Optimization of Friction Welding Condition of S45C Carbon Steel by Response Surface Method

Optimization of Friction Welding Condition of S45C Carbon Steel by Response Surface Method

Hiroshi YAMAGUCHI*, Koichi OGAWA* and Kazuhiko SAKAGUCHI**

(Received June 15, 1991)

The brake type friction welding condition was investigated using response surface method. The optimum welding condition to yield maximum tenstile strength at the weld was attained through a steepest ascent path. Factors of brake type friction welding are friction pressure, up-set pressure, friction time, rotating speed and braking time. Considering these welding parameters as independent variables, and tensile strength as dependent variable, some experiments were carried out to obtain the optimum welding condition. As a result, a second-order equation for predicting tensile strength was established without significant lack of fit to the data. Several confirmatory tests at the optimum welding condition were performed and excellent results were obtained. A successful weld has an average tensile stregth 20 percent higher than that of the base metal.

1. Introduction

Friction welding is a welding process which effectively utilizes frictional heat, and subsequently forms a solid state bond at the interface. The process works on a wide variety of similar and dissimilar metals. The principle of the process is illustrated schematically in Fig. 1. In friction welding, one of the two members to be welded is rotated relative to the other, and the two are then brought together by an axial force. The friction between the two rubbing surfaces generates heat causing the materials at the interface to soften and then deform radially. The parameters in the process which dictate the quality of the weld are friction pressure, up-set pressure, friction time, up-set time, rotating speed, braking time, and others. Determination of these parameters has been primarily based on experiences attained through extensive experimental runs becouse of lack of theoretical formula. It has been found that each conbination of materials and each joint design has its own peculiar range of optimum parameter levels. Consequently, a considerable amount of effort is generally needed in attaining an operating condition for a new of material or a new joint design. Previous study[1] of the thermal aspects of the process showed that the specific frictional heat generated at the interface can be expressed

$$q(t)=K \cdot \mu \cdot p \cdot r \cdot n \tag{1}$$

* Course of Instrument Science, College of Integrated Arts and Sciences.

** Faculty of Engineering, Doshisha University.

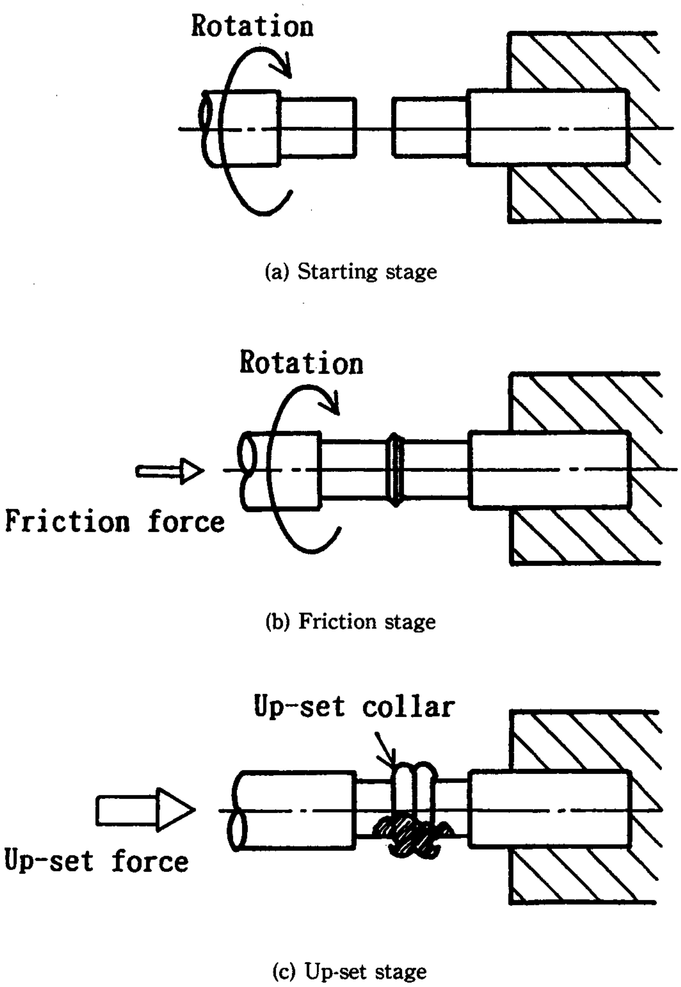

Fig. 1 Schematic diagram of the brake type frictlon welding process.

where K is a constant, μ is the coefficient of friction, p is the unit axial pressure in MPa, r is the radial distance in millimeter from the axis of rotation, and n is the rotating speed in rpm. However, it has not been made clear whether the equation can be use for employing of optimum friction welding condition. The major concern of a welding process is the joint efficiency usually based on tensile or fatigue strength of the weld as compared with the base material. Since a thoretical relationship between weld strength and welding parameters is not available, the purpose of this paper is to establish empirical model where maximum weld strength or successful weld are included. Especially, in this paper the fitted equation can be used to predict tensile strength of a weld within the optimum region explored, and the response surface method used in this study to attain such an optimum region is presented in detail.

2. Experimental setup and procedures

The friction welding apparatus which takes a brake type and a inertia type process was specifically fabricated for this study. The material used in this study is S45C carbon steel for machine construction. The chemical composition and mechanical properties of the material are shown in Table 1 and Table 2. After performing the friction welding, a tensile test was carried out to investigate the strength of the weld using the test specimen having shape and dimensions as shown in Fig. 2.

Table 1 Chemical composition of base material

Material	C	Si	Mn	P	S
S45C	0.46	0.13	0.66	0.012	0.09

Table 2 Machanical properties of base material

Material	Tensile Strength σ_B(MPa)	Elongation δ(%)	Vickers Hardness H_V
S45C	670	22.8	210

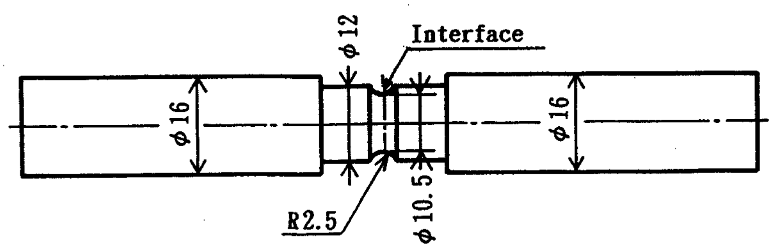

Fig. 2 Shape and dimensions of tensile test specimen.

3. Exploration of the response surface

3. 1 Out line of response surface

Conventionally, a sure way to find the optimum conditions would be to explore the whole experimental region. This would involve carrying out experiments on a grid of points throughout the region. A multi-dimensional grid search as required in this case would become economically prohibitive. The technique used in this study is the response surface method[2] by which the optimum region is reached through a steepest ascent path. The method essentially conbines the techniques of factorial experiments and regression analysis in a sequential manner. A regression equation can be established to fit the response surface in the optimum region when it is reached.

3. 2 First factorial design

To start exploring the response surface a two-level, five-factor factorial design was first implemented[3]. The five factors refer to the controllable variables in friction welding; the friction pressure(P_1), the up-set pressure(P_2), the friction time(t_1), the rotating speed (N) and the braking time(t_B). Considering these welding parameters as dependent variables and tensile strength as independent variable, the response surface method runs was carried out. Based on the performance of the friction welding apparatus, the center of the factorial design and the factor levels were selected as shown in Table 3. For convenience in calculation, factors or

Table 3 Factor levels for the 1st factorial design.

Variables			Levels Set for Experiments			
Friction welding factors ζ_k		coded	Low(-1)	High(+1)	Center(0)	Unit size
Friction pressure	P_1(MPa)	X_1	20	30	25	5
Up-set pressure	P_2(MPa)	X_2	30	45	37.5	7.5
Friction time	t_1(s)	X_3	0.5	1.5	1.0	0.5
Rotating speed	N (rpm)	X_4	1500	2500	2000	500
Braking time	t_B(s)	X_5	0.5	1.8	0.8	+1.0, -0.3
Starting time of up-set pressure Ps(s)=0 , Up-set time t_2(s)=10.0						

independent variables in physical units were transformed into standardized or coded variables x_1,x_2,x_3,x_4,x_5[4]. The low and high levels of all five variables were set at -1 and $+1$ unit, respectively, are in dimensionless unit used in the design with its size depending on the scale selected for each factor. The design matrix and the corresponding responses are given in Table 4. Assuming the response surface of the subregion can be approximated by a sloping plane the slopes $\beta_1,\beta_2,\beta_3,\beta_4,\beta_5$ of the plane in the direction of x_1,x_2,x_3,x_4,x_5, respectively, may be obtained by fitting the first order equation

$$\eta=\beta_0 x_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3+\beta_4 x_4+\beta_5 x_5 \tag{2}$$

to the data of the experiment. η in eq.(1) is the response at a given set of factor levels, and x_0 is a dummy variable which is always equal to one. The least-square estimates for the regression coefficients β's of eq.(2)are:

$b_0=397.3,\ b_1=155.4,\ b_2=160.3$

$b_3=12.9,\ b_4=136.8,\ b_5=106.3$

The standard error for these estimates was computed to be ± 63.5 based on the data from the repeated experiments. The regression equation which represents the sloping plane in the sub-region becomes

$$\hat{y}=397.3+155.4x_1+160.3x_2+12.9x_3+136.8x_4+106.3x_5 \tag{3}$$

Table 4 Design matrix and experimental results of the lst factorial experiment.

Trial No.	Design Matrix X_1	X_2	X_3	X_4	X_5	Tensile Strength σ_B(MPa)
1	1	1	1	1	1	858.62
2	1	1	1	-1	-1	499.16
3	1	1	-1	1	-1	639.78
4	1	1	-1	-1	1	528.03
5	1	-1	1	1	-1	60.02
6	1	-1	1	-1	1	522.28
7	1	-1	-1	1	1	778.62
8	1	-1	-1	-1	-1	------
9	-1	1	1	1	1	806.70
10	-1	1	1	-1	-1	------
11	-1	1	-1	1	-1	593.9
12	-1	1	-1	-1	1	------
13	-1	-1	1	1	-1	------
14	-1	-1	1	-1	1	------
15	-1	-1	-1	1	1	------
16	-1	-1	-1	-1	-1	------

The constant term of eq.(3) represents the response when the variables are all set at the base level, that is, at the center of the factorial design. Since b_1, b_2, b_3, b_4, b_5 are all positive, increasing any one of the variables would improve the response. Accordingly, the direction in which maximum improvement in response tends upward, namely, the next factorial design was carried out with the direction would increase.

3. 3 Second factorial design

Based on the information obtained from the first experiment, a second factorial design was selected in the direction of improvement in response with the factor levels given in Table 5. From the second factorial experiment, the first-order regression equation was obtained as the following.

$$\hat{y} = 762.1 + 0.774x_1 + 18.3x_2 + 40.6x_3 + 0.929x_4 + 75.4x_5 \tag{4}$$

Further, the standard error for these estimates was computed to be ± 19.4 based on the data from the repeated experiments. Here, in order to decide the adequency of the eq.(4) the comparison of estimated value and observed value was carried out and

Table 5 Factor levels for the 2nd factorial design.

Variables		Levels Set for Experiments			
Friction welding factors ζ_k	coded	Low(-1)	High(+1)	Center(0)	Unit size
Friction pressure P_1(MPa)	X_1	25	45	35	10
Up-set pressure P_2(MPa)	X_2	25	67.5	45	+22.5, -20
Friction time t_1(s)	X_3	0.5	1.5	1.0	0.5
Rotating speed N (rpm)	X_4	2500	3500	3000	500
Braking time t_B(s)	X_5	0.5	1.8	0.8	+1.0, -0.3
Starting time of up-set pressure Ps(s)=0 , Up-set time t_2(s)=10.0					

the result is shown in Table 6. As there is little difference in the results, the calculation of steepest ascent path can be performed using eq.(4). The calulated of the steepest ascent path are summarized in Table 7.

Table 6 Comparison of estimated value and observed value.

Trial No.	Variables					Estimated Value	Observed Value	Ratio
	P_1	P_2	t_1	N	t_B	$\hat{y}$	σ_B(MPa)	$\sigma_B/\hat{y}$
33~36	35	55	3.0	3000	0.8	757.5	780.2	1.03

Table 7 Calculation of steepest ascent path and subsequent trials on the path.

Variables	P_1	P_2	t_1	N	t_B	Estimated Value $\hat{y}$	Observed Value σ_B(MPa)
Base level	35	45	1.0	3000	0.8		
Unit (x)	10	22.5	0.5	500	1.0		
Estimated slope (b)	0.774	18.3	40.6	0.929	75.4		
Unit x•b	7.74	411.75	20.3	464.5	75.4		
Possible trials	37	49.5	1.1	3100	1.2	784.7	-----
on the path of	39	54.0	1.2	3200	1.6	811.9	-----
steepest ascent	41	58.5	1.3	3300	2.0	839.1	-----
Trial No. 37T	40	56	1.25	3250	1.8	830.1	810.64
38T	45	68	1.5	3500	2.8	898.1	837.66
39T	50	79	1.75	3750	3.8	961.5	863.41

3.4 Third factorial design

As the regression values of x_1 and x_4 are fairly low in value than the standard error, their levels were stabilized as center conditions. Then, two-level, three-factor factorial design was implemented. Table 8 gives the design matrix and the results of the third factorial experiment. The fitted equation is

$$\hat{y} = 879.0 - 2.00x_2 + 5.08x_3 - 1.93x_5 \quad (5)$$

The standard error for the estimates of the coefficients is ± 13.6. In light of the

Table 8 Design matrix and experimental results of the 3rd factorial experiment.

Trial No.	Design Matrix			Tensile Strength
	X_2	X_3	X_5	σ_B(MPa)
41	1	1	1	831.14
42	1	1	-1	876.40
43	1	-1	1	817.83
44	1	-1	-1	788.25
45	-1	1	1	820.04
46	-1	1	-1	860.85
47	-1	-1	1	866.81
48	-1	-1	-1	780.86

experimental error, it is apparent that none of the effects can be significant. This is an indication that a stationary region of the response surface may have been reached.

3.5 Central composite design

The emphasis of the exploration at this point was turned to further investigate the characteristics of the stationary region. For this purpose the fourth factorial design was augmented to include three more levels with an attempt to bring out quadratic or even higher order effects if they do exist. The levels chosen include the center of the factorial design and two more levels at 1.68 times units extended from the center on each side of the three axes. The design consists of a total of 18 trials.

Table 9 gives the experimental results and the corresponding design matrix for the additional trials.

To examine the fitness of the model, an analysis of variance (ANOVA) was computed as shown in Table 10. From the result of ANOVA, the lack of fit does not signify for second-order model when they are tested by F-Observation at 5% significant level, then, the regressions of first-order and second-order are fit for second-order model. Accordingly, pooling the lack of fit, ANOVA was calculated as shown in Table 11. Then, least-square estimates for the regression coefficients β's with their respective standard errors were computed to be

$b_0=869.5$ $\quad$ $b_1=3.50\pm9.92$

$b_2=5.32\pm9.92$ $\quad$ $b_3=-1.03\pm9.92$

$b_{11}=6.55\pm10.3$ $\quad$ $b_{22}=-16.3\pm10.3$

$b_{33}=10.6\pm10.3$ $\quad$ $b_{12}=23.0\pm13.0$

$b_{13}=3.86\pm13.0$ $\quad$ $b_{23}=-17.2\pm13.0$

Table 9 Extended experimental points and results for the composite design.

Trial No.	Design Matrix			Tensile Strength σ_B (MPa)
	X_2	X_3	X_5	
49	0	0	0	818.79
50	0	0	0	880.45
51	0	0	0	836.09
52	0	0	0	827.54
53	1.68	0	0	887.22
54	-1.68	0	0	849.25
55	0	1.68	0	813.32
56	0	-1.68	0	794.34
57	0	0	1.68	883.76
58	0	0	-1.68	875.74

Table 10 Analysis of variance for the composite design.

Factors	Sum of Squares	Degree of Freedom	Mean Squares	F Ratio F_0
Regression	1.544×10^4	9	1.715×10^3	5.41
(1st order)	2.010×10^3	3	6.699×10^2	2.11
(2nd order)	1.343×10^4	6	2.238×10^3	7.06
Lack of fit	9.797×10^3	5	1.960×10^2	6.18
Pure error	9.509×10^3	3	3.170×10^2	----
Total	2.618×10^5	17		

Based on the foregoing estimates the second-order model for the region becomes

$$\hat{y}=869.5+3.50x_2+5.32x_3-1.03+x_5 6.55x_2^2$$
$$-16.3x_3^2+10.6x_5^2-23.0x_2x_3+3.86x_2x_5-17.2x_3x_5 \quad (6)$$

Table 11 Analysis of variance for the composite design pooled to lack of fit.

Factors	Sum of Squares	Degree of Freedom	Mean Squares	F Ratio F_0
Regression	1.544×10^4	9	1.715×10^3	1.28
(1st order)	2.010×10^3	3	6.699×10^2	0.50
(2nd order)	1.343×10^4	6	2.238×10^3	1.67
Pure error	1.075×10^4	8	1.344×10^3	----
Total	2.618×10^5	17		

3. 6 Canonical reduction of the model

Equation(6) is in a form of a general quadratic equation which it is rather difficult to characterize the nature of the surface represented. A quadratic surface could have a unique center which represents a maximum or it may form a ridge without a center at all[5]. However, it is known that a general quadratic equation can be reduced to a standard or canonical form by eliminating all linear and mixed terms[6]. In the case of a three-variable quadratic surface, its canonical form is

$$Y-Y_0=B_{11}X_2^2+B_{22}X_3^2+B_{33}X_5^2 \qquad (7)$$

where Y_0 is the response at center of a central quadratic surface and B_{11}, B_{22} and B_{33} are new coefficients for the quadratic terms.

Geometrically, canonical reduction is achieved by transformation of the coordinate system. The new coordinate system has its origin at the center of the quadratic and the surface is symmetrical to the new axes X_1,X_2 and X_3.

The canonical form of the response surface after transformation of the axes is then

$$Y-866.0=16.8X_2^2+68.0X_3^2-2.27X_5^2 \qquad (8)$$

If the stoppage point of eq.(8) is to be the optimum welding condition, they can be transformed back in terms of physical variables, P_2, t_1 and t_B.

After transformation, the optimum welding conditions can be represented as follows:

$$P_1=35\text{MPa},\ P_2=55\text{MPa},\ t_1=4\text{s},$$
$$t_2=10\text{s},\ t_B=2.7\text{s},\ N=3000\text{rpm}$$

Several confirmatory tests at the optimum welding conditions were performed and the excellent results were obtained.

4. Conclusions

The main conclusions obtained in this study are as follows:

1) Response surface method provided a faster way to attain an optimum welding

condition for the brake type friction welding process over a wide experimental range.

2) The optimum region for brake type friction welding of S45C carbon steel is rather broad, i.e., a relatively wide range of operating conditions would produce successful welds.

3) The tensile strength at the weld is about 20 percent higher than the base material.

References

1) K.K. Wang and P. Nagappan, Welding Journal, Welding Research Supplement, Sept. pp.419 -426 (1970).

2) G.E.P. Box and K.B. Wilson, Journal of the Royal Statistical Society, Series B, Vol.13, No.1, pp.1-38 (1951).

3) O.L. Davis (Editer), The Design and Analysis of Industrial Experiments, Hafner Publishing Cl., 1963.

4) S.M. Wu, Journal of Engineering for Industry, Trans. ASME, Series B,Vol.86,No.2,May, pp.105 -116(1964).

5) G.E.P. Box, Biometries, Mar., pp.16-60(1954).

6) N.V.Yefimov, "Quadratic Forms and Matrices", Academic Press, (1964).