Generalized Energy-Type Lyapunov Functions for the Transient Stability Analysis of Power Systems

Generalized Energy-Type Lyapunov Functions for the Transient Stability Analysis of Power Systems

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This paper attempts to generate energy-type Lyapunov functions for the single-machine power system taking into account saliency and variable damping, on the basis of the concept of generalized momentum and potential force. The generated Lyapunov functions give better estimations of the stability region than those obtained by the other methods.

1. Introduction

For the transient stability analysis of a power system, many methods have been proposed and studied such as (a) direct simulation method, (b) eigenvalue method, and (c) the second method of Lyapunov. Especially, it is necessary to generate the so-called Lyapunov function for applying (c) to the analysis. For example, the Zubov method[1)], the Luré-Popov method[2,3)] and the method based on energy concept[4-7)] seem to be the most general ones to construct Lyapunov functions.

The authors have presented that a Lyapunov function for a power system can be generated through the generalized energy-type state function consisting of generalized momentum and potential force[8-11)]. In this paper, we propose a more general procedure to construct Lyapunov functions by introducing arbitrary functions $\alpha(x)$ and $\beta(x)$. Also, estimations of the stability region are numerically calculated, and they are compared with those obtained by the other methods[1,4,5,10,12-14)].

2. A Generalized Energy-Type State Function

From the physical viewpoint, it is well-known that the total energy of a mechanical system is given by the sum of the kinetic energy $T(x, \dot{x})$ and the potential energy

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$U(x)$[7], that is,

$$T(x, \dot{x}) = \int_0^{\dot{x}} p(x, \dot{x}) \mathrm{d}\dot{x}\,, \tag{1}$$

$$U(x) = \int_0^x f(x) \mathrm{d}x\,, \tag{2}$$

where $p(x, \dot{x})$ denotes the generalized momentum, $f(x)$ the generalized potential force, x the position, and $\dot{x}$ the velocity with $\cdot$ $= \mathrm{d}/\mathrm{d}t$.
Based on this concept, we define

$$p(x, \dot{x}) \triangleq \dot{y} = p(\dot{y}), \tag{3}$$

and compose modified energy-type state function $V(x, \dot{x})$ as

$$\begin{aligned} V(x, \dot{x}) &= \int_0^{\dot{y}} \dot{y} \mathrm{d}\dot{y} + \int_0^x f(x) \mathrm{d}x \\ &= \frac{1}{2} p^2(x, \dot{x}) + \int_0^x f(x) \mathrm{d}x. \end{aligned} \tag{4}$$

Since $p(x, \dot{x})$ is the generalized momentum, $\dot{y}$ can be interpreted to be a momentum on the y-axis. Here, it should be emphasized that the definition Eq.(3) of $\dot{y}$ is, as a matter of convenience, introduced for the construction of the state function $V(x, \dot{x})$ given by Eq. (4).

3. Generalized Energy-Type Lyapunov Functions

Under the usual assumptions of (i) consant input, (ii) constant field flux linkage, and (iii) constant angular momentum[12], we treat the following swing equation of a single-machine power system with saliency and variable damping;

$$\ddot{x} + D(x)\dot{x} + g(x) = 0, \tag{5}$$

where

$$x = \delta - \delta_0, \tag{6}$$

$$D(x) = a + b\cos 2(x + \delta_0) > 0,\ a > b > 0; \tag{7}$$

the generalized damping coefficient

$$\begin{aligned} g(x) = K_1\ &\{\sin(x+\delta_0) - \sin\delta_0\} \\ &- K_2\ \{\sin 2(x+\delta_0) - \sin 2\delta_0\}\ ,\ K_1 > 2K_2 > 0; \end{aligned} \tag{8}$$

the generalized nonlinear restoring force

δ_0; the stable equilibrium value of δ

a, b, K_1, K_2; constants.

Letting x_c be the unstable equilibrium point of x near the origin ($x = \dot{x} = 0$), in a limited range

$$\Gamma = \{x \mid x_c - 2\pi \leqq x \leqq x_c\}\ , \tag{9}$$

we find that this system has properties

$$D(x) > 0, \tag{10}$$

$$xg(x) \geqq 0,\ g(0) = 0, \tag{11}$$

and

$$R(x)g(x) \geqq 0 \tag{12}$$

with

$$R(x) \triangleq \int_0^x D(x)\mathrm{d}x. \tag{13}$$

It is important to notice how we can reflect the damping term $D(x)\dot{x}$ of Eq.(5) into the kinetic energy Eq.(1) and the potential energy Eq.(2) for constructing a energy-type state function Eq.(4). Introducing arbitrary functions $\alpha(x)$ and $\beta(x)$, and rewriting Eq.(5) yield

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\{\dot{x} + \alpha(x)R(x)\} + \frac{\mathrm{d}}{\mathrm{d}x}\{(1-\alpha(x))R(x)\}\ \dot{x} \\ + \{\beta(x) + g(x)\} = \beta(x). \end{aligned} \tag{14}$$

Since the first and the third terms of the left hand side of Eq.(14) are corresponding to the generalized momentum $p(x,\dot{x})$ and the generalized potential force $f(x)$ respectively, Eq.(4) enables us to construct a generalized energy-type state function $V(x,\dot{x})$ as

$$V(x,\ \dot{x}) = \frac{1}{2}\{\dot{x} + \alpha(x)R(x)\}^2 + \int_0^x \{\beta(x) + g(x)\}\ \mathrm{d}x. \tag{15}$$

The time derivative along a trajectory of Eq.(5) becomes

$$\begin{aligned} \dot{V}(x,\ \dot{x}) = -\phi(x)\dot{x}^2 - \{\alpha(x)R(x)\phi(x) - \beta(x)\}\ \dot{x} \\ -\alpha(x)R(x)g(x), \end{aligned} \tag{16}$$

where

$$\phi(x) \triangleq \frac{\mathrm{d}}{\mathrm{d}x}\{(1-\alpha(x))R(x)\}\ . \tag{17}$$

Using Eqs.(10)−(13), and choosing $\beta(x)$ to make a squared form of Eq.(16) such as

$$\beta(x) = \alpha(x)R(x)\phi(x) + 2\ \{\alpha(x)R(x)g(x)\phi(x)\}^{1/2} \tag{18}$$

with $\alpha(x)\phi(x) \geqq 0$, we find that Eqs.(15)−(18) give

$$\begin{aligned} V(x,\ \dot{x}) = \frac{1}{2}\{\dot{x} + \alpha(x)R(x)\}^2 + \int_0^x \{\alpha(x)R(x)\phi(x) + g(x)\}\ \mathrm{d}x \\ +2\int_0^x \{\alpha(x)R(x)\phi(x)g(x)\}^{1/2}\mathrm{d}x, \end{aligned} \tag{19}$$

and

$$\dot{V}(x, \dot{x}) = -\left[\{\phi(x)\}^{1/2}\dot{x} - \{\alpha(x)R(x)g(x)\}^{1/2}\right]^2, \tag{20}$$

which is non-positive in Γ with

$$\alpha(x) \geqq 0 \text{ and } \phi(x) \geqq 0. \tag{21}$$

Moreover, it follows

$$xf(x) = \begin{cases} x\left[\{\alpha(x)R(x)\phi(x)\}^{1/2} + \{g(x)\}^{1/2}\right]^2, & x>0, \quad (22) \\ -x\left[\{\alpha(x)(-R(x))\phi(x)\}^{1/2} + \{-g(x)\}^{1/2}\right]^2, & x<0. \quad (23) \end{cases}$$

This guarantees that $V(x,\dot{x})$ is positive definite in Γ. Thus, it is found that the state function $V(x,\dot{x})$ given by Eq.(19) is a generalized energy-type Lyapunov function for Eq.(5).

On the other hand, since the condition Eq.(21) is

$$\phi(x) = \frac{d}{dx}\{(1-\alpha(x))R(x)\} \triangleq \frac{d}{dx}F(x) \geqq 0, \tag{24}$$

where

$$\alpha(x) = 1 - F(x)/R(x) \geqq 0, \tag{25}$$

$F(\gamma)$ must be a monotone increasing function. Then, as examples of $\alpha(x)$, $F(x)$ satisfying Eq.(25) gives

1) $\alpha(x) = \exp(-k_1x^2)$
2) $\alpha(x) = \text{sech}(k_1x)$
3) $\alpha(x) = \cos(k_1x)$
4) $\alpha(x) = 1 - k_1x^2$
5) $\alpha(x) = 1 - k_1R^2(x) - k_2G^2(x)$,

where

$$G(x) \triangleq \int_0^x g(x)dx \geqq 0, \tag{26}$$

and k_1 and k_2 are constants chosen to satisfy the condition Eq.(25) in the region Γ. For example, the case 5) gives

$$0 \leqq \max_{X \epsilon \Gamma}\{k_1R^2(x) + k_2G^2(x)\} \leqq 1. \tag{27}$$

However, the problem to obtain theoretically optimal values of k_1 and k_2 seems to be interesting.

4. Numerical Results

In Fig. 1, the true stability region and other cases are numerically calculated. The

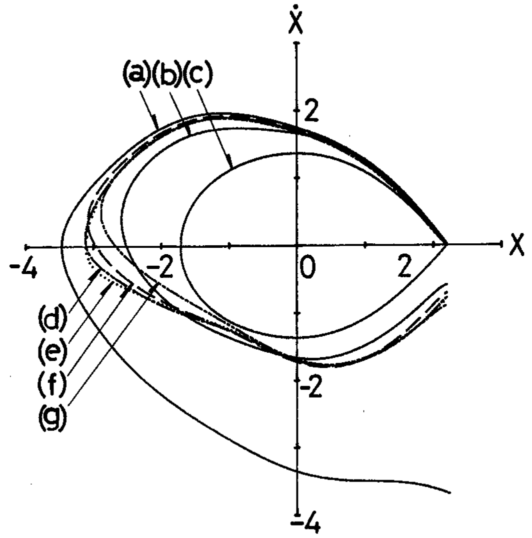

Fig. 1 Comparison of stability regions;
(a) true region, (b) $\alpha(x)=0.5$, (c) $\alpha(x)=0$, (d) $\alpha(x)=1-R^2(x)$, (e) $\alpha(x)=1-0.8R^2(x)-0.03G^2(x)$, (f) $\alpha(x)=\text{sech}(0.6x)$, (g) $\alpha(x)=1-0.05x^2$, and $K_1=1.0$, $K_2=0.2$, $a=0.3$, $b=0.1$, $\delta_0=0.6325$.

case (c) corresponds to the simplest one constructed by setting $\alpha(x)=0$ in Eq.(19), and the effect of the damping term in Eq.(5) is not included. Cases (b), (d), (e), (f) and (g) give better estimations than that of the case (c). The case (b) has been represented based on the energy concept[1,4,5,10,12], and the case (g) has been developed recently by using the operational transform technique[13,14].

5. Conclusions

Introducing arbitrary functions $\alpha(x)$ and $\beta(x)$ into the generalized momentum and potential force for considering the effect of the damping term in Eq.(5), we have proposed the method to generate generalized energy-type Lyapunov functions for the power system, and have numerically shown better estimations of the stability region than those obtained by the other methods. Hence, we can conclude that the proposed method is more generalized than the other ones, the procedure to construct the Lyapunov function Eq.(19) is systematic, and the choice of arbitrary functions $\alpha(x)$ and $\beta(x)$ is quite simple. As we pointed out in Section 3, it is interesting to find an optimal function $\alpha(x)$ theoretically. Also, an application of the proposed method to multi-machine power systems is under investigation.

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