mathbf{f}(x)$ to $\mathbf{g}(x)$ by using the diagonalizer $\mathbf{T}$ as

$$\mathbf{f}(x) = \mathbf{T}g(x) \tag{32}$$

where $g(\mathrm{x})$ is the $4(2\mathrm{M}+1)\times 1$ column matrix. The general solutions of the coupling equation (24) are given by

$$\mathbf{f}(x) = \mathbf{T}\ [\delta_{mn}\exp\ \{j\kappa_m(x\text{-}x_0)\}]\ \mathbf{g}(x_0) \tag{33}$$

where $[\delta_{mn}\exp\ \{j\kappa_m(x\text{-}x_0)\}]$ is the $4(2\mathrm{M}+1)\ \times\ 4(2\mathrm{M}+1)$ diagonal matrix, and x_0 is the arbitrary fixed point. This representation of the solutions in terms of $\mathbf{g}(x)$ may be useful to describe the electromagnetic fields inside not only the grating region, but also the external region such as the air and the substrate, since a column matrix $\mathbf{g}(x)$ has a significant physical meaning described below.

The first order coupled-wave equations (24) still hold in the uniform regions whose relative permittivity tensor is a constant. In this case, the sub-matrices $[\varepsilon_{ij}]$ are diagonal because the Fourier series has the only zero-order term. Then all submatrices constituting the coupling matrix $\mathbf{C}$ are also diagonal. We can decompose the eigenvalue problem of the coupling matrix $\mathbf{C}$ into the $2\mathrm{M}+1$ eigenvalue problems of 4×4 matrices. The eigenvalue problems of 4×4 matrices must be solved $2\mathrm{M}+1$ times, but this is advantageous to the computation-time in comparison with that of the $4(2\mathrm{M}+1)\times 4(2\mathrm{M}+1)$ matrix $\mathbf{C}$. The elements g_m of the column matrix $\mathbf{g}$ mean the complex amplitude of the plane wave with the propagation constants κ_m along the x-direction since the sub-matrices composing of $\mathbf{T}$ are all diagonal. From the sign the eigenvalues $\{\kappa_m\}$, it is easy to determine whether the amplitudes g_m are incoming or outgoing waves with respect to the x-direction. Therefore the eigenvalues $\{\kappa_m\}$ and the amplitudes $\{g_m\}$ can be arranged in the order of the direction of propagation as

$$\kappa = \begin{pmatrix} \kappa^+ \\ \kappa^- \end{pmatrix}, \qquad \mathbf{g} = \begin{pmatrix} \mathbf{g}^+ \\ \mathbf{g}^- \end{pmatrix} \tag{34}$$

where superscripts $^{(+)}$ and $^{(-)}$ indicate the outgoing and the incoming waves along the x axis. respectively. However, for $m<0$ and Re $\{s_m\}>0$ we must chose the improper waves $(Im\ \{s_m\}>0)$[14].

For the general anisotropic media without the grating, the eigenvalue problems of the 4×4 matrices have to be determined by the numerical implementation. In the isotropic uniform region such as region 1, the eigenvalue problems of the 4×4 matrices yield the solutions in closed form. The sorting in a similar way described above is necessitated for κ and $\mathbf{T}$. In addition, when the optical axis of the uniaxial crystal is coincided with the coordinate axis, if necessary, we can obtain the eigenvalue solutions in the closed form such as the isotropic media.

4. Characteristic Equations of Leaky waves

At the boundary surfaces ($x=x_i,\ i=1,2$), tangential components of the fields are continuous across these boundaries, that is,

$$\mathbf{f}_1(x_1)=\mathbf{f}_2(x_1),\quad \mathbf{f}_2(x_2)=\mathbf{f}_3(x_2). \tag{35}$$

For the wave guiding problems by dielectric waveguides the incoming waves do not exist in region 1 and 3, and we can put as

$$\mathbf{g}_1^-=\mathbf{g}_3^+=\ [0\cdots0\cdots0\cdots0\cdots0\cdots0]^t. \tag{36}$$

Therefore the boundary conditions at the each surface yield the homogeneous equation as

$$\mathbf{T}_1\begin{bmatrix}\mathbf{g}_1^+(x_1)\\ \mathbf{0}\end{bmatrix}=\mathbf{T}_2\ [\delta_{mn}\exp\{j\varkappa_{2.m}(x_1-x_2)\}]\ \begin{bmatrix}\mathbf{g}_2^+(x_2)\\ \mathbf{g}_2^-(x_2)\end{bmatrix}, \tag{37}$$

$$\mathbf{T}_2\begin{bmatrix}g_2^+(x_2)\\ g_2^-(x_2)\end{bmatrix}=\mathbf{T}_4\begin{bmatrix}\mathbf{0}\\ g_3^-(x_2)\end{bmatrix} \tag{38}$$

where, unknown valuables are $\mathbf{g}_1^+(x_1)$, $\mathbf{g}_2^\pm(x_2)$ and $\mathbf{g}_3^-(x_2)$. Then the dimension of the homogeneous equation is $4(2M+1)\times2$. We can systematically calculate this large system of equations by using the algorithm of the successive elimination, even if the number of layered regions increases more than three regions. Consequently the characteristic equation which determines the normalized complex propagation constant s_0 is given by

$$det.\ [W]=0,\qquad\qquad [W]\begin{bmatrix}\mathbf{g}_1^+\\ \mathbf{g}_2^+\\ \mathbf{g}_2^+\\ \mathbf{g}_3^-\end{bmatrix}=\mathbf{0}. \tag{39}$$

The determinant of the matrix $[W]$ is a function of the complex value s_0, then we must search for det. $[W]=0$ on the s_0 complex plane by using the numerical iteration methods such as the two-dimensional Newton method.

5. Numerical calculations

Numerical calculations were performed for the anisotropic dielectric waveguides with the film and substrate consisting of lithium niobate($LiNbO_3$). We assumed that the principal relative permittivity of $LiNbO_3$ for an optical wavelength of 0.633 μ m is given by

$$\varepsilon_{fo}=5.27,\quad \varepsilon_{fe}=4.88\ \text{(film)}, \tag{40}$$

$$\varepsilon_{so}=5.23,\quad \varepsilon_{se}=4.84\ \text{(substrate)}. \tag{41}$$

The relative permittivity in the film region is only a little percentages larger than that in the substrate in order to fabricate the waveguides. The optical axis of the uniaxial crystal consisting of the film and substrate makes an angleϕ to the z axis on the y-z plane, and the inclination angle of the propagating direction of the light is set to be θ from the z axis as shown Fig.1(b) and (c).

We first show the dispersion curves of the anisotropic waveguide without the acoustic modulation($\delta=0$) in Fig.2(a), and the variation of phase consants for the inclination angle of crystal axis in Fig.2(b). In this case the guided modes are propagating along the z axis ($\theta=0$), so that $q_0=0$ and $\beta/k_0=Re\{s_0\}$. It can be seen that the phase constants of TM-like modes do not fluctuate for a change of the inclination angle of the c-axis, while those TE-like modes result in large variations. Although we use the notations of TE-like and TM-like for weakly hybrid modes in Fig.2, it is worth noting that two modes become the complete hybrid modes at the near point of the degeneration.

Figures 3(a) and (b) show the propagatios constants of leaky waves as a function of modulation factor for the angle of propagation $\theta=0°$. Here, the inclination angle of crystal axis is $\phi=5°$, and the thickness of the grating region is $d/\lambda=3.0$.

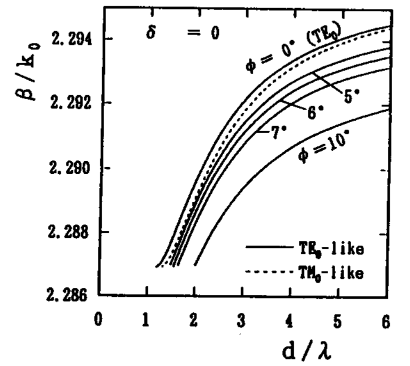


Fig.2 (a) Dispersion curves of anisotropic slab waveguides.

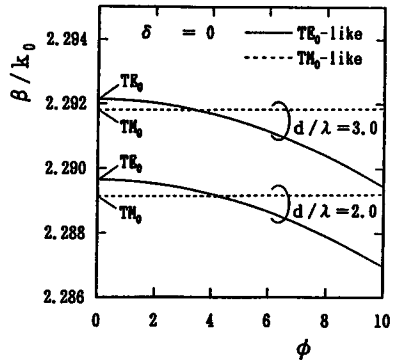


Fig.2 (b) Variations of phase constants for inclination angles of c-axis.

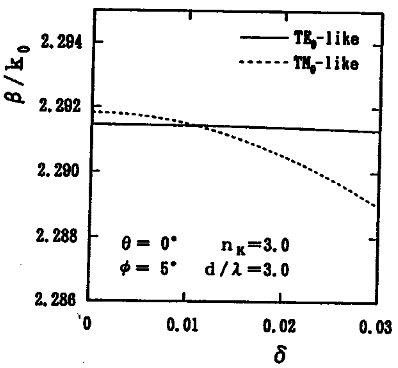


Fig.3 (a) Phase constants as a function of modulation factor for $\theta=0°$.

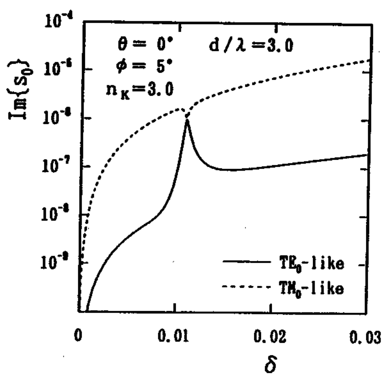


Fig.3 (b) Attenuation constants as a function of modulation factor for $\theta=0°$.

The normalized grating vector is set to be $n_K = \lambda/\Lambda = 3.0$, so that leaky waves radiate a single beam from the film to the substrate. Moreover, it is worth noting that the modulation factor δ is the normalized particle displacement of the acoustic wave. At the degenerate point of two phase constants, the attenuation constants of two hybrid leaky waves change abruptly and exchange these values each other.

Figures 4(a) and (b) show the propagatios constants for the oblique propagation of leaky waves. Here, we use the normalized phase constant β/k_o along the direction of the propagation and the attenuation constant $Im(s_o)$ along the z axis, since the leaky waves are obliquely propagating under the condition $Im(q_o) = 0$.

Next we show the variations of the propagation constants for the period of acoustic waves for the case of the angle of propagation $\theta = 0°$ in Fig.5(a) and(b).

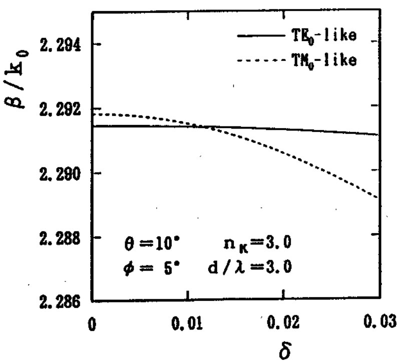


Fig.4 (a) Phase constants as a function of modulation factor for $\theta = 10°$

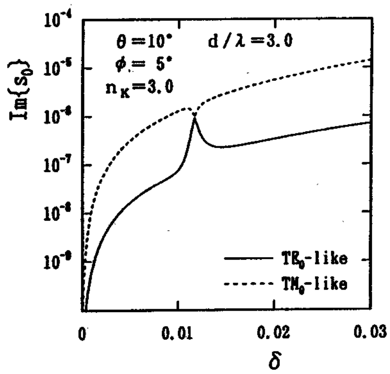


Fig.4 (b) Imaginary part of propagation constants along z axis as a function of modulation factor for $\theta = 10°$

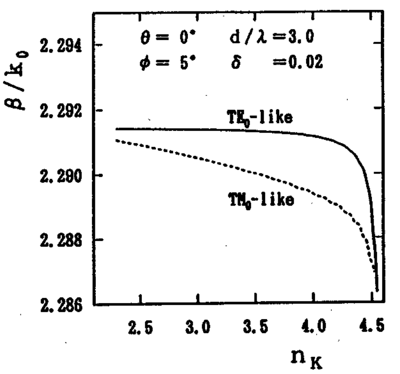


Fig.5 (a) Phase constants as a function of the period of acoustic waves for $\theta = 0°$.

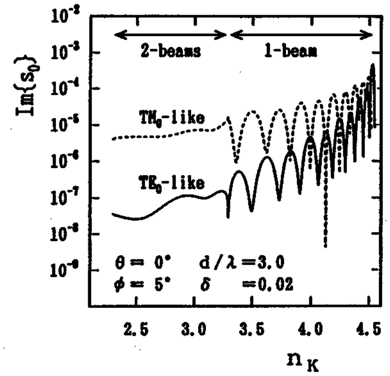


Fig.5 (b) Attenuation constants as a function of the period of acoustic waves for $\theta = 0°$.

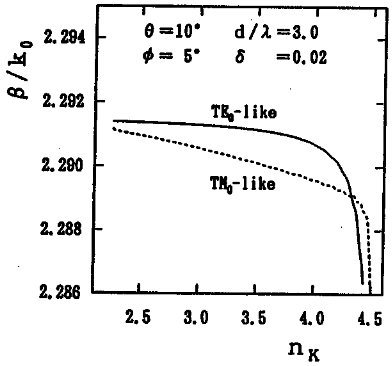


Fig.6 (a) Phase constants as a function of the period of acoustic waves for $\theta = 10°$.

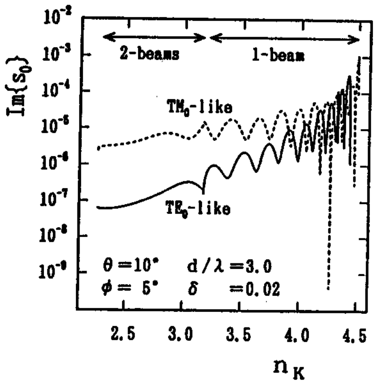


Fig.6 (b) Imaginary part of propagation constants along z axis as a function of the period of acoustic waves for $\theta = 10°$.

Similarly for the obique propagation of leaky waves the curves are shown is Fig.6(a) and (b). Where the modulation factor is $\delta = 0.02$, the inclination angle of crystal axis is $\phi = 5°$ and the thickness of the grating region is $d/\lambda = 3.0$. It can be seen that the phase constants decrease near the stop band $n_K \fallingdotseq 4.505$, while the attenuation constants change rapidly in the 1-beam radiation region.

We finally show the accuracy of the solution by the truncation size of the field expansion in table 1. Here,the parameters correspond to the grating stuctures used in Fig.4 except for the fixed value $\delta = 0.02$. The convergence of the propagation constants for truncation size is very fast for such a small modulation factor.

Table 1. Accuracy of the solutions by the truncation size of the field expansions.

$\delta = 0.02$, $\theta = 10°$, $\phi = 5°$, $d/\lambda = 3.0$, , $n_K = 3.0$

	(TE$_0$-like)		(TM$_0$-like)	
2M+1	β/k_0	Im(s_0)	β/k_0	Im(s_0)
3	2.29128594476,	3.309E-07	2.29056561679,	5,535E-06
5	2.29128578932,	3.309E-07	2.29056593424,	5.539E-06
7	2.29128578934,	3.309E-07	2.29056593414,	5.539E-06
9	2.29128578934,	3.309E-07	2.29056593414,	5.539E-06
11	2.29128578934,	3.309E-07	2.29056593414,	5.539E-06
13	2.29128578934,	3.309E-07	2.29056593414,	5.539E-06

6. Conclusions

The formulation of rigorous numerical method on grating waveguides has been presented under the most general conditions such as anisotropic waveguides described by arbitrary tensor permittivities and oblique propagation with respect to the periodic variation of the grating. This formulation has been applied to the analysis for oblique propagating leaky waves in anisotropic dielectric waveguides with the index-modulation caused by the acoustic shear wave. In this method any arbitrary level of accuracy can be obtained by increasing the number of space harmonics retained in the numerical calculations. Although this analysis has been performed only for the guidance of light waves, this method may be applied to the analysis for the three dimensional scattering problems from the anisotropic media.

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References

1) E.N.Glytsis and T.K.Gaylord. Applied Optics. 27, No.24/15,pp.5031(1988).
2) D.Marcuse. IEEE J.Quantum Electron, QE-11,pp.759(1977).
3) K.Araki and T,Itoh. IEEE Trans. Microwave theory Tech., MTT-29,pp. 911(1981).
4) S.Erkin, N.S.Chang, H.Maheri and M.Tsutsumi. IEEE, Microwave theory Tech., MTT-36,pp.568(1988).
5) K.Rokushima and J.Yamakita. J.Opt.Soc.Am. 73, pp.901(1983).
6) R.Petit and G.Tayeb. SPIE Proc.815,pp.11(1987).
7) E.N.Glytsis and T.K.Gaylord, J.Opt.Soc.Am., A, 4, pp.2061(1987).
8) J.Yamakita, K.Matsumoto, S.Mori and K.Rokushima. IEICE of Japan, J73-C-I, 1,pp.1(1990).
9) J.Yamakita, K.Matsumoto and K.Rokushima. SPIE Proc.1319, ICO-15,pp.337 (1990).
10) S.T,Peng. J.Opt.Soc.Am.A, 6,No.12, pp.1869(1989).
11) M.Matsumoto, M.Tsutsumi and N.Kumagai, IEEE Trans. Microwave theory Tech., MTT-35, pp.1033(1987).
12) Jin Li, J.Yamakita and K.Rokushima. Proc. of OFSET'90, EMT90-98-112, pp. 115(1990)
13) A.Yariv and P.Yeh ; *Optical Waves in Crystals,* John Wiley & Sons, Chapter 9, 1984.
14) R.E.Collin and F. J. Zucker, Eds. ; *Antenna Theory,* part 2, McGraw-Hill, Chapter 19, 1969.