Design of Rectifying Inspection Plans by Variables

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Although the inspection is to control the fractition defective in the lot inspected, there often appears that the lot quality characteristic obeys a continuous distribution. Two kinds of rectifying inspection plans, called LTPD plans and AOQL plans, are considered for which the average total inspection is to be minimized subject to particular side conditions. In this paper we propose the rectifying inspection plans by variables with a sampling plan having sample size and acceptance coefficient under the condition that the lot quality characteristic obeys a normal distribution. It is illustrated that the average total inspection can be economized by using the proposed inspection plans.

1. Introduction

Two kinds of rectifying sampling plans, called LTPD (lot tolerance percent defective) and AOQL (average outgoing quality limit) plans which have been given extensive treatment by Dodge and Romig[1], are considered for which the average total inspection (ATI) is to be minimized subject to particular side conditions. The third kind of plans developed by Hall and Hassan[2] are called outgoing quality (OQ) plans. All of those three types of plans are rectifying inspection plans by attributes with a sampling plan (n,c) having sample size n and acceptance number c.

Although the inspection is to control the fraction defective in the lot inspected, there often appears that the lot quality characteristic obeys a continuous distribution. When the lot quality characteristic obeys a normal distribution with unknown variance, Bender[3] produced a table of variables sampling plans, fitted to the attribute plans of Table II-A of MIL-STD-105D, by means of an iterative computer program based on the noncentral t-distribution. Hamaker[4] demonstrated that the OC curve for s-method plans could be adequately derived from a normal approximation and that the more complicated use of the noncentral t-distribution could be avoided. Recently, by using Hamaker's approximations, Govindaraju[5] provided procedures and tables for the selection of variables sampling plans for given AQL (acceptable quality level) and AOQL, whenever rejected lots were 100% inspected for replace-

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ment of nonconforming units. However, the rectifying sampling plans provided by Govindaraju do not always satisfy the requirement of minimizing ATI. In this paper we propose procedures for determining variables sampling plans for the two kinds of rectifying inspection plans, called LTPD and AOQL plans, under the condition that the lot quality characteristic obeys a normal distribution. It is illustrated that the average total inspection can be economized by using the proposed inspection plans.

2. Variables Sampling Plans

Suppose that the quality characteristic x is measurable on a continuous scale and normally distributed with mean μ and standard deviation σ. The following notations will be used:

N : lot size
S_u : upper specification limit
$\bar{p}$: process average
p_1 : lot tolerance percent defective
$\phi(\cdot)$: p. d. f. of N(0, 1)
$\Phi(\cdot)$: c. d. f. of N(0, 1)
z_p : upper p-quantile value of N(0, 1)
n : sample size
k : acceptance coefficient
$L(p)$: probability of acceptance
ATI : average total inspection

where we put the 'σ' and 's' on foot to indicate the cases in which the standard deviation σ is known(σ-method) and unknown(s-method), respectively.

The acceptance criterion for the σ-method plan is

$$\begin{cases} \text{if } \bar{x}+k_\sigma\sigma \leqq S_u, \text{ then accept the lot,} \\ \\ \text{otherwise reject the lot,} \end{cases} \qquad (1)$$

where $\bar{x}$ is the average quality characteristic derived from the sample. Under this criterion, the probabillity of acceptance will be

$$L_\sigma(p) = \int_{-\infty}^{\omega} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\,\mathrm{dz} = \Phi(\omega) \qquad (2)$$

with

$$\omega = \sqrt{n_\sigma}(S_u - k_\sigma - \mu)/\sigma. \qquad (3)$$

Notice that the fraction defective p in the lot is described as

$$p=\int_{(S_u-\mu)/\sigma}^{\infty}\phi(z)\,dz, \tag{4}$$

therefore, from Eqs. (3) and (4), ω is found by

$$\omega=\sqrt{n_\sigma}(z_p-k_\sigma) \tag{5}$$

When the standard deviation σ is unknown, the acceptance criterion of Eq. (1) is replaced by

$$\begin{cases}\text{if } \bar{x}+k_s s\leqq S_u, \text{ then accept the lot,}\\ \text{otherwise reject the lot,}\end{cases} \tag{6}$$

where s is the customary estimate of σ. Under this criterion, the probability of acceptance will be based on the noncentral t-distribution. To avoid the complication of using the noncentral t-distribution, we use the approximations given by Hamaker[4]. Since the relationships between parameters are

$$k_\sigma=k_s(4n_s-5)/(4n_s-4) \quad , \tag{7}$$

$$\frac{1}{n_\sigma}=\frac{1}{n_s}+\frac{k_s^2}{2(n_s-1)} \quad , \tag{8}$$

then we may expect that the σ-method and s-method plans will pass nearly identical OC curve as follows

$$L_s(p)\simeq L_\sigma(p). \tag{9}$$

3. Design of Rectifying Inspection Plans When the Variance is Known

When all nonconforming units found in the rejected lots are replaced by conforming units in a rectifying inspection plan, the average total inspection (ATI) is given by

$$\mathrm{ATI}_\sigma=n_\sigma L_\sigma(\bar{p})+N[1-L_\sigma(\bar{p})]=N-(N-n_\sigma)L_\sigma(\bar{p}), \tag{10}$$

where $\bar{p}$ is the process average.

3.1 LTPD Plans

When standard deviation σ is known, the design problem of LTPD plans is to minimize the avarage total inspection ATI defined by Eq. (10), subject to

$$L_\sigma(p_1) \leqq \beta, \tag{11}$$

where β is a given consumer's risk. From the economic viewpoint, the sampling plans should be designed under the condition of $L_\sigma(p_1)=\beta$. In this case, the auxiliary variable ω_1 corresponding to $L_\sigma(p_1)=\beta$ will be fixed at $\omega_1 = z_{1-\beta}$ by using Eq.(2). Hence, based on Eqs.(3) ~ (5) and given (p_1, β), we can obtain the following relations:

$$\bar{\omega}=z_{1-\beta}+\sqrt{n_\sigma}\ (z_{\bar{p}}-z_{p_1}), \tag{12}$$

$$k_\sigma=z_{p_1}-z_{1-\beta}/\sqrt{n_\sigma} \tag{13}$$

where $\bar{\omega}$ is the auxiliary variable corresponding to the value of $L_\sigma(\bar{p})$. Then from Eq.(12), it can be found that the probability of acceptance is a function of one variable of sample size n_σ, so that ATI_σ is a function of n_σ too. Hence we can find the optimal sample size n_σ minimizing ATI_σ in a simple way. In fact, when n_σ is considered as a real number in the region of $0<n_\sigma \leqq N$, if $\beta \geqq 0.023$, that is $z_{1-\beta}^2 \leqq 4$ and logically $\bar{p}<p_1<0.5$, we find

$$\frac{\mathrm{dATI}_\sigma}{\mathrm{d}n_\sigma}=\Phi(\bar{\omega})-\frac{(N-n_\sigma)}{2\sqrt{n_\sigma}}\phi(\bar{\omega})(z_{\bar{p}}-z_{p_1}) \tag{14}$$

and

$$\frac{\mathrm{d}^2\mathrm{ATI}_\sigma}{\mathrm{d}n_\sigma^2}=\frac{N-n_\sigma}{4\sqrt{n_\sigma}}\left\{\left(z_{\bar{p}}-z_{p_1}+\frac{z_{1-\beta}}{2\sqrt{n_\sigma}}\right)^2+\frac{4-z_{1-\beta}^2}{4n_\sigma}+\frac{4}{N-n_\sigma}\right\}(z_{\bar{p}}-z_{p_1})\phi(\bar{\omega})>0 \tag{15}$$

so that ATI_σ is a convex function of n_σ. In another way, we have:

$$\lim_{n_\sigma\to+0}\frac{\mathrm{dATI}_\sigma}{\mathrm{d}n_\sigma}=-\infty$$

$$\left.\frac{\mathrm{dATI}_\sigma}{\mathrm{d}n_\sigma}\right|_{n=N}>0$$

then there is only one n_σ satisfying $\frac{\mathrm{dATI}_\sigma}{\mathrm{d}n_\sigma}=0$ in region of $0<n_\sigma<N$. Thus, the optimal n_σ can be found easily and further acceptance coefficient k_σ will be calculated by using Eq.(13).

3.2 APQL Plans

If the fraction defective of the accepted lot is p and all defectives found in the rejected lots are replaced by nondefectives, the average outgoing quality (AOQ) will be

$$\mathrm{AOQ}_\sigma(p)=pL_\sigma(p) \tag{16}$$

where p is defined by Eq.(4). The problem is to minimize ATI_σ subject to

$$\sup_p \mathrm{AOQ}_\sigma(p)\leqq \mathrm{AOQL}. \tag{17}$$

Here we also consider the case of the equal condition of Eq.(17). A value of p^* is assumed and AOQ_σ at p^* will be

$$p^*L_\sigma(p^*)=\mathrm{AOQL}. \tag{18}$$

Therefore p^* must be satisfied by

$$\left.\frac{\mathrm{dAOQ}_\sigma}{\mathrm{d}p}\right|_{p=p^*}=\Phi(z_{p*})\Phi(\omega^*)/\sigma-\sqrt{n_\sigma}\ \phi(\omega^*)\ \{1-\Phi(z_{p*})\}/\sigma=0. \tag{19}$$

From Eqs. (18) and (19)

$$\sqrt{n_\sigma}\ \phi(\omega^*)(p^*)^2/\phi(z_{p*})=\mathrm{AOQL} \tag{20}$$

and

$$\omega^*=z_{1-\mathrm{AQOL}/p*} \tag{21}$$

where ω^* is the auxiliary variable corresponding to value of $L_\sigma(p^*)$. By using Eqs. (20) and (21)

$$n_\sigma=\frac{(\mathrm{AOQL})^2}{(p^*)^4}\ \exp\ \{z_{1-\mathrm{AOQL}/p*})^2-(z_{p*})^2\} \tag{22}$$

$$k_\sigma=z_{p*}-z_{1-\mathrm{AOQL}/p*}/\sqrt{n_\sigma}. \tag{23}$$

So that we can obtain p^* by using Eq.(22) at fixed n_σ with numerical methods, then calculate k_σ with Eq.(23) and calculate ATI_σ with Eq.(10). Following this process, we can get the optimal solution $(n_\sigma,\ k_\sigma)$ by using iterative computer programs in which ATI_σ values are compared.

4. Design of Rectifying Inspection Plans When the Variance is Unknown

When σ is unknown, under the relationship between $(n_\sigma,\ k_\sigma)$ and $(n_s,\ k_s)$ satisfying Eqs.(7) and (8), the average total inspection can be calculated approximately as

$$\mathrm{ATI}_s=N-(N-n_s)L_s(\bar{p})\simeq N-(N-n_s)L_\sigma(\bar{p}). \tag{24}$$

4. 1 LTPD plans

The problem of a LTPD plan is

$$\begin{aligned} &\mathrm{ATI}_s = N - (N - n_s) L_\sigma(\bar{p}) \rightarrow \min \\ &\text{subject to} \quad L_s(p_1) = \beta. \end{aligned} \tag{25}$$

From Eqs.(7), (8) and (13), the relationship between n_σ and n_s will be

$$a(\sqrt{n_\sigma})^2 + b\sqrt{n_\sigma} + c = 0 \tag{26}$$

where

$$a = 2(n_s - 1)(4n_s - 5)^2 + n_s(4n_s - 4)^2 z_{p_1} \tag{27}$$

$$b = -2n_s(4n_s - 4)^2 z_{p_1} z_{1-\beta} \tag{28}$$

$$c = n_s(4n_s - 4)^2 z_{1-\beta} - 2(n_s - 1)(4n_s - 5)^2. \tag{29}$$

Generally in the case of $p_1 < 0.5$ and $\beta = 10\%$, the negative solution of Eq.(26) is ignored and only one n_σ can obtained from equation Eq.(26) at fixed n_s. Hence we can calculate k_σ by using Eq.(13) and then calculate ATI_s with Eq.(24) at this fixed n_s, so that we can find the optimal sample size n_s by comparing ATI_s with a computer program and then find k_s by Eq.(7).

4. 2 AOQL plans

When σ is unknown, the problem of an AOQL plan is

$$\begin{aligned} &\mathrm{ATI}_s = N - (N - n_s) L_\sigma(\bar{p}) \rightarrow \min \\ &\text{subject to} \sup_{p} \ \mathrm{AOQ}_s(p) = \mathrm{AOQL}. \end{aligned} \tag{30}$$

Dealing with Hamaker's formulae Eqs.(7), (8) and Eqs.(22), (23), the relationship between p^* and n_s can be drawn out by eliminating the k_s, n_σ k_σ, so that we can proceed in the same way to find p^* at fixed n_s with numerical methods and then obtain n_σ, p_σ from Eqs.(22) and (23). Therefore $L_\sigma(\bar{p})$ can be found by using Eqs. (2) and (5), then ATI_s can be calculated by Eq.(24). The problem will be solved with the help of iterative computer programs in which ATI_s values are compared too.

5. Examples

Table 1 gives some examples of LTPD variable plans compared with corresponding attribute plans and Table 2 gives those examples of AOQL plans. By using the information of normal distribution, the average total inspection by using variable plans can be economized in comparison with using attribute plans.

Table 1 Single Samling LTPD Plans by Variables with p_1 = 1 % and β=10%

$\bar{p}$(%)	$\bar{p}$=0.05%				Attribute plans	$\bar{p}$=0.1%				Attribute plans
N	n_σ n_S	k_σ k_S	ATI_σ ATI_S	$L_\sigma(\bar{p})$ $L_S(\bar{p})$	n, c, ATI	n_σ n_S	k_σ k_S	ATI_σ ATI_S	$L_\sigma(\bar{p})$ $L_S(\bar{p})$	n, c, ATI
500	16 53	2.647 2.725	18.43 65.07	0.9950 0.9730	180,0,207.54	23 70	2.594 2.665	27.12 89.30	0.9914 0.9551	180,0,232.71
1000	18 62	2.629 2.690	20.45 74.14	0.9975 0.9871	205,0,282.45	26 85	2.578 2.629	30.38 104.00	0.9955 0.9792	205,0,352.36
5000	22 82	2.600 2.636	24.98 93.60	0.9994 0.9976	385,1,460.29	33 117	2.550 2.580	37.72 135.17	0.9991 0.9963	530,2,604.94
10000	24 90	2.588 2.620	26.89 101.58	0.9997 0.9988	530,2,554.11	36 130	2.540 2.565	40.81 147.84	0.9995 0.9982	530,2,688.76

Table 2 Single Samling AOQL Plans by Variables with AOQL = 0.5%

$\bar{p}$(%)	$\bar{p}$=0.05%				Attribute plans	$\bar{p}$=0.1%				Attribute plans
N	n_σ n_S	k_σ k_S	ATI_σ ATI_S	$L_\sigma(\bar{p})$ $L_S(\bar{p})$	n, c, ATI	n_σ n_S	k_σ k_S	ATI_σ ATI_S	$L_\sigma(\bar{p})$ $L_S(\bar{p})$	n, c, ATI
500	8 23	2.332 2.377	9.65 28.53	0.9967 0.9884	70,0,84.80	11 29	2.327 2.356	13.77 38.09	0.9943 0.9807	70,0,99.07
1000	9 27	2.329 2,361	10.94 32.90	0.9980 0.9939	70,0,63.36	13 35	2.328 2.346	15.94 45.28	0.9970 0.9893	70,0,95.60
5000	12 37	2.327 2.344	14.10 43.79	0.9996 0.9986	165,1,180.58	18 52	2.335 2.340	21.34 63.79	0.9993 0.9976	165,1,224.01
10000	13 41	2.328 2.341	15.58 48.80	0.9997 0.9992	165,1,196.68	20 60	2.338 2.341	23.82 72.47	0.9996 0.9987	270,2,296.10

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