Fast Transient Stability Evaluation by Estimation of Electrical Power Output

Fast Transient Stability Evaluation by Estimation of Electrical Power Output

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This paper describes a new approach for fast evaluation of power system transient stability, taking into account the voltage dependence of system load. The method is based on the direct method using energy function. In the method, we propose to calculate the potential energy by polynomial expression of electrical power output, for the realization of fast evaluation. The method is applied to a 3 and a 9 generators systems.[1,2]

The time required for calculation of the proposed method is considerably reduced than that of the usual direct method, and in most cases, the accuracy of evaluation is also satisfactory.

1. Introduction

It is a very important problem to construct the reliable power system that prevents interruption of power supply which is caused by large disturbances. One of the important items which should be investigated at this juncture is transient stability, and fast analysing method for transient stability is indispensable for its on-line evaluation.

Recently, the voltage dependence of system load markedly affects the dynamic behavior of a power system. However, on-line evaluation for power system transient stability with consideration of load characteristics may not be realized, because much computing time is required, even if the usual direct method is used.

In this paper, a new approach is presented, which is based on the direct method using energy function. In the method, it is proposed to calculate the potential energy by polynomial expression of electrical power output, taking into account the voltage dependence of system load.

The above method is applied to a 3 machine 9 buses system[1)] and to a 9 machine 25 buses system[2)] as numerical examples, and the time for required calculation of the proposed method is compared with that of the usual direct method.

2. Voltage Dependence of Composite Load

For several years, much effort has been devoted to the modeling of load characteristic and some load models have been proposed for simulation studies. Among these models, a V^n-type model is the most fundamental.

In this paper, the following dynamic load model[3)] is employed.

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$$P^{(k)} = P_0 \left\{ \left(\frac{E^{(k)}}{E_0}\right)^{n_p} + A_p \sum_{j=1}^{k} \left(\frac{E^{(j)}}{E_0}\right)^{n_p} \left(\frac{E^{(j)} - E^{(j-1)}}{E^{(j-1)}}\right) e^{\frac{-(k-j)\Delta t}{T_p}} \right\} \quad (1)$$

$$Q^{(k)} = k_0 + (Q_0 - k_0) E_0^{\,n_q} \left\{ \left(\frac{E^{(k)}}{E_0}\right)^{n_q} + A_q \sum_{j=1}^{k} \left(\frac{E^{(j)}}{E_0}\right)^{n_q} \cdot \left(\frac{E^{(j)} - E^{(j-1)}}{E^{(j-1)}}\right) e^{\frac{-(k-j)\Delta t}{T_q}} \right\} \quad (2)$$

where

P_0, Q_0, E_0: initial values of active power, reactive power and load voltage, respectively,
$P^{(k)}, Q^{(k)}, E^{(k)}$: values at $t_k = (k-1)\cdot\Delta t$ of active power, reactive power and load voltage, respectively,
n_p, n_q: load parameters of active and reactive power respectively,
T_p, T_q: time constants of active and reactive power respectively,
Δt: time interval.
Specially, this model is static load model at $A_p = A_q = 0$, and

$n_p = n_q = 2$: constant-impedance load,
$n_p = n_q = 1$: constant-current load,
$n_p = n_q = 0$: constant-power load.

In the Eq. (1), when load voltage changes from E_0 to E at step response, the change of P is shown in Fig. 1. The characteristic of Q in the Eq. (2) is also like Fig. 1.

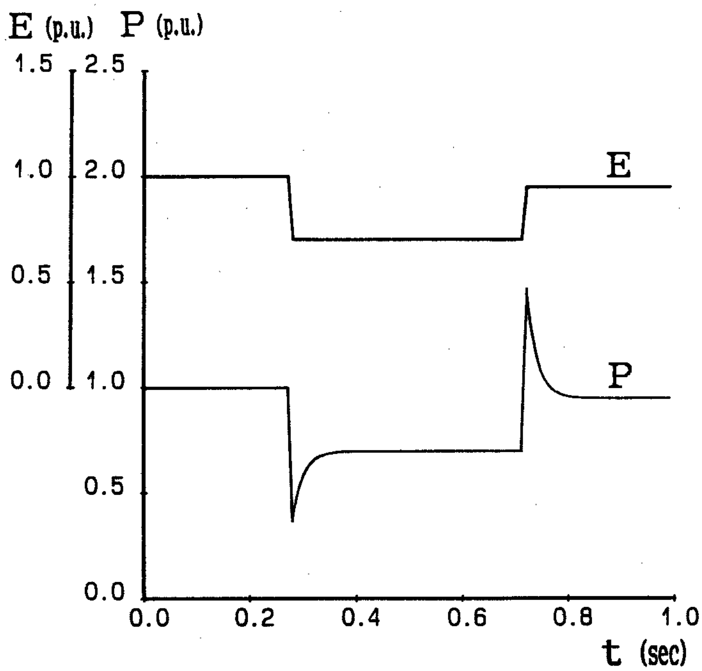

Fig. 1 Power response of load with voltage change

3. Mathematical Formulation

3.1. System equation

The following assumptions are made for multimachine systems.

(1) Each synchronous machine is represented by a constant voltage behind its transient reactance.

(2) Mechanical power input is constant and the governor action is not taken into account.

(3) Each synchronous machine is a round-rotor machine.

Under the above assumptions, the motion of the ith machine is generally described by the following differential equation.

$$\begin{aligned} M_i \frac{d^2\delta_i}{dt^2} + D_i\left(\frac{d\delta_i}{dt} - \frac{d\delta_0}{dt}\right) &= P_{mi} - P_{gi} \\ &= P_{mi} - |E_i|^2 G_{ii} - \sum_{j=1}^{n} (C_{ij} \sin \delta_{ij} + D_{ij} \cos \delta_{ij}) \\ &\qquad (i = 1, 2, 3, \ldots\ldots n) \end{aligned} \tag{3}$$

n : number of machine in the system,
D_i : damping coefficient of ith machine,
P_{gi} : electrical power output of ith machine,
δ_i : rotor angle in the reference frame,
δ_{ij} : $\delta_i - \delta_j$,
δ_0 : center of angle, $\delta_0 = \sum_{i=1}^{n} M_i \delta_i / \sum_{i=1}^{n} M_i$
M_i : inertia constant of ith machine,
P_{mi} : mechanical power input of ith machine,
E_i : internal voltage behind transient reactance of ith machine.

The case of constant impedance load, C_{ij}, D_{ij} are constants, but under considering load characteristics, C_{ij}, D_{ij} change with load voltage, and resultant electrical power output varies. For this reason, in solving the system's differential equations, sequential calculations for loads are required.

3.2. Load calculation

The nodal equation which gives the current-voltage relation of a multi-machine system is written approximately at the (k)th interval as follows:

$$\begin{bmatrix} I_G{}^{(k)} \\ I_L{}^{(k)} (= 0) \\ I^{(k)} (= 0) \end{bmatrix} = Y^{(k-1)} \begin{bmatrix} E_G{}^{(k)} \\ E_L{}^{(k)} \\ E^{(k)} \end{bmatrix} \tag{4}$$

where

$E_G{}^{(k)}, E_L{}^{(k)}, E^{(k)}$: internal voltage vector of generators, load-bus voltage vector and floating-bus voltage vector, respectively

$I_G{}^{(k)}, I_L{}^{(k)}, I^{(k)}$: generator current vector, load-bus current vector, floating-bus current vector, respectively

$Y^{(k-1)}$: modified nodal admittance matrix involving load admittance $Y_L^{(k-1)}$

By elimination of floating buses,

$$\begin{bmatrix} I_G^{(k)} \\ I_L^{(k)} (=0) \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL}^{(k-1)} \end{bmatrix} \begin{bmatrix} E_G^{(k)} \\ E_L^{(k)} \end{bmatrix} \tag{5}$$

and Eq. (5) is written in the following hybrid form[4]

$$\begin{bmatrix} I_G^{(k)} \\ E_L^{(k)} \end{bmatrix} = \begin{bmatrix} h_{11}^{(k-1)} & h_{12}^{(k-1)} \\ h_{21}^{(k-1)} & h_{22}^{(k-1)} \end{bmatrix} \begin{bmatrix} E_G^{(k)} \\ I_L^{(k)} (=0) \end{bmatrix} \tag{6}$$

$$h_{11}^{(k-1)} = Y_{GG} - Y_{GL} (Y_{LL}^{(k-1)})^{-1} Y_{LG}$$
$$h_{12}^{(k-1)} = Y_{GL} (Y_{LL}^{(k-1)})^{-1}$$
$$h_{21}^{(k-1)} = -(Y_{LL}^{(k-1)})^{-1} Y_{LG}$$
$$h_{22}^{(k-1)} = (Y_{LL}^{(k-1)})^{-1}$$

Then, admittance representation of ith load at (k)th interval is given by Eq. (7).

$$Y_{Li}^{(k)} = (P_i^{(k)} - jQ_i^{(k)}) \,/\, (E_{Li}^{(k)})^2 \tag{7}$$

We now perform the matrix reduction on the Y matrix at the generator-buses as $I_G^{(k)} = Y_G^{(k)} E_G^{(k)}$. Let elements of this $Y_G^{(k)}$ matrix be denoted by $Y_{ij}^{(k)}$, and define $D_{ij}^{(k)} = |E_i| |E_j| G_{ij}^{(k)}$, $C_{ij}^{(k)} = |E_i| |E_j| B_{ij}^{(k)}$.

By repeating this process for each interval, the loads are calculated with every time.

3.3. Energy type Lyapunov function and stability evaluation

The transient energy function V, which is always defined for the post fault system, can be given in Eq. (9)[5,6].

$$\begin{aligned} V &= V_k + V_p \\ &= \sum_{i=1}^{n} M_i \tilde{\omega}_i^2 / 2 && \ldots V_k \\ &\quad + \sum_{i=1}^{n} P_i (\theta_i^0 - \theta_i) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij} E_i E_j (\cos\theta_{ij}^0 - \cos\theta_{ij}) \\ &\quad + \sum_{i=1}^{n} \sum_{j=1}^{n} \int_{\theta_i^0+\theta_j^0}^{\theta_i+\theta_j} G_{ij} E_i E_j \cos\theta_{ij}\, d(\theta_i + \theta_j) && \ldots V_p \end{aligned} \tag{9}$$

where

$Y_{ij} = G_{ij} + jB_{ij}$,

$\theta_i = \delta_i - \delta_o$: new rotor angle with respect to center of angle δ_o,

θ_i°: initial stable point of θ_i

$\theta_{ij} = \theta_i - \theta_j$

ω_i: angular velocity of ith machine,

$\omega_o = d\delta_o/dt$,

$\dot{\theta}_i = \tilde{\omega}_i = \omega_i - \omega_o$,

V_k: kinetic energy due to the relative angular velocities,

V_p: potential energy due to the deviations of angles from their stable points.

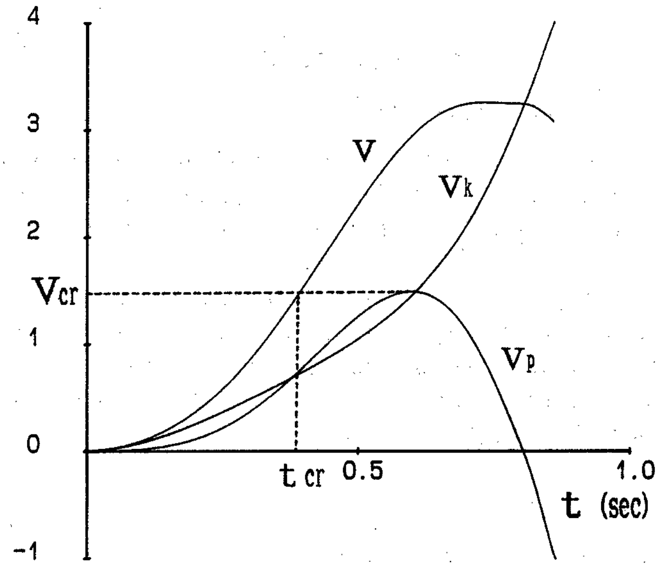

Fig. 2 Concept of the decision for critical clearing time

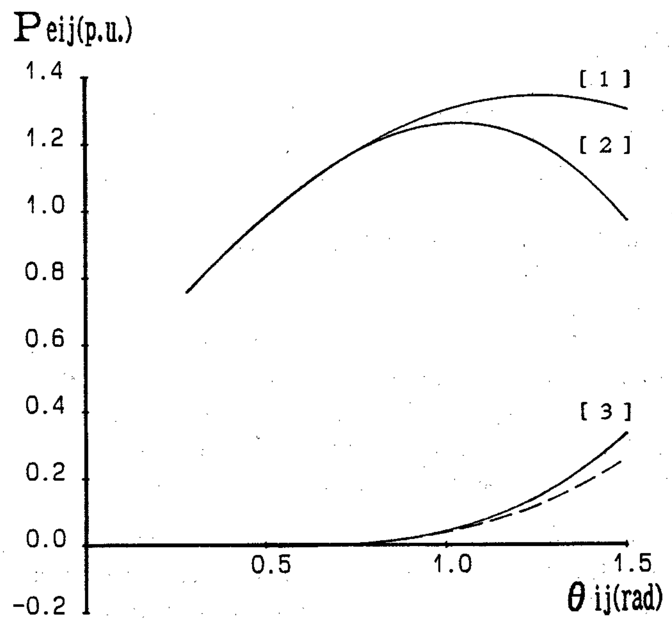

Fig. 3 Curve of electrical power output

The value of energy function gradually increases during the fault, and by finding the time where this value equals to the critical value V_{cr}, the critical clearing time for stability can be determined.[7)]

This procedure is as follows:

(1) Construct an energy function $V(t) = V_k(t) + V_p(t)$,
(2) Find the time t^* where $dV_p(t)/dt = 0$,
(3) Set $V_p(t^*) = V_{cr}$,
(4) Find the critical clearing time t_{cr} by finding the time where $V_{cr} = V(t)$.

4. Proposed Method

4.1. Estimation of electrical power output

In this section, we investigate the estimation of electrical power output of generator which consider the nonlinearity of the load. Figure (3) shows an example of the electrical power output curve along the sustained fault trajectory in a multimachine system.

Curve [1] is the power-angle considering the constants load. Curve [2] is the power-angle considering voltage dependence of the load. Curve [3] is $\Delta P_{eij}(\theta_{ij})$ which shows curve [1]−curve [2]. Hence, using this electrical power output P_{ei} of ith machine can be identified by the forms of Eq. (10) and Eq. (11). The curve identified by Eq.(10) is illustrated a dotted line in Fig.3.

$$\Delta P_{eij}(\theta_{ij}) = K1(\theta_{ij} - \theta_{ij}{}^0)^{1/3} + K2(\theta_{ij} - \theta_{ij}{}^0) + K3(\theta_{ij} - \theta_{ij}{}^0)^3 \tag{10}$$

$$P_{ei} = \sum_{j=1}^{n} P_{eij}$$

$$P_{eij} = C_{ij} \sin \theta_{ij} + D_{ij} \cos \theta_{ij} - \left\{K1(\theta_{ij} - \theta_{ij}{}^0)^{1/3} + K2(\theta_{ij} - \theta_{ij}{}^0) + K3(\theta_{ij} - \theta_{ij}{}^0)^3\right\} \tag{11}$$

The first and second term of right hand side in the Eq. (11) are the power considering constant impedance load, and the third term indicates the effect of the voltage dependence of load. The Constants $K1$, $K2$, $K3$ are identified with the method of least squares.

The merit of Eq. (11) is as follows:

(1) The effect of the voltage dependence of load can be dealt with correctively, because the nonlinearity of electrical power output is almost expressed by Eq. (10).
(2) Energy function is analytically calculated, because electrical power output is expressed by polynomial of relative rotor angle of generators.
(3) Approximate value of critical energy is easily obtained.

4.2. Approximation of swing

In the case, considering the voltage dependence of load, sequential computation of system load is required for the process of calculation of the sustained fault trajectory. So we propose an approximate solution of the swing equation, for the fast determination of the sustained fault trajectory and for the estimation of electrical power output.

The approximate solution derives as follows:

(1) Estimate the values of α_i^0, β_i and T_i of ith machine, by solving the Eq. (3) for a few steps after the fault occurred.

(2) Using these values,

$$\omega_i = \frac{d\delta_i}{dt} = \alpha_i t,\ \alpha_i = \alpha_i^0 + \sum_{k=1}^{m} \beta_i e^{\frac{-k*\Delta t}{T_i}} \tag{12}$$

(3) Making use of the above ω_i,

$$\delta_i = \delta_i^0 + \int \omega_i dt \tag{13}$$

where

α_i: angular acceleration of ith machine,
β_i: correction constant of angular acceleration of ith machine,
T_i: time constant of ith machine,
δ_i^0: initial rotor angle of ith machine,

$m = t/\Delta t$.

Figure 4 shows δ and ω for the sustained fault. Each process of simulation, direct and proposed method in this paper is illustrated in Fig. 5. Double line of Fig. 5 is the part that the sequential computation of load is required.

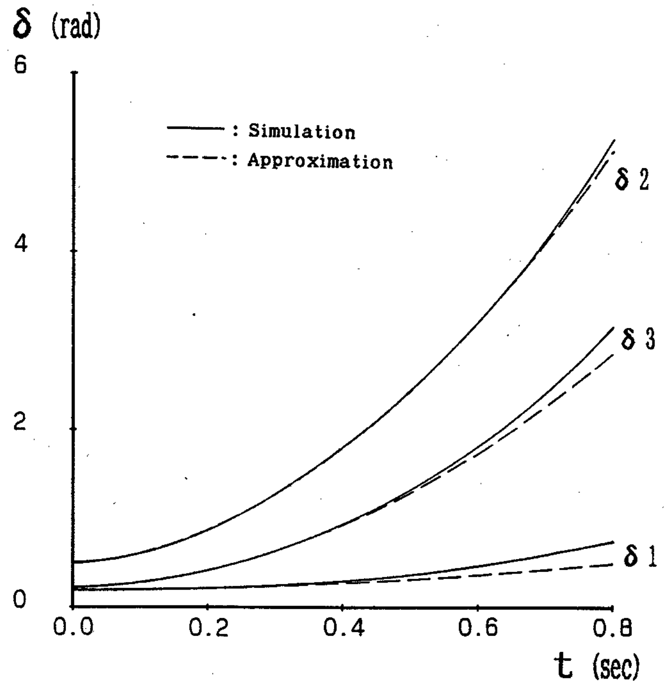

Fig. 4-1 Approximation of δ_i for a 3 machine system

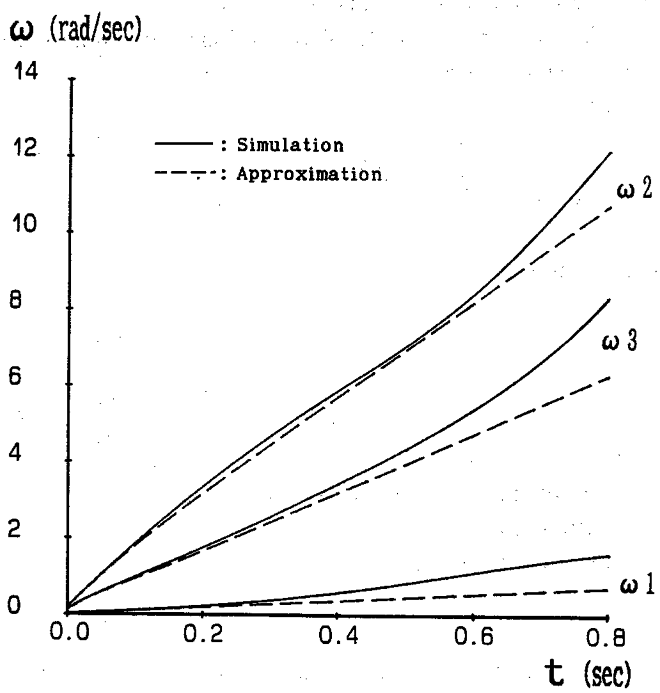

Fig. 4-2 Approximation of ω_i for a 3 machine system

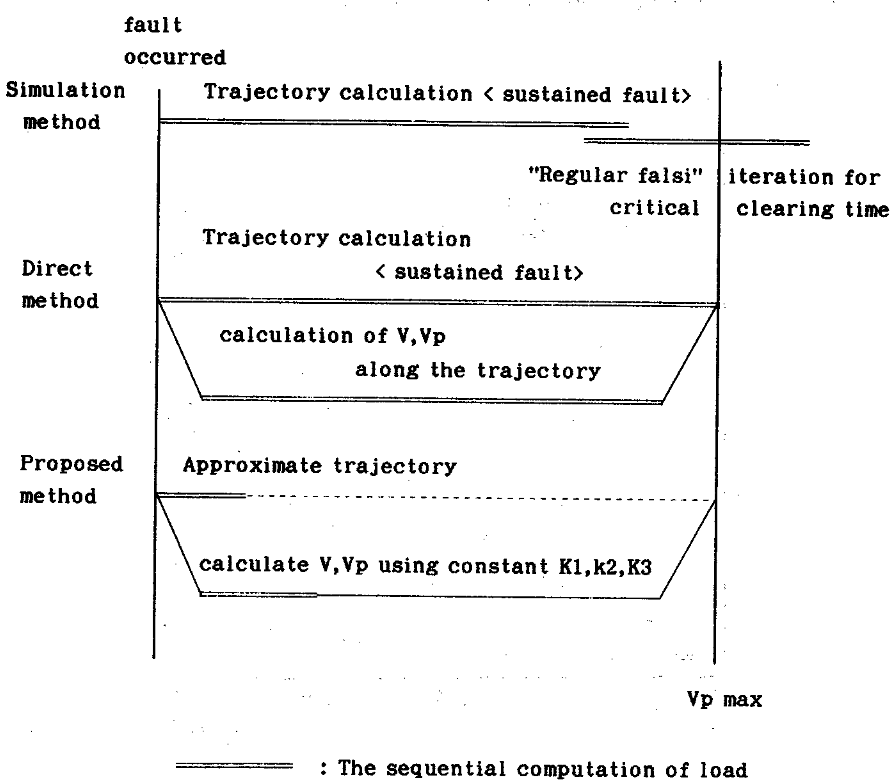

Fig. 5 The process of estimation for transient stability

5. Simulation

The usual direct and proposed method under considering the voltage dependence of load are applied to the transient stability analysis of two model systems which consist of 3 generators 9 buses and 9 generators 25 buses. The assumed disturbance at F is the 3-phase short circuit which occurs one of two lines, and the faulted line is cleared by opening the circuit breaker of faulted line after a certain lapse of time.

Table 1 shows the value of load parameter used in this analysis, and Table 2 shows the critical clearing time in each method. The comparison of the time required to calculate is demonstrated by the ratio, based on the time required for the usual direct method and the results are written in brackets of Table 2.

Further on the proposed method (1), first 5 steps are used for calculating the trajectory and every 5 steps sampling data are used for calculating the electrical power output. On the proposed method (2), first 3 steps for the trajectory and every 10 steps for the electrical power output.

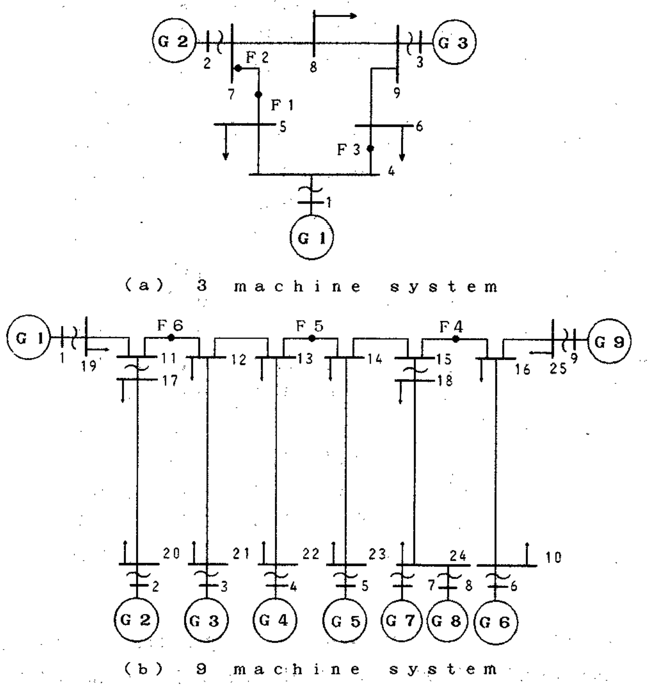

Fig. 6 Configuration of a 3 and a 9 machine systems

Table 1 Load parameter

	P characteristics np	Tp	Ap	Q characteristics nq	Tq	Aq
①	2.0	–	0	2.0	–	0
②	1.27	.018	1.17	1.19	.017	1.20
③	1.06	.020	1.00	1.29	.025	0.89
④	0.95	.025	1.58	1.53	.032	1.32
⑤	0.81	.010	1.77	0.91	.040	1.74

Table 2 Estimation of the critical clearing time (sec) for a 3 and a 9 machine systems

System	Fault	Load Parameter	simulation method	direct method	Proposed method (1)	Proposed method (2)
3 machine	F1	①	0.44 – 0.45	0.42	0.38	0.37
		②	0.40 – 0.41	0.40	0.38	0.38
		③	0.40 – 0.41	0.39	0.36	0.38
		④	0.37 – 0.38	0.41	0.39	0.41
		⑤	0.38 – 0.39	0.37	0.36	0.39
				[1]	[0.14]	[0.09]
	F2	①	0.26 – 0.27	0.24	0.24	0.24
		④	0.21 – 0.22	0.23	0.22	0.21
		⑤	0.21 – 0.22	0.22	0.20	0.20
				[1]	[0.18]	[0.12]
	F3	①	0.42–0.43	0.42	0.39	0.39
		②	0.44 – 0.45	0.46	0.44	0.44
		③	0.47 – 0.48	0.49	0.47	0.46
				[1]	[0.12]	[0.08]
9 machine	F4	①	0.3 – 0.33	0.35	0.33	0.32
		②	0.31 – 0.32	0.33	0.32	0.33
		③	0.29 – 0.30	0.32	0.31	0.32
		⑤	0.27 – 0.28	0.29	0.30	0.31
				[1]	[0.15]	[0.09]
	F5	①	0.23 – 0.24	0.19	0.18	0.18
		②	0.22 – 0.23	0.17	0.17	0.18
		③	0.18 – 0.19	0.16	0.16	0.16
		⑤	0.16 – 0.17	0.16	0.15	0.16
				[1]	[0.18]	[0.10]
	F6	①	0.14 – 0.15	0.14	0.15	0.15
		②	0.19 – 0.20	0.15	0.15	0.15
		⑤	0.17 – 0.18	0.15	0.14	0.14
				[1]	[0.16]	[0.10]

From the results of table 2, it is proved that the proposed method gives the good approximation of the direct method, and that the calculating time of proposed method is about 10% of that of the usual direct method.

6. Conclusion

In this paper, we demonstrated the method for fast evaluation of transient stability, considering the voltage dependence of load.

From the result, the time required for calculation is considerably reduced than that of the usual direct method and the accuracy is satisfactory in most cases. In the future, it is expected that the power system might be larger and system load characteristics might be more various. In the sense, the proposed method seems to be very suitable and adaptable for fast evaluation of the developed power system.

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