Flow Field Measurement by Pathline Method

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A quantitative analysis of visualized flow picture has been enhanced with the versatility of modern computer.
Generally, the time integration value of light intensity of each tracer particle during exposure is recorded in the film or VTR of flow image. When the tracer moves, the so-called pathline is formed in the picture. This means that the both ends can be determined by differentiation of image data in principle.
This paper describes an analysis method of visualized pathline image. Using several conventional image processing techniques, derivation of image data, correlation technique, two-dimensional FFT, and so on, a two-dimensional velocity field is obtained.

1. Introduction

Flow visualization techniques are widely used in fluid dynamics to observe and measure properties of a flow field such as velocity, density or temperature variations. Many of these techniques have been previouisly applied to qualitative observation and identification of structures in flows. Further research, however, in quantitative means is also of great value in guiding intuition.

One of the means is to picture the motion of tracer particles for the measurement of the flow velocities. Such methods in the past, using scalars to determine the displacement of indivudual particles, require enormous time consumption. Recently, digital analysis of the visualized flow image, referred to as computer aided flow visualization (CAFV), has been implemented with the development of varioius computer techniques.

CAFV includes the following three categories. The first is a description of solutions using numerical simulation. In this method, the results such as velocity, pressure, vorticity and so on, are visualized for clear understanding of the flow phenomena. Development of financially feasible computers is crucial for analyzing

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the turbulent flow.

The second is a flow visualization with indirect point-by-point measurement results, in which the velocity is measured by Pitot-static tubes, wind turbine anemometers, hot wire anemometers or hot film velocimetries. Computers are used for the expression of the flow phenomenon using a number of measured data. In these cases, the probe itself disturbs the flow and many probes are required to measure the flow field instantaneoiusly.

The third is the so called digital image processing of the flow field. This method enables determining a large volume of the instantaneous velocity by visualized flow image without disturbing the flow. The investigation of flow fields by the analysis of the records of the trajectories of tracer particles is recently the most popular for this measurement.

Two essentially independent approaches have been attempted as to extracting the quantitative velocity data. The first approach is tracking of individual tracer particles on two sequential single exposed images. Jonas[1)] identified the positions of individual particle of both pictures by scanning the successive images and derived the velocity field from the tracer particle coordinates. In this case, a high resolution scanner, a high cost computer and sophisticated techniques are required for the analysis of the flow image. Recently, however, a number of researchers employ this kind of methods because of its great possibilities in the measurement.

The other method is the velocity field measurement by identifying the edge of each pathline of tracer particles. Uemura[2)] has initially suggested the possibility of using a micro-computer and actually used a 8 bit small computer. Unfortunately, the pathlines are not always recorded clearly and separately. The crossing and overlapping of the pathlines complicates the subsequent data analysis. Directional ambiguity in the velocity field due to lack of the phase information ,which is inherent in the single image analysis, is not resolved.

To solve these problems Kobayashi[3)] and Ohmi[4)] have developed a computer system that automatically determines the two dimensional flow field using a component labeling. This technique essentially requires the identification of beginning and terminal positions of the pathline by three synchronized cameras and a large amount of computer time.

In this paper the authors develop the analysis method of pathline picture by combing conventional image processing techniques. Using a CCD camera with electric shutter, pathlines are recorded in a single-exposure image, and then it is divided into adequate number and size of sampling windows. The statistical analysis is applied in each area. This technique enables the fast and simple analysis in fairly high velocity field.

2. The Length of the Pathline

Image fields are generally represented by intensity of the light reflection distributions. Pathline images can be considered as an image degradation or a blurred image. Let $o(x,y)$ represents an original undistorted picture. This image is to be considered that the input of a system of which the output $i(x,y)$ corresponds to the blurred image. The system itself consists of two components, a point spread function (PSF) $h(x,y)$ and an additive noise process $n(x,y)$. Here, the point spread function is the response of the blurred image to a two-dimensional unit impulse $\delta(x)\delta(y)$. A description relating output o and input i can be given as,

$$i(x,y)=g(x,y)+n(x,y) \tag{1}$$

where

$$g(x,y)= \iint h(x,y,x',y')o(x',y')\mathrm{d}x'\mathrm{d}y' \tag{2}$$

Thus Eq. (1) can be simplified to the convolution integral denoted as $*$.

$$i(x,y)=o(x,y)* h(x,y)+n(x,y) \tag{3}$$

By neglecting noise in Eq. (3),

$$i(x,y)=o(x,y)* h(x,y) \tag{4}$$

It can be understood that the image processing in this case means solving Eq. (4).

Image degradation occurs due to the motion of tracer during exposure. The picture of the film is formed by integration of the instantaneous exposure during the time interval of which the shutter is open. Thus, it can be written as,

$$i(x,y)= \iint_0^T o(x-x(t),\ y-y(t))\mathrm{d}t \tag{5}$$

where $x(t)$ and $y(t)$ represent x and y components of the displacement, respectively and T the duration of the exposure.

Therefore, the two-dimensional pathline image caused by a linear motion in arbitrary directions is described as,

$$i(x,y)= \iint_0^T o(x-at/T,\ y-bt/T)\mathrm{d}t \tag{6}$$

where $o(x,y)$ represents the originally undistorted particle image field, t represents the time, T the duration of the exposure, a the x component of the velocity of the particles in the region and b the y component of the velocity.

Let us denote the directional derivative of i at (x,y), in the direction of the unit vector u as $D_u i(x,y)$. Here u is defined as the component of the unit vector u are

$$(x,y)=\left(\frac{a}{\sqrt{a^2+b^2}},\ \frac{b}{\sqrt{a^2+b^2}}\right), \text{ respectively.}$$

$D_u i(x,y)$ can be expressed in terms of the gradient of the $i(x,y)$, which gives

$$D_u i(x,y)=\frac{di(x,y)}{du}=\nabla\, i(x,y)u \tag{7}$$

Now we consider the partial derivative of $i(x,y)$ in the direction of x and y, respectively.

By replacing $x\text{-}at/T$ by τ, which gives (3) as

$$i(x,y)=\int_0^T o(\tau,\ y-\frac{b}{a}(x-\tau))\frac{t}{a}\,\mathrm{d}t \tag{8}$$

The partial derivative of $i(x,\ y)$ with respect to x gives

$$\frac{di}{dx}=\frac{t}{a}\ \{o(x,y)-o(x-a,\ y-b)\} \tag{9}$$

and also

$$\frac{di}{dy}=\frac{t}{b}\ \{o(x,y)-o(x-a,\ y-b)\} \tag{10}$$

We have

$$D_u i(x,y)=\frac{2}{\sqrt{a^2+b^2}}\,T\ \{o(x,y)-o(x-a,\ y-b)\} \tag{11}$$

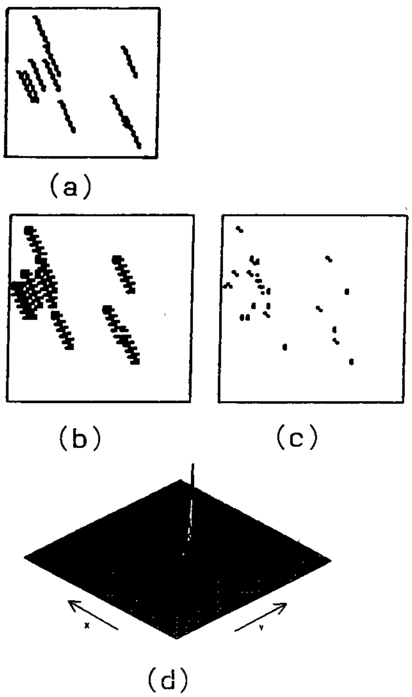

Fig. 1 The directional derivative to the randomized 10 lines with a displacement of $(\Delta x, \Delta y)=(5, 10)$ pixels; (a) simulated pathline image, (b) processed data, (c) distribution of derivative values in 7 levels, and (d) spike function obtained by Fourier Transform.

Each pair of positive and negative values gives the length of each pathline. By separating negative values from the processed images, we can obtain two sequential images. It is possible to apply the so-called FFT method[5] to these two images for evaluating the pathline length. This method has developed originally for two sequential images to obtain the velocity vectors and it has a characteristic to yield the image displacement statistically, instead of identifying each tracer particle.

In order to confirm the presented principle, the pathline images are made in a active area of 64 by 64 grid pixels on a computer display. A random function is applied to randomly space pathlines in the area. These images are considered to be a binary images without any noise. Figures 1(*a*) to (*d*) show the analysis of 10 pathlines with a displacement of (Δx, Δy)=(5,10) in pixel, where Δx and Δy is the horizontal and vertical components of the pathline length, respectively. The spatial derivation in the direction of pathline is shown in Fig. 1(b). Spike function appears at both ends of the pathline. The maximum and the minimum values of the gray levels are shown in Fig. 1(c).1 These both values are separated into two pictures. This set of the pictures gives the displacement of the particle moving along the pathline. In the result on Fig. 2(d), a peak location away from the center point indicates the average spatial displacement of tracer particles moving along the pathline.

When the orientation of the pathline is known, the beginning and terminal points of the pathline can be found by taking differentiation with respect to the pathline direction. The single pathline picture is separated into two image and the so called FFT method is applied to obtain a velocity field without a sense of the vector. Consideration of the radii of particles moving along the pathline is inevitable for the determination of collect displacement of particles.

3. The Orientation of the Pathline

The derivation of the orientation of the pathline is investigated here for two cases, a local line cross-correlation and a local spatial auto-correlation.

3.1. Local line cross-correlation

A cross-correlation is implemented on a sample pair of lines which is parallel to each other with a certain distance to obtain an orientation of a pathline. In case a sample pair of lines intersects the same pathline, the location of peak value of cross-correlation function represents the spatial shift of the pathline between the sample pair of lines, which gives an angle of the pathline. This method is basically similar in principle to the one described by Kirita[6]. Figure 2 shows the coordinate system for this method. A sample region is taken between +A to −A on the axis, which limits the analysis range between +45° to −45°. The analysis is performed on both x axis and y axis; this covers the whole angle of the pathline.

In the cross correlation, the resolution of the angle, in a principle, can be defined

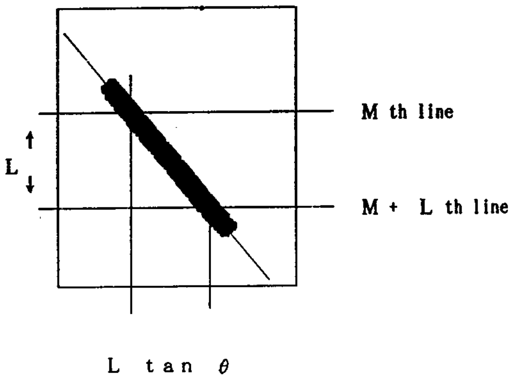

Fig. 2 Conceptional arrangement of the local line cross-correlation.

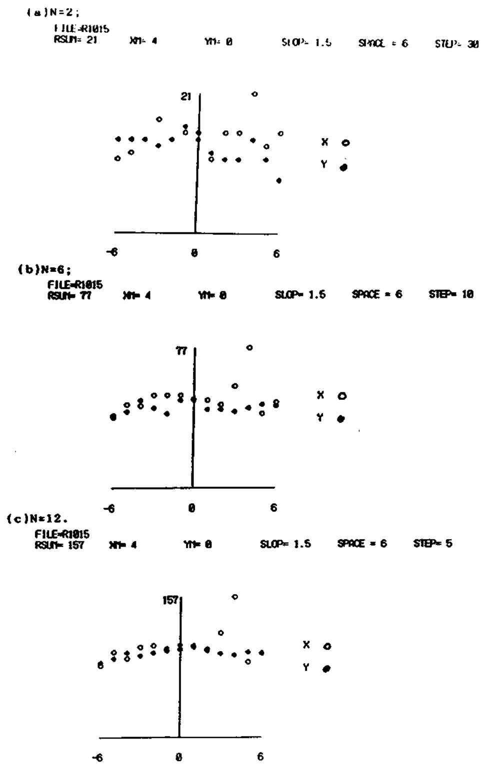

Fig. 3 The effect of numbner of sampled pairs (N) on line cross-correlation values (a)N=2, (b)N=6, and (c)N=12

arbitrarily. The angle along the axis, can be determined as

$$d\theta_{axis}=\tan^{-1}(-\frac{-1}{L}) \tag{12}$$

In order to examine the effect of number of sampled pairs, the cross-correlation value is shown on an idealized 64 by 64 pixels grid. The analysis of 50 randomized lines with the displacement of x and y as 10 and 15 pixels, respectively is shown in Fig. 3. The sampling space is 6 pixels and the number of sampled pairs is examined on 2, 6 and 12 pairs. With the increase of the number of sampled pairs, the correlation peak value becomes large. Thus, an accurate analysis is possible by the increaisng number of sample pairs.

3.2. Local spatial auto-correlation function

As is shown in Fig. 4 and spatial auto-correlation technique is applied to a fixed sampling region A and a sampling region B which is a certain distance away from the former region A. Maximum movement of a sampling region B away from the fixed region A should be smaller than a side pixel of the sample region B. A pathline direction is determined by fitting a linear function to the correlation value. The statistical technique of a spatial auto-correlation for the discrete case can be written as

$$R_{FG}=\frac{\sum_{i=1}^{n} f_i g_i-\frac{(\sum_{i=1}^{n} f_i \cdot \sum_{i=1}^{n} g_i)}{n}}{\sqrt{(\sum_{i=1}^{n} f_i^2-\frac{(\sum_{i=1}^{n} f_i)^2}{n})(\sum_{i=1}^{n} g_i^2-\frac{(\sum_{i=1}^{n} g_i)^2}{n})}} \tag{13}$$

Here f_i and g_i represent the spatial region.

The effect of an increase in the size of the sampling window on determination of the orientation is examined in a numerical modeling of 64 by 64 pixels grid that are already applied in the former section. The sampling windows had a size of 16 by 16 pixels and a auto-correlation value is taken between −5 to 5 in both X and Y axis. The obtained auto-correlation function is presented in 8 gray levels. The peak value of auto-correlation function locates always on the center of course and plotted certain values of the correlation coefficients show the direction of pathlines.

The linear equation that goes through an origin is fitted to the auto-correlation values by means of a least square method. Figure 5 shows the results of this method. In this examination, the number of the correlation value is totally 36 and the angle is 11.3 degree. The threshold values of correlation coefficient affects to the evaluated angle as shown in Fig. 5.

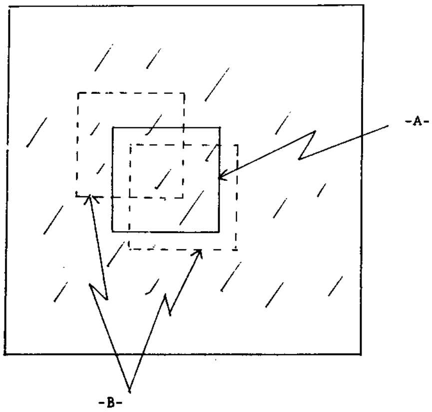

Fig. 4 Conceptional arrangement of local spatial correlation.

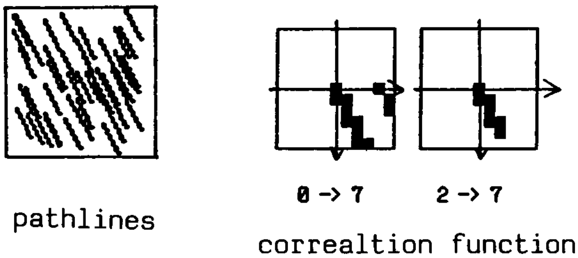

Fig. 5 The local spatial auto-correlation values in the analysis of randomized data.

4. Velocity Measurement by the Pathline Method

The present method is implemented to measure the velocity in a compact water channel flow. The tracer particle has neutral buoyancy and is around 50 μm phosphorescent polystyrene. Illumination is provided by a continuous wave Argon Laser of which the beam is spread to 1 mm thick sheet using a cylindrical lens. The image analysis system consists of the CCD camera with a electric shutter and PC9800 XL^2 with high resolution mode BASIC language and a image processing interface board. The shutter is used to change the time between image capturing from 1/60 to 1/4000. A digital image data has an active area of 256 pixels horizontally by 256 pixel vertically and is stored to a diskette. A total of 16 images can be stored. Acquired images are then downloaded, processed to binary data by threshold in adequate value. In this experiment, the shutter speed is set by 1/100 second. The example of analyzed data is shown in Fig. 6. The flow is simple steady flow. The sampling region in this case is 64 by 64 grids. Thus the velocity vector of which the value is over 32 is regarded as a noise and eliminates from the image.

In the visualized picture of the water channel flow, there are a number of tracers. However, the pathlines disappear from the computer screen due to an adequate

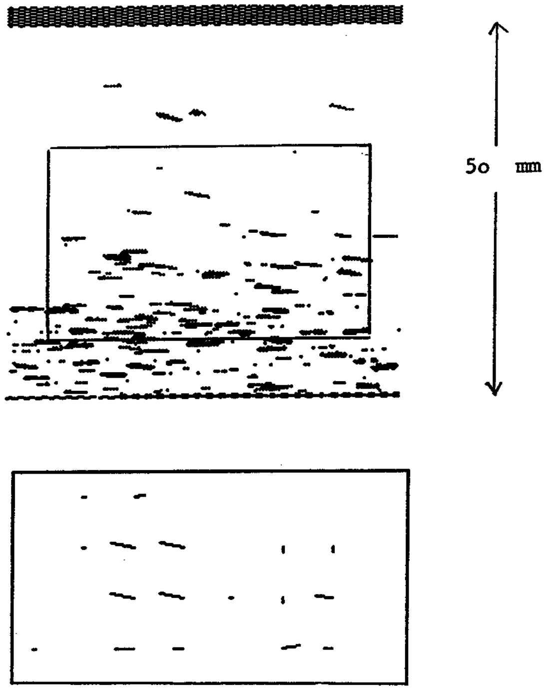

Fig. 6 The flow field image and the result of the measurement.

threshold value. Even if pathlines exist in a region, most of pixels do not connect to each other to form lines. This is because the peaks of the derivative does not correspond to the length of the pathline. Consideration of the selection of the tracer and the technique of the threshold is crucial for the flow measurement. From the obtained vector in Fig. 6, it can be found the uniform flow velocity is around 50 cm/s.

5. Conclusions

Pathline method is introduced as a fast and simple analysis by combing conventional image processing techniques and implementing the simple systems using a personal computer. The characteristics of a pathline method is summarized as follows.

(1) The spatial derivative of the pathline image field separates the pathline into beginning and terminal point, which gives the length of the pathline.
(2) The statistical analysis of the pathline method gives the average velocity in a sampling region and eliminates the noise which is a result of the analysis of crossing and overlapping pathline.
(3) For the method of determination of pathline orientation, the line cross-correlation is better than spatial auto-correlation.

(4) The consideration of the adequate density and length of pathlines, together with the threshold value in a sampling window is important for the analysis.
(5) The flow field measurement of arouind 1 m/s is possible with the elimination of noise.

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