An Edge Detector Using Variable Median Filter

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Yoichi JYO* and Kazuo NOHARA*

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In this paper we present a new edge detector for two dimensional image data. The algorithm is based upon the sorting procedure of variable median filter. This edge detector has the ability of noise elimination and edge detection in one algorithm.Therefore, the process of noise elimination before edge detection is unnecessary any more.

Because variable median filter eliminates impulse noise completely, our proposed algorithm is a useful tool specially for images corrupted with impulse noise.

Applications to a two dimensional test image are demonstrated.

1. Introduction

Edge detecting is one of the fundamental image processing operations. There are some requirements to be satisfied by an ideal edge detector operating on an image corrupted by noise. The classical edge detectors such as Sobel or Prewitt types are still widely used[1]. Realistic image signals may sometimes contain impulse-like structures. Sobel and Prewitt edge detectors cannot avoid the detecting false edges caused by noise. For example, grained edges are extracted for impuilse noises. Therefore, such noises should be eliminated before edge detection. Noise filtering becomes a particularly difficult task when the original signal contains sharp edges and impulses. In image processing it is very important to preserve these discontinuities while filtering out noises, since the human visual system is very sensitive to edge information.

The application of median filter has been made in many areas of digital signal processing, including speech processing and image processing. The median filter takes the values of image data from an input window and writes them into a list after sorting. One useful property of median filter is the ability to eliminate impule noise while preserving signal edges. Miyazawa and Ejima proposed an edge detector (ME detector) with sort matching based upon regular median filtering procedure[2]. The regular median filters often exhibit blurring for large window sizes,or insufficient noise elimination for small window sizes.

* Department of Electronics, College of Engineering

In this paper we propose an edge detector (VME detector) using variable median-filtering procedure[3]. Variable median filter preserves edges and eliminates impulse noise completely. The proposed edge detector is a suitable tool for images which have been corrupted with impulse noise.

2. Edge Detecting Procedure

2.1 Regular and variable median filters

For two dimensional image data,we let x_i denote the value of the interested pixel. As a window a $n \times n$ (n is odd) square mask is used as shown in Fig. 1. The window moves across the image, for example line by line, and each value of the center pixel (marked with * in Fig. 1) in the window is substituted by the filter output.

Let $X=(x_1, x_2, \cdots\cdots, x_i, \cdots\cdots, x_m)$ be the odd numbered list of the values in the window, where $m=n^2$.

Denote the sorted list by

$$\begin{aligned}\tilde{X} &= (x_{p(1)}, \cdots\cdots, x_{p(m)}) \\ &= (y_1, \cdots\cdots, y_m),\end{aligned} \tag{1}$$

whereby the permutation p sorts the values according to their magnitude:

$$x_{p(1)} \geqq \cdots\cdots\cdots \geqq x_{p(m)}. \tag{2}$$

The median is defined as the value with the center index:

$$\mathrm{med}(x_1, x_2, \cdots\cdots, x_i, \cdots\cdots, x_m) = y_{(m+1)/2}. \tag{3}$$

For the regular median filter each value of the center pixel in the window is

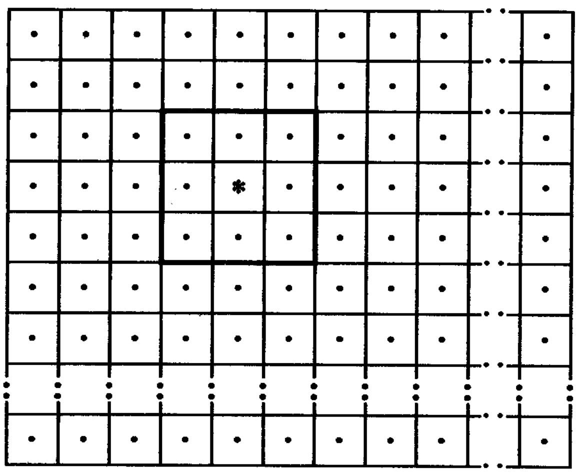


Fig. 1 Two dimensional image data and a $n \times n$ window (for $n=3$).

substituted by $y_{(m+1)/2}$.

On the other hand, the sorted list of the variable median filter is obtained as follows.

Let y_c be the value of the center pixel of the input window. Construct the sorted list with multiplicity M containing M copies of y_c:

$$\begin{aligned}\tilde{X}_M &= (y_1, \cdots\cdots, y_{c-1}, y_c, \cdots, y_c, \cdots\cdots, y_{c+1}, \cdots\cdots, y_m) \\ &= (z_1, \cdots\cdots\cdots, z_{m+M-1})\end{aligned} \tag{4}$$

The variable median with multiplicity M is defined as the median of this list:

$$\begin{aligned}\text{var med } (x_1, \cdots, x_i, \cdots, x_m) &= z_{(m+M-1+1)/2} \\ &= z_{(m+M)/2}.\end{aligned} \tag{5}$$

The median of this longer sorted list is the variable median and the original value of the center pixel in the window is substituted by $z_{(m+M)/2}$. Comparison of two sorted lists of regular and variable median filters is shown in Fig. 2. In 3) the properties of variable median filter are investigated in detail and it is demonstrated that the variable median filter does not only destroy pixel noise but also clears off noise

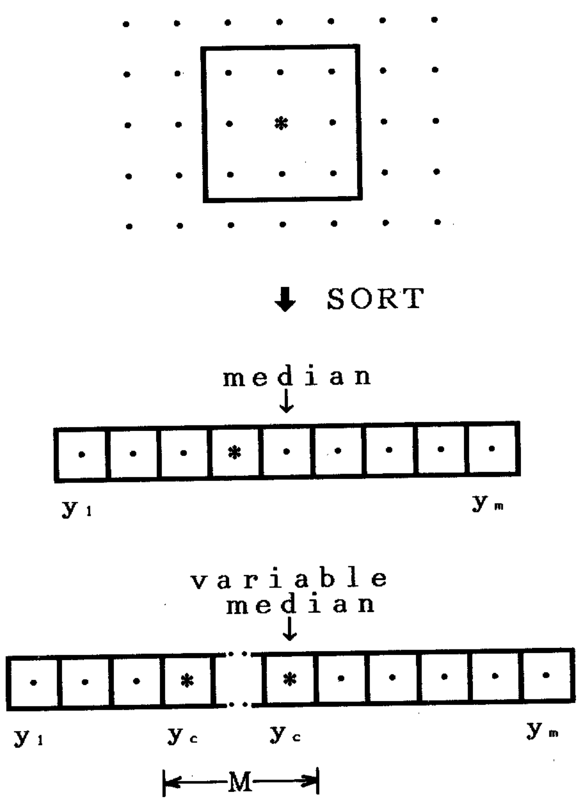


Fig. 2 Comparison of two sorted list (for $n=3$).

grains.

2.2 Edge detecting algorithm

The edge detecting algorithm with the variable median filtering procedure is presented bellow.

As shown in Fig. 3(a), we construct a set of $n \times n$ square window (A_1, A_2). The window A_2 is symmetric with respect to A_1 in the horizontal direction. Note that the interested pixel marked with * is the point of symmetry and not the center pixel in the window.

In much the same way as (A_1, A_2), we construct three other sets of windows (A_3, A_4) in the vertical direction, (A_5, A_6) and (A_7, A_8) in the diagonal directions as shown in Figs. 3(a) and (b).

Let $A_1(k)$ be the longer list containing $n(=M)$ copies of y_c, where $k(k=1, 2, \cdots\cdots, n^2+n-1)$ is the rank order index. Similarly the sorted list $A_2(k)$ for the window A_2 can be obtained. Then partition the sorted list in decending order into three subsets as shown in Fig. 4. Subset I has n highest level elements and subset III has n lowest level elements. For edge detecting only the elements of subset II are employed so as to minimize the influence of noise.

We introduce a new function S_{12} defined by the next equation.

$$S_{12}=\sum_{k=n+1}^{n^2-1} | A_1(k)-A_2(k) | . \tag{6}$$

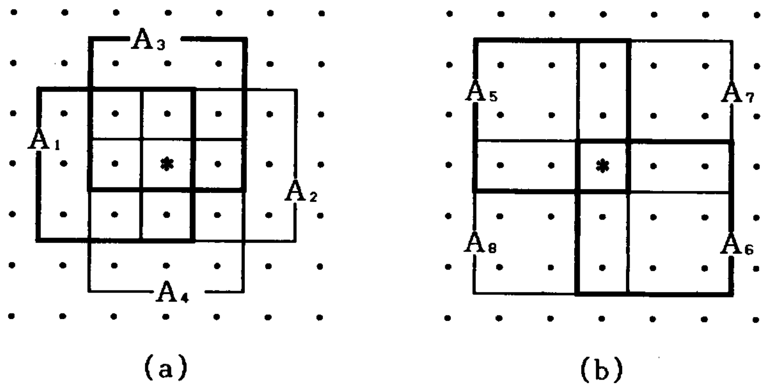


Fig. 3 The way of constructing the $n \times n$ windows $A_1 \sim A_8$ (for $n=3$).

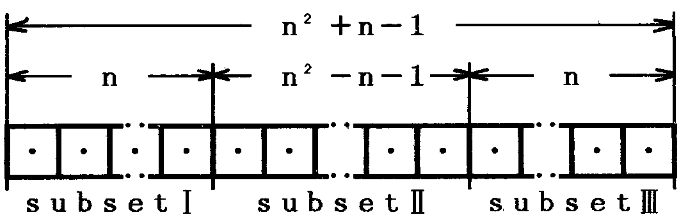


Fig. 4 The longer sorted list partitioned into three subsets.

Equation (6) means that S_{12} is the sum of absolute value of difference of image data which have the same rank index k in the sorted lists of $A_1(k)$ and $A_2(k)$.

Similarly S_{34}, S_{56} and S_{78} are defined as follows.

$$S_{34}=\sum_{k=n+1}^{n^2-1} \mid A_3(k)-A_4(k) \mid . \tag{7}$$

$$S_{56}=\sum_{k=n+1}^{n^2-1} \mid A_5(k)-A_6(k) \mid . \tag{8}$$

$$S_{78}=\sum_{k=n+1}^{n^2-1} \mid A_7(k)-A_8(k) \mid . \tag{9}$$

Using Eqs. (6)~(9),we define the edge density D by

$$D=(S_{12}+S_{34}+S_{56}+S_{78})/(n^2-n-1). \tag{10}$$

If an edge exists among the windows $A_1 \sim A_8$, the edge density D becomes large. Therefore, using an appropriate threshold level T the output value Y of the original interested pixel can be determined as follows.

$$Y=\begin{cases} 255, & \text{if } D \geqq T \\ 0, & \text{if } D<T. \end{cases} \tag{11}$$

A simple example is given in Fig. 5 to illustrate the advantage of the algorithm of VME detector compared with that of ME detector. It might be an extreme case, we suppose that the pixel values of upper part above the edge line are all 1 and those of lower part under the edge line are all 0.

For example, we construct a set of windows (A, B) in accordance with rules mentioned above, where the interested pixel is the point of symmetry. As a window 3×3 square mask is used. Figure 5(a) is the case that the interested pixel marked with ○ of original image data is on a boundary line and (d) is the other case. For the sorted list containing three copies of the value 1, total six elements in subset I and subset III are removed (Figs. 5(b) and (e)).

Due to this operation, the number of elements of the sorted lists reduces to five elements. Using only five elements in subset II the next equation can be obtained.

$$S_{AB}=\begin{cases} 3, & \text{for the case of Fig. 5(a)} \\ 0, & \text{for the case of Fig. 5(d).} \end{cases} \tag{12}$$

On the other hand, the result of algorithm using the sorted lists of Figs. 5(c) and (f) is given as follows.

$$S_{AB}'=\begin{cases} 4, & \text{for the case of Fig. 5(a)} \\ 1, & \text{for the case of Fig. 5(d).} \end{cases} \tag{13}$$

To calcurate S_{AB}' of Eq.(13), only five elements in subset II are used in the same manner as calculating S_{AB} of Eq.(12).

Equation (12) means that if the interseted pixel is the boundary pixel of the edge

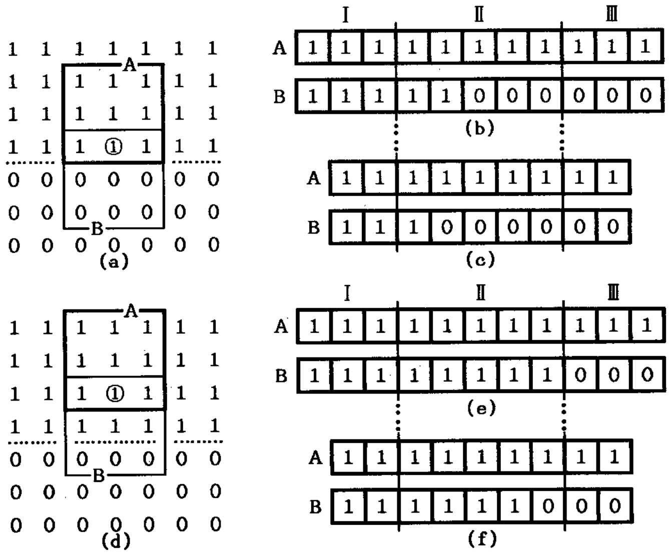


Fig. 5 The difference of the sorted lists for the case that the interested pixel marked with ○ is the boundary pixel or not (for $n=3$).
(a) An example for the case that the marked pixel is the boundary pixel of the edge.
(b) The longer sorted lists for (a).
(c) The sorted lists for (a).
(d) An example for the case that the marked pixel is not the boundary pixel of the edge.
(e) The longer sorted lists for (d).
(f) The sorted lists for (d).

S_{AB} has a positive finite value and if the interested pixel is far from the edge line S_{AB} is 0. This shows that VME detector can draw a clear distinction between the two cases of Figs. 5(a) and (d). On the other hand, S_{AB}' of Eq.(13) for ME detector has positive values for two cases.

For this reason the algorithm using variable median procedure is better than the algorithm using regular median procedure.

3. Simulation

In order to verify the performance of the VME detector, we perform several tesets with 256×256 GIRL image on PC-286VE computer. In all simulations, we use as a test image GIRL of 256 gray levels shown in Fig. 6. As the windows 3×3 square masks are used. In the following, the processed images are thresholded visually to achieve the best separation of edges from noise ensurring at the same time a low number of drop-outs from edges.

Fig. 6 The original 256×256 GIRL image.

(a) VME detector, $T=70$.

(b) ME detector, $T=90$.

(c) Sobel detector, $T=100$.

Fig. 7 Edge detecting results for the image of Fig. 6.

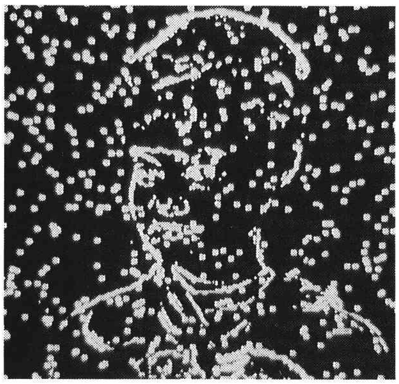
First, we study the ability of VME, ME and Sobel detectors for the original image of Fig. 6. The detecting results from three different detectors are shown in Figs. 7(a), (b) and (c). No large differences appear from the simulated results except that the response of ME detector is thicker at edges than the two other detectors.

Figure 8(a) shows the GIRl image with added impulse noises of the height 256. The impulse noises locate at random on the two dimensional image data. A result from processing by VME detector is shown in Fig. 8(b), and from ME and Sobel detectors in Figs. 8(c) and (d), respectively. Sobel detector detects many false edges caused by impulse noises. On the other hand, VME and ME detectors give fairly good agreement with the results of Figs. 7(a) and (b) except that a few false edges are detected due to adjacent impule noises. The response of ME detector has a tendency to be thicker at edges than VME detector.

(a) The GIRL image with added impulse noises of the height 256.

(b) VME detector, $T=80$.

(c) ME detector, $T=100$.



(d) Sobel detector, $T=115$.

Fig. 8 The GIRL image with added impulse noises and edge detecting results.

The main reseults, which emerges from the inspection of these pictures, is the good performance of the new VME detector.

4. Conclusions

An edge detector using the concept of the longer sorted list of variable median filter is proposed. Since the process of noise eliminating before edge detecting can be omitted, this algorithm improves the efficiency of the edge detecting.

Simulations show that this algorithm is useful for edge detecting of images corrupted with impuilse noises.

The decision method of threshold level T is the subject for a future study.

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