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A Modification of Siskos' Multicriteria Decision-Making Methodology Using Fuzzy Outranking Relations

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This paper presents a modified version of Siskos' methodology for multicriteria decision-making. A modification we propose is based on non-additive measures in fuzzy systems theory. Substitutive and non-substitutive decision models are proposed. The rank reversal of actions when powerful dependent criteria are introduced is hard to occur by our proposed modification of Siskos' multicriteria decision-problem using fuzzy outranking relations.

For a comparative study a numerical example of the evaluation of radiological protection system in nuclear power plants is quoted.

1. Introduction

Multicriteria decision-making problems are very complex systems, especially when the decision-making criteria give rise to uncertainty and ambiguity. A multicriteria decision-making methodology which allows the analyst to integrate fuzzy outranking relations into a domination structure has been proposed by Siskos *et al.*¹⁾.

The procedure is divided into two phases¹⁾: in the first phase, a method of assessing fuzzy outranking relations is established using partial fuzzy relations, where each is considered as a model of monocriterion uncertainty. In the second phase, the fuzzy outranking relation established in the first phase is used to define the fuzzy set of non-dominated actions. In the first phase the weights of criteria are assumed to be given. But the assessment of weights of criteria is also an important issue in the decision problems. The Analytic Hierarchy Process by Saaty²⁾ is a very convenient method for this purpose and various applications in the real world decision problems are reported in the literature. Hence we assume to adopt Saaty's ratio (proportional) scale as weights of criteria for multicriteria decision problems using fuzzy outranking relations.

In Siskos' methodology, however, under the assumption of ratio scale the rank reversal may occur by introducing an additional powerful dependent criterion. Thus, in this paper, we propose to use lower probability³⁾ P_* and upper probability³⁾ P^* generated from the possibility measure π which is a non-additive measure proposed by Zadeh⁴⁾. By our proposed approach we have the appearance of rank reversal only in comparatively few cases.

In Section 2 we briefly survey Siskos' methodology using fuzzy outranking relations. In Section 3 we present the modified methods. The theoretical background for applying non-additive measures is given and we point out the critical point in Siskos' methodology. The validity of the modified method is shown by examples. Lastly, in

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Table 1 Performance of actions on the n criteria

Action	Multicriteria evaluation					
	1	2	-----	i	-----	n
a	$g_1(a)$	$g_2(a)$	-----	$g_i(a)$	-----	$g_n(a)$
b	$g_1(b)$	$g_2(b)$	-----	$g_i(b)$	-----	$g_n(b)$
c	$g_1(c)$	$g_2(c)$	-----	$g_i(c)$	-----	$g_n(c)$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Section 4 we compare Siskos' methodology and the modified methods by quoting the problem in the evaluation of radiological protection system in nuclear power plants.

2. Siskos' Methodology

The general multicriteria choice problem is formulated as follows. If $A = \{a, b, c, \dots\}$ is a finite set of actions evaluated according to n criteria, noted as g_1, g_2, \dots, g_n , the problem consists in choosing the best action in A . The real number $g_i(a)$ represents the performance of action a for the i -th criterion. It is assumed that the higher the number $g_i(a)$, the more this action satisfies the preferences of the decision-maker. The multicriteria evaluation of an action $a \in A$ is defined by the vector $g(a) = [g_1(a), g_2(a), \dots, g_n(a)]$ which is comprised of the performances of this action on the n criteria (see Table 1). Each decision-making criterion is given a weighting factor p_i . These factors sum to one, i.e.

$$\sum_{i=1}^n p_i = 1 \quad (1)$$

where p_i represents the relative importance which the decision-maker gives to i -th criterion. The fuzzy outranking relation in $A \times A$ is represented by a membership function $d : A \times A \rightarrow [0, 1]$ in which the different values $d(a, b)$ denote the strength of the relationship between any two actions a and b in A . Thus, $d(a, b)$ is the degree of credibility of the outranking of the action b by the action a . The fuzzy outranking relation is reflexive [$d(a, a) = 1, \forall a \in A$] and

$$a \text{ is preferred to } b \Leftrightarrow d(a, b) > d(b, a), \quad (2)$$

$$a \text{ is indifferent to } b \Leftrightarrow d(a, b) = d(b, a) > 0, \quad (3)$$

$$a \text{ is incomparable to } b \Leftrightarrow d(a, b) = d(b, a) = 0. \quad (4)$$

$d(a, b)$ is based on $g(a), a \in A$. It can be written for any pair in $A \times A$:

$$\begin{aligned} d(a, b) &= d[g(a) - g(b)] \\ &= d[g_1(a) - g_1(b), \dots, g_n(a) - g_n(b)]. \end{aligned} \quad (5)$$

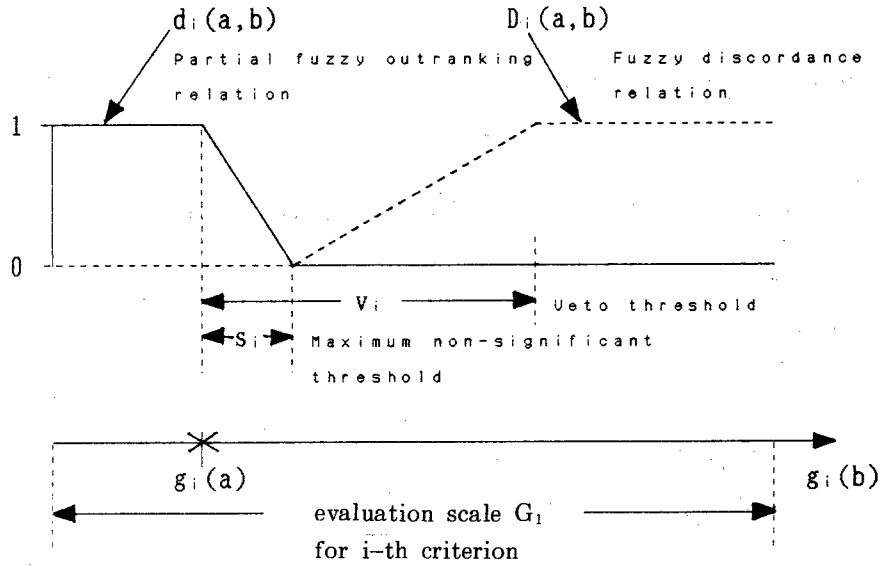


Fig. 1 Partial fuzzy outranking relation $d_i(a, b)$ and fuzzy discordance relation $D_i(a, b)$ for fixed $g_i(a)$ and variable $g_i(b)$, $b \in A$, for the i -th criterion.

To obtain an explicit form for $d(a, b)$, $(a, b) \in A \times A$, n pairs of partial fuzzy relations $[d_i(a, b) \text{ and } D_i(a, b), i = 1, 2, \dots, n]$ are defined. The first fuzzy relation $d_i(a, b)$ is the partial fuzzy outranking relation of b by a for the i -th criterion. The second fuzzy relation $D_i(a, b)$, called discordance, is designed to take into account the degree of incomparability of these two actions due to i -th criterion. These two fuzzy relations, shown in Fig. 1, depend directly on the sign and amplitude of the difference $g_i(a) - g_i(b)$ for each pair in $A \times A$. Let s_i denote the maximum non-significant threshold, beyond which the comparison can be made with certainty.

For any pair in $A \times A$ the partial fuzzy outranking relation is given by the following membership function:

$$d_i(a, b) = \begin{cases} 1; & \text{if } g_i(b) - g_i(a) \leq 0 \\ 0; & \text{if } g_i(b) - g_i(a) \geq s_i \\ \text{between 0 and 1;} & \text{otherwise.} \end{cases} \quad (6)$$

For $g_i(b) - g_i(a) \in [0, s_i]$ the decrease of $d_i(a, b)$ can be determined by linear interpolation as

$$d_i(a, b) = 1 - \frac{g_i(b) - g_i(a)}{s_i}.$$

v_i denotes the veto threshold beyond which a can in no case outrank b . $D_i(a, b)$ designates the discordance degree of criterion i for pair (a, b) according to the following definition:

$$D_i(a, b) = \begin{cases} 1; & \text{if } g_i(b) - g_i(a) \geq v_i \\ 0; & \text{if } g_i(b) - g_i(a) \leq s_i \\ \text{between 0 and 1;} & \text{otherwise.} \end{cases} \quad (7)$$

For $g_i(b) - g_i(a) \in [s_i, v_i]$ the precise values of $D_i(a, b)$ can be determined by linear interpolation as

$$D_i(a, b) = \frac{g_i(b) - g_i(a) - s_i}{v_i - s_i}$$

Having obtained n partial fuzzy outranking relations and taking into account the weights of criteria p_i , we have a fuzzy concordance relation $C : A \times A \rightarrow [0, 1]$ such as

$$C(a, b) = \sum_{i=1}^n p_i d_i(a, b). \quad (8)$$

Thus, for each pair $(a, b) \in A \times A$, $C(a, b)$ indicates the criteria concordance degree for the outranking of b by a .

To obtain the fuzzy outranking relation $d(a, b)$, the fuzzy concordance relation $C(a, b)$ and the n fuzzy discordance relations $D_i(a, b)$ are linked. The fuzzy outranking relation is given by the membership function $d : A \times A \rightarrow [0, 1]$ such as

$$d(a, b) = \begin{cases} C(a, b); & \text{if } C(a, b) \geq D_i(a, b), \forall i \\ \frac{C(a, b)}{1 - C(a, b) \prod_{i^*} [1 - D_{i^*}(a, b)]} & \text{with } i^* \in \{i \mid D_i(a, b) > C(a, b)\} \\ \text{; otherwise} & \end{cases} \quad (9)$$

with $d_i(a, b)$, $D_i(a, b)$ and $C(a, b)$ defined, respectively, by Eqs. (6), (7) and (8).

Given the global outranking structure that synthesizes situation of preference, indifference and incomparability, the fuzzy domination relation is given as a membership function $d^D : A \times A \rightarrow [0, 1]$ such as

$$d^D(a, b) = \begin{cases} d(a, b) - d(b, a); & \text{if } d(a, b) \geq d(b, a) \\ 0; & \text{otherwise.} \end{cases} \quad (10)$$

Thus, for a fixed $b \in A$ action, the membership function $d^D(b, a)$ is the fuzzy set of actions $a \in A$ that are dominated by b . It is now easy to obtain the non-domination structure by using the complementation operation in the fuzzy set theory, $d^{ND} : A \times A \rightarrow [0, 1]$ such as

$$d^{ND}(a, b) = 1 - d^D(a, b). \quad (11)$$

Similarly, for a fixed $b \in A$, $d^{ND}(b, a)$ is the fuzzy set of actions in A that are not

dominated by b . This fuzzy set, called the fuzzy set of non-dominated actions, $\mu^{ND} : A \rightarrow [0, 1]$, is determined by the intersection operation between fuzzy sets in the following way:

$$\mu^{ND}(a) = 1 - \max_{b \in A} [d(b, a) - d(a, b)] \quad (12)$$

where $\mu^{ND}(a)$ represents the non-domination degree of action a simultaneously by all the other actions. This induces the following decision-making rule: choose $a^* \in A$ with

$$\mu^{ND}(a^*) = 1 - \min_{a \in A} \max_{b \in A} [d(b, a) - d(a, b)]. \quad (13)$$

3. Modified Methods Using Non-additive Measures

3.1. Possibilistic weights and expectations

In a conventional decision method such as the simple additive weighting method, the grade of importance of each decision-making criterion is represented by the additive weights. Hence the

weights p_1, p_2, \dots, p_n are normalized as $\sum_{i=1}^n p_i = 1$.

In this paper we assume that the relative importance of each criterion is given as non-additive possibilistic weights, i.e. a possibility measure³⁾ denoted as $\pi_1, \pi_2, \dots, \pi_n$. And, the weights are normalized as $\max \pi_i = 1$.

As was shown by Dubois and Prade⁵⁾, the distribution π_i simply relates to the underlying basic probability assignment in the theory of evidence⁶⁾. Let Ω denote finite universe set and, A and B be subsets of Ω . $\pi(A)$ denotes the possibility measure of a subset A .

A lower probability is a mapping P_* from 2^Ω to $[0, 1]$. A lower probability is uniquely defined through the specification of basic probability assignment m , satisfying;

$$m(\phi) = 0, \sum_{B \subset \Omega} m(B) = 1 \quad (14)$$

and we have

$$P_*(A) = \sum_{B \subset A} m(B), \forall A \subset \Omega. \quad (15)$$

A set A such that $m(A) > 0$ is called a focal element. The upper probability $P^*(A) = 1 - P_*(A)$ is also defined as

$$P^*(A) = \sum_{B \cap A \neq \phi} m(B). \quad (16)$$

Assuming, without loss of generality, the π_i 's are decreasingly ordered ($\pi_1 = 1 \geq \pi_2 \geq \dots \geq \pi_n \geq \pi_{n+1} = 0$) and n is the density of Ω . Defining $A_i = \{e \mid e \in \Omega, \pi(e) \geq \pi_i\}$ and

$$\begin{aligned}
 m(A) &= 0 && \text{if } A \neq A_i \text{ for all } i \\
 m(A_i) &= \pi_i - \pi_{i+1} && \text{for } i = 1, \dots, n,
 \end{aligned} \tag{17}$$

we have

$$P^*(A) = \pi(A) \text{ for any } A \subset \Omega. \tag{18}$$

The class of upper probability measures includes possibility measures as a special case. This property was shown by Banon⁷⁾. The concept of expectation is basic, and useful in the conventional probability theory. Dempster's³⁾ framework enables it to be carried over to upper probability measures which include Zadeh's possibility measures. Let g be a real valued function over Ω . From the knowledge of upper probability, an upper distribution function of g can be defined by

$$F(v) \triangleq P^*(g \leq v). \tag{19}$$

The expectation $E(g)$ with respect to upper probability is the Lebesgue-Stieljes integral,

$$\begin{aligned}
 E(g) &= \int_{-\infty}^{\infty} v \, dF(v) \\
 &= \sum_{i=1}^n g_i \cdot [F(g_i) - F(g_{i-1})]
 \end{aligned} \tag{20}$$

where it is assumed that $0 = g_0 \leq g_1 \leq g_2 \leq \dots \leq g_n$ and $F(g_0) = 0$. Let B_i 's be the nested sequence of the sets such that $B_i = \{e \mid g(e) \leq g_i, e \in \Omega\}$ and $\phi = B_0 \subset B_1 \subset B_2 \subset \dots \subset B_n$. A_i 's are also nested sequence as $A_1 \subset A_2 \subset \dots \subset A_n$ by the definition.

[Proposition 1] The lower expectation E_* of the function $g: \Omega \rightarrow \mathbb{R}^1$ with respect to possibility can be written as

$$E_*(g) = \sum_{i=1}^n [(\pi_i - \pi_{i+1}) \cdot \min_{e \in A_i} g(e)]. \tag{21}$$

Though we can readily see Eq. (21) from the property shown by Smets⁸⁾, we will show a direct proof.

(Proof)

Since $P^*(A) = \pi(A)$ for all $A \subset \Omega$.

$$\pi(B_i) = \sum_{(B_i \cap A_j) \neq \phi} m(A_j), \tag{22}$$

$$\begin{aligned}
 E(g) &= \sum_{i=1}^n g_i \cdot [\pi(B_i) - \pi(B_{i-1})] \\
 &= \sum_{i=1}^n \left[\sum_{(B_i \cap A_j) \neq \phi} m(A_j) - \sum_{(B_{i-1} \cap A_j) \neq \phi} m(A_j) \right] \cdot g_i
 \end{aligned}$$

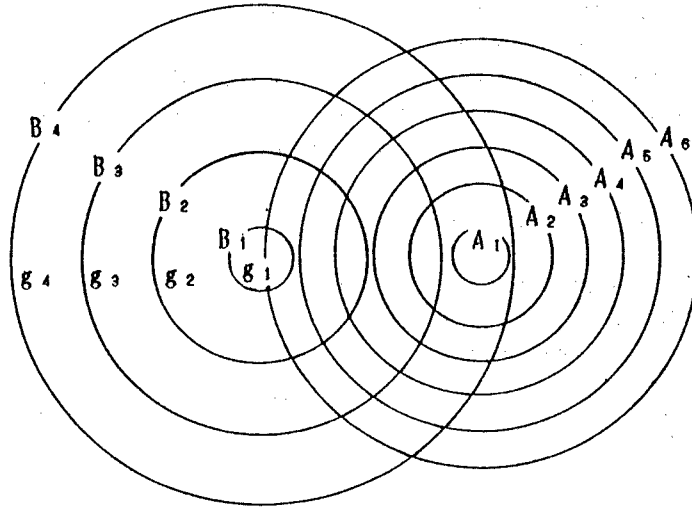


Fig. 2 Nested sequences of A_i 's and B_i 's.

$$= \sum_{i=1}^n \left[\frac{\sum_j m(A_j)}{\substack{(B_i \cap A_j) \neq \phi \\ (B_{i-1} \cap A_j) = \phi}} \right] \cdot g_i. \quad (23)$$

Subsets B_i 's and A_j 's are the nested sequence as shown in Fig. 2. Hence,

$$\begin{aligned} E(g) &= \sum_{i=1}^n [m(A_i) \cdot \min_{e \in A_i} g(e)] \\ &= \sum_{i=1}^n [(\pi_i - \pi_{i+1}) \cdot \min_{e \in A_i} g(e)]. \end{aligned} \quad (24)$$

[Corollary 1] When the possibilistic weights are given as $\pi_1 = \pi_2 = \dots = \pi_n = 1$, then

$$E(g) = \min_{e \in \Omega} g(e) = \min_i g_i. \quad (25)$$

Similarly, we have the following formula for the upper expectation E^* .

$$E^*(g) = \sum_{i=1}^n [(\pi_i - \pi_{i+1}) \cdot \max_{e \in A_i} g(e)]. \quad (26)$$

3.2. Modified methods

We call Siskos' method S-model, and the modified versions using upper probability and lower probability are called U-model and L-model, respectively.

As the method by Siskos *et al.*¹⁾, our modified version is divided into two phases. The first phase consists in assessing the fuzzy outranking relation from the fuzzy partial relation $d_i(a, b)$ and $D_i(a, b)$. The proposed fuzzy outranking relation represents the non-compensatory effect, i.e. non-substitutive evaluation of alternatives, taking into account the weights of attributes. The second phase is the same as Siskos' method.

The first phase is as follows. Let us assume that we have n criteria for a decision problem. And we have n partial fuzzy outranking relations. We can aggregate these fuzzy relations into a single one taking into account the grade of possibilistic importance of criteria by adopting possibility theory and Dempster's expectation with respect to upper probability. We propose to use Eq. (21) as an aggregation formula instead of Eq. (8) by Siskos *et al.*¹⁾. Let I_{A_i} denote the index set such as

$$I_{A_i} = \{j \mid j \in \{1, \dots, n\}, \pi_j \geq \pi_i\} \quad (27)$$

where π_i denotes the possibilistic weight of i -th criterion. $d_i(a, b)$ is the partial fuzzy outranking relation. Replacing $g(e)$ of $E(g)$ in proposition 1 by $d_i(a, b)$, the aggregation formula proposed by Siskos is modified as

$$C'(a, b) = \sum_{i=1}^n [(\pi_i - \pi_{i+1}) \cdot \min_{j \in I_{A_i}} d_j(a, b)]. \quad (28)$$

For each pair $(a, b) \in A \times A$, $C'(a, b)$ indicates the complementary (non-substitutive) degree of criteria concordance for the outranking of b by a , taking into account the weights of criteria. The fuzzy outranking relation is defined as

$$d(a, b) = \begin{cases} C'(a, b); & \text{if } C'(a, b) \geq D_i(a, b), \forall i \\ \frac{C'(a, b)}{1 - C'(a, b)} \prod_{i^*} [1 - D_{i^*}(a, b)] & \\ \text{with } i^* \in \{i \mid D_i(a, b) > C'(a, b)\}. \end{cases} \quad (29)$$

In the case of L-model, i.e. the model using lower probability P_* , we define

$$C''(a, b) = \sum_{i=1}^n [(\pi_i - \pi_{i+1}) \cdot \max_{j \in I_{A_i}} d_j(a, b)], \quad (30)$$

$$d(a, b) = \begin{cases} C''(a, b); & \text{if } C''(a, b) \geq D_i(a, b), \forall i \\ \frac{C''(a, b)}{1 - C''(a, b)} \prod_{i^*} [1 - D_{i^*}(a, b)] & \\ \text{with } i^* \in \{i \mid D_i(a, b) > C''(a, b)\}. \end{cases} \quad (31)$$

Other procedures are the same as in the case of upper probability P^* .

In the second phase, given the outranking structure $d(a, b)$, we define the fuzzy domination relation as

$$d^D(a, b) = \begin{cases} d(a, b) - d(b, a); & \text{if } d(a, b) \geq d(b, a) \\ 0; & \text{otherwise.} \end{cases} \quad (32)$$

We obtain the non-domination structure d^{ND} such as

$$d^{ND}(a, b) = 1 - d^D(a, b). \quad (33)$$

The fuzzy set of non-dominated actions μ^{ND} is determined by

$$\mu^{ND}(a) = 1 - \max_{b \in X} [d(b, a) - d(a, b)], \quad (34)$$

$$\mu^{ND}(a^*) = \max_{a \in X} \mu^{ND}(a) = 1 - \min_{a \in X} \max_{b \in X} [d(b, a) - d(a, b)]. \quad (35)$$

3.3. Illustrative examples of Siskos' methodology and the modified versions

The concordance index of Eq.(8) can be thought assuming additivity of the weight of criteria. Hence the dependent criterion has undesirable influence to the decision. We show it by a numerical example.

A decision maker has to choose among three alternatives A, B and C. The choice is based on information related to two criteria. Evaluations of these criteria are the result of psychological tests and vary between 0 and 10. 10 denotes the optimal solution (see Table 2). The maximum non-significant threshold s_i is equal to 2 for the two criteria, i.e. $s_1 = s_2 = 2$. The veto threshold v_i is equal to 5 for the two criteria, i.e. $v_1 = v_2 = 5$. The weights are given as $p_1 = 0.4$ and $p_2 = 0.6$.

By Siskos' methodology we have $\mu^{ND}(a) = \{0.68/A, 0.73/B, 1.00/C\}$. The optimal solution is alternative C.

To examine the occurrence of rank reversal by assuming ratio scale in assessing the weight p_i , we introduce an additional criteria which is powerfully dependent to criterion 1 (see Table 3). Here we assume ratio scale²⁾. The maximum non-significant thresholds s_i are all equal to 2 for the three criteria, i.e. $s_1 = s_2 = s_3 = 2$. The veto thresholds v_i are equal to 5 for three criteria, i.e. $v_1 = v_2 = v_3 = 5$. The weights p_i are $p_1 = 0.285$, $p_2 = 0.430$ and $p_3 = 0.285$.

By Siskos' methodology, we have $\mu^{ND}(a) = \{0.69/A, 1.00/B, 0.86/C\}$. The

Table 2 Values of evaluation for two criteria

Alternative	Multicriteria evaluation	
	1	2
A	5.2	5.6
B	7.1	4.5
C	3.8	8.0

Table 3 Values of evaluation for three criteria

Alternative	Multicriteria evaluation		
	1	2	3
A	5.2	5.6	5.2
B	7.1	4.5	7.1
C	3.8	8.0	3.8

optimal solution is alternative B. Thus, the rank reversal appeared when introduced the additional dependent criterion.

Let us now solve the problem using the modified methodology. In case of two criteria we have $\mu^{ND}(a) = \{0.47/A, 0.71/B, 1.00/C\}$ by U-model and $\mu^{ND}(a) = \{0.67/A, 0.67/B, 1.00/C\}$ by L-model, respectively. The optimal solutions is alternative C in both cases.

Introducing an additional criterion, we have $\mu^{ND}(a) = \{0.46/A, 0.83/B, 1.00/C\}$ by U-model and $\mu^{ND}(a) = \{0.66/A, 0.66/B, 1.00/C\}$ by L-model, respectively. The optimal solution is alternative C.

As shown above, by using fuzzy concordance relations with non-additive measures, the rank reversal is hard to appear.

4. A Comparative Study in the Choice of a Radiation Protective System

We here quote the data from the paper by Siskos' *et al.*¹⁾ to compare the computational results between their methodology and ours (see Table 4).

Choosing a protective system for a nuclear power plant is a problem of choosing a treatment option for each effluent pathway. Since there are several possible options for each of the six categories of effluents (PT, DR, LV, TEG, BR and BAN), there will be as many systems or actions as there are combinations. In fact we have $3 \times 4 \times 2 \times 4 \times 2 \times 2 = 384$ actions. We shall choose the best combination from 384 actions.

Decision making criteria are as follows:

- (1) investment cost (in 10^3 francs);
- (2) annual operating cost (in 10^3 francs);
- (3) short-term public health impact indicator (in man-Sieverts);
- (4) long-term public health impact indicator (in man-Sieverts);
- (5) health impact indicator for critical group (in man-Sieverts);
- (6) occupational health impact indicator (in man-Sieverts);
- (7) adaptability of the system to abnormal operation of the reactor (qualitative criterion).

As shown in Table 5, the best options by S-model and by U-model are the same (110311). But, as to the second to the best in S-model the inpreferable options in the seventh criterion (12 and 13 is small compared with 16) were chosen, because in other criterion the evaluated values are relatively better. Whereas, the option (130311) which has no specific defect was chosen by U-model. Hence, these results show the non-substitutive decision in U-model.

In the case of L-model, three options were chosen as the best, since they were evaluated as the best in the seventh criterion (the evaluated value is 16) which is the most important one. This implies the substitutive decision by L-model.

5. Conclusion

Siskos' methodology requires sufficient attention if we determine the ratio scale weights of criteria when some criteria are dependent each other. We have demonstrated that the rank reversal may occur by introducing a powerful dependent criterion when the weights are estimated as a ratio scale.

The advantage of our modification is that we have substitutive and non-substitutive evaluations of the alternatives, which are apart from those of additive utility ap-

Table 4 Data base for the problem of choosing a protective system for a French nuclear power plant of the PWR-1300 MW (e) type. Quoted from Siskos *et al.*¹⁾.

Effluent pathways	Treatment options (symbols)	Treatment characteristics	Investment cost (10 ³ F)	Annual operating cost (10 ³ F)	Public health impact indicator in short term (M-Sv)	Public health impact indicator in long term (M-Sv)	Health impact indicator on critical group (Sievert)	Occupational health impact indicator (M-Sv)	Safety criterion (qualitative)
Tritium purge	PT-0	no treatment	0	0	0.036	1.3*10 ⁻⁴	1.9*10 ⁻⁸	0	0
	PT-1	St (10 days)	698	28	0.027	1.6*10 ⁻⁴	1.5*10 ⁻⁸	0.10	2
	PT-2	St (10 days)+Dem [DF=5]	1459	142	0.0054	3.3*10 ⁻⁵	3.0*10 ⁻⁹	0.30	2
Drains	DR-0	no treatment	0	0	15	2.6*10 ⁻²	7.8*10 ⁻⁶	0	0
	DR-1	St (5 days)+Fi+Ev [DF=100]	8330	400	0.12	2.9*10 ⁻⁴	6.0*10 ⁻⁸	0.20	2
	DR-2	St (5 days)+Fi+Ev+Dem [DF=1000]	8711	456	0.012	2.9*10 ⁻⁵	6.0*10 ⁻⁹	0.35	2
	DR-3	St (10 days)+Fi+Ev+Dem [DF=1000]	10412	524	0.0093	3.2*10 ⁻⁵	4.9*10 ⁻⁹	0.65	3
Laundry	LV-0	no treatment	0	0	0.009	1.6*10 ⁻⁵	4.5*10 ⁻⁹	0	0
	LV-1	St (30 days)+Fi	431	51	0.0033	2.9*10 ⁻⁵	2.0*10 ⁻⁹	0.05	0
Steam generator blowdown	GV-0	no treatment	0	0	0.021	5.1*10 ⁻⁵	1.1*10 ⁻⁸	0	0
Turbine building's floor drains	FPE-0	no treatment	0	0	0.0069	1.3*10 ⁻⁵	3.6*10 ⁻⁹	0	0
	TEG-0	no treatment	0	0	84	4.4*10 ⁻⁴	2.5*10 ⁻³	0	0
Gaseous effluents	TEG-1	St (20 days)	2058	82	3.6	4.6*10 ⁻⁴	1.1*10 ⁻⁴	0.20	1
	TEG-2	St (40 days)	2235	89	1.6	4.6*10 ⁻⁴	4.8*10 ⁻⁵	0.25	2
	TEG-3	St (60 days)	2413	96	1.5	4.6*10 ⁻⁴	4.5*10 ⁻⁵	0.30	3
Reactor building ventilation	BR-0	no treatment	0	0	0.9	5.0*10 ⁻⁶	2.5*10 ⁻⁵	0	0
	BR-1	Fi [DF=10]	86	24	0.75	4.8*10 ⁻⁶	2.2*10 ⁻⁵	0.10	4
Auxiliary building ventilation	BAN-0	no treatment	0	0	0.51	2.7*10 ⁻⁶	1.5*10 ⁻⁵	0	0
	BAN-1	Fi [DF=10]	320	58	0.48	2.7*10 ⁻⁶	1.4*10 ⁻⁵	0.10	4
Condenser air ejector	EX-0	no treatment	0	0	0.57	3.0*10 ⁻⁶	1.6*10 ⁻⁵	0	0
Turbine building steam leakage	EVPE-0	no treatment	0	0	0.0045	0	9.7*10 ⁻⁸	0	0
Average criteria weights			0.130	0.110	0.150	0.050	0.090	0.230	0.240
Criteria evaluation scales for the protective systems			from	0	0	3.4	0.00062	0.0001	0
			to	15121	895	100	0.0026	1.5	16
Maxima non-significant thresholds			100	10	50%	100%	30%	100%	3
Veto thresholds			15000	1000	75	0.02	0.003	0.5	10

Notes: St=storage; Fi=filtration; Ev=evaporation; Dem=demineralization; DF=decontamination factor

Table 5 Results obtained by using three methods

	Rank	Protective system proposed						Non-domination degree	Multicriteria consequences of actions						
		PT	DR	LV	TEG	BR	BAN		1 (10 ³ F)	2 (10 ³ F)	3 (M-Sv)	4 (M-Sv)	5 (Sv)	6 (M-Sv)	7 (Qual)
Siskos' methodology	1	1	1	0	3	1	1	0.876	11847	606	3.5	0.00093	0.000081	0.80	15
	2	0	1	0	2	1	1	0.874	10971	571	3.6	0.00090	0.000084	0.65	12
	2	0	1	0	3	1	1	0.874	11149	578	3.5	0.00090	0.000081	0.70	13
The method by using lower probability	1	2	3	0	3	1	1	1.000	14690	844	2.8	0.00055	0.000081	1.45	16
	1	1	3	1	3	1	1	1.000	14360	781	2.8	0.00069	0.000081	1.30	16
	1	2	3	1	3	1	1	1.000	15121	895	2.7	0.00056	0.000081	1.50	16
The method by using upper probability	1	1	1	0	3	1	1	1.000	11847	606	3.5	0.00093	0.000081	0.80	15
	2	1	3	0	3	1	1	0.988	13929	730	2.8	0.00068	0.000081	1.25	16
	3	1	3	1	3	1	1	0.965	14360	781	2.8	0.00069	0.000081	1.30	16
The weights of criteria									0.130	0.110	0.150	0.050	0.090	0.230	0.240

proach⁹⁾. Especially in the decision problems such as choosing protective systems for the nuclear power plants, even if a system is the very best for public health, it is not a best selection if the system sacrifices the occupational health for instance. Hence the non-substitutive decision seems necessary, since the problem is concerned with the lives of human beings.

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