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Application of the Position Angle Matrix to Analysis of the Boundary-Value Problems of the Dissymmetrical Many Stage Cascade Connected Polyphase Transmission Lines with the Initial Conditions

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In this paper the line potential and the line current along the combined dissymmetrical multi-conductor transmission system are calculated by making use of the position angle matrix. These are the general solutions of the system, which have been solved under arbitrary and initial conditions of system.

1. Introduction

In the previous papers[1,2] we dealt with the boundary-value problem in the multiple-stage cascade connected polyphase transmission system by making use of the position angle matrix and showed that the line potential[1] and the line current[2] at any point were given in the compact form by using the position angle matrix with some numerical examples for the practical case. But in those papers we considered only the electric sources at the sending ends of the transmission lines and ignored the initial values of voltages or currents distributed on the transmission lines.

In this paper we take into consideration not only the electric sources at the sending ends of the transmission lines but also the arbitrary initial values of potentials and currents on the lines. The construction of the transmission system is substantially the same as that in the previous paper[2], and it is generally assumed that different lines are connected in series and the arrangement of conductors at any stage is dissymmetrical, so-called the combined dissymmetrical multi-conductor transmission system.

Although the system has the initial values, we can easily obtain the solutions, potentials and currents at any point on the transmission line, under the arbitrary boundary conditions by making use of the position angle matrix. This leads to get the solutions of potentials and currents of such a system from the diagram directly using the position angle matrices without solving the initial and boundary value problem of simultaneous partial differential equations.

2. Solutions of Potentials and Currents on the 2-Stage Combined Multi-Conductor System

Consider a 2-stage combined n-conductor transmission system having lengths l_1 and l_2 as shown in Fig. 1. A group of voltage sources $[e_0]$ and a group of impedances $[z_1]$

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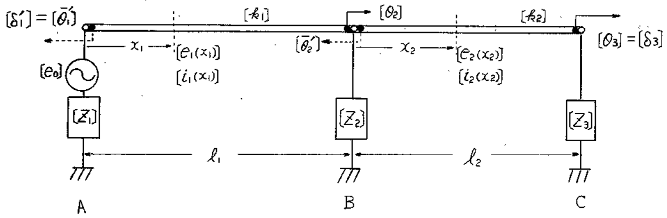

Fig. 1 An equivalent circuit of cascade connected polyphase transmission system.

are in series at the sending terminal A. Groups of impedances $[z_2]$ and $[z_3]$ are at the point B and at the receiving terminal C, respectively. Matrices $[e_i(x_i)]$ and $[i_i(x_i)]$ are respectively traveling voltage and current waves at an arbitrary point x_i on the i-th stage line. Now these solutions may be expressed[3] in the operational forms as follows:

$$[e_i(x_i)] = \sinh [k_i] x_i \cdot [\alpha_i] + \cosh [k_i] x_i \cdot [\beta_i] + [k_i]^{-1} \int_0^{x_i} \sinh [k_i] (x_i - \xi_i) [Q_i(\xi_i)] \, d\xi_i \tag{1}$$

$$[i_i(x_i)] = - [k_i^*]^{-1} [k_i] (\cosh [k_i] x_i \cdot [\alpha_i] + \sinh [k_i] x_i \cdot [\beta_i]) + [k_i^*]^{-1} \Big\{ p [L_i] [I_i(x_i, 0)] - \int_0^{x_i} \cosh [k_i] (x_i - \xi_i) [Q_i(\xi_i)] \, d\xi_i \Big\} \tag{2}$$

$$[k_i^*] = [pL_i + R_i] \tag{3}$$

$$[k_i]^2 = [k_i^*] [pC_i + G_i] \tag{4}$$

$$[Q_i(x)] = p [L_i] \frac{d}{dx} [I_i(x, 0)] - p [k_i^*] [C_i] [E_i(x, 0)] \tag{5}$$

where $[\alpha_i]$ and $[\beta_i]$ are the constants of the integration to be determined by boundary conditions, $[E_i(x_i, 0)]$ and $[I_i(x_i, 0)]$ are the initial values of voltages and currents on the lines, $[L_i]$, $[C_i]$, $[R_i]$ and $[G_i]$ are $n \times n$ matrices representing the inductances, capacitances, resistances and leakances per unit length of the transmission lines respectively and p is Heaviside's operator.

For such a case of Fig. 1, the boundary conditions are written as

$$[e_1(l_1)] = [e_2(0)] = [z_2] ([i_1(l_1)] - [i_2(0)]) \tag{6}$$

$$[e_2(l_2)] = [z_3] [i_2(l_2)] \tag{7}$$

We can get integral constants $[\alpha_1]$, $[\beta_1]$, $[\alpha_2]$ and $[\beta_2]$ from Eqs. (1), (2), (6) and (7). At $i = 2, x_2 = 0$ in Eq. (1), we find

$$[e_2(0)] = [\beta_2] \tag{8}$$

From Eqs. (1), (2) and (7), we obtain the next equation

$$\begin{aligned}
&\sinh [k_2]\, l_2 \cdot [\alpha_2] + \cosh [k_2]\, l_2 \cdot [\beta_2] \\
&+ [k_2]^{-1} \int_0^{l_2} \sinh [k_2]\,(l_2 - \xi_2)\,[Q_2(\xi_2)]\, d\xi_2 \\
&= [z_3]\,(-[k_2^*]^{-1}\,[k_2]\,(\cosh [k_2]\, l_2 \cdot [\alpha_2] \\
&\quad + \sinh [k_2]\, l_2 \cdot [\beta_2]) + [k_2^*]^{-1} \left\{ p\,[L_2]\,[I_2(l_2, 0)] \right. \\
&\quad \left. - \int_0^{l_2} \cosh [k_2]\,(l_2 - \xi_2)\,[Q_2(\xi_2)]\, d\xi_2 \right\})
\end{aligned} \tag{9}$$

Now, introducing the position angle matrix $[\delta_3]$ that satisfies the relation $\tanh [\delta_3] = [z_3]\ [k_2^*]^{-1}\,[k_2]$ and taking Eq. (8) into account, Eq. (9) becomes as follows:

$$\begin{aligned}
&(\sinh [k_2]\, l_2 + \tanh [\delta_3]\, \cosh [k_2]\, l_2)\,[\alpha_2] \\
&= -(\cosh [k_2]\, l_2 + \tanh [\delta_3]\, \sinh [k_2]\, l_2)\,[e_2(0)] \\
&\quad - \int_0^{l_2} \sinh [k_2]\,(l_2 - \xi_2)\,[k_2]^{-1}\,[Q_2(\xi_2)]\, d\xi_2 \\
&\quad + \tanh [\delta_3] \cdot [k_2]^{-1} \left\{ p\,[L_2]\,[I_2(l_2, 0)] \right. \\
&\quad \left. - \int_0^{l_2} \cosh [k_2]\,(l_2 - \xi_2)\,[Q_2(\xi_2)]\, d\xi_2 \right\}
\end{aligned}$$

Therefore

$$\begin{aligned}
[\alpha_2] = &-\left\{ \mathrm{SH}([\delta_3] + [k_2]\, l_2) \right\}^{-1} \mathrm{CH}([\delta_3] + [k_2]\, l_2) \cdot [e_2(0)] \\
&-\left\{ \mathrm{SH}([\delta_3] + [k_2]\, l_2) \right\}^{-1} \int_0^{l_2} \mathrm{SH}\left\{ [\delta_3] + [k_2]\,(l_2 - \xi_2) \right\} \\
&\times [k_2]^{-1}\,[Q_2(\xi_2)]\, d\xi_2 + \left\{ \mathrm{SH}([\delta_3] + [k_2]\, l_2) \right\}^{-1} \\
&\times \sinh [\delta_3] \cdot [k_2]^{-1}\, p\,[L_2]\,[I_2(l_2, 0)]
\end{aligned} \tag{10}$$

Substitution of Eqs. (8) and (10) in Eq. (1) results

$$\begin{aligned}
[e_2(x_2)] = &\,\mathrm{SH}\left\{ [k_2]\,(l_2 - x_2) + [\delta_3] \right\} \left\{ \mathrm{SH}([k_2]\, l_2 + [\delta_3]) \right\}^{-1} \\
&\times ([e_2(0)] - \int_0^{x_2} \sinh [k_2]\, \xi_2 \cdot [k_2]^{-1}\,[Q_2(\xi_2)]\, d\xi_2) \\
&+ \sinh [k_2]\, x_2 \cdot \left\{ \mathrm{SH}([\delta_3] + [k_2]\, l_2) \right\}^{-1} \\
&\times (\sinh [\delta_3] \cdot [k_2]^{-1}\, p\,[L_2]\,[I_2(l_2, 0)] \\
&- \int_{x_2}^{l_2} \mathrm{SH}\left\{ [\delta_3] + [k_2]\,(l_2 - \xi_2) \right\} [k_2]^{-1}\,[Q_2(\xi_2)]\, d\xi_2)
\end{aligned} \tag{11}$$

3. Solutions of Potentials and Currents on the *m*-Stage Combined Multi-Conductor System

By the same method as Section 2, the equation of the line potentials for all cases of one-, two- and three-stage transmission lines may be written in the same style as follows:

$$\begin{aligned}[e_i(x_i)] = &\mathrm{SH}\left\{[k_i]\,(l_i - x_i) + [\theta_{i+1}]\right\}\left\{\mathrm{SH}\,([k_i]\,l_i + [\theta_{i+1}])\right\}^{-1} \\ &\times ([e_i(0) - [Q_i\,(0, x_i)]) + \sinh\,[k_i]\,x_i \\ &\times\left\{\mathrm{SH}\,([\tilde{\theta}_{i+1}] + [k_i]\,l_i)\right\}^{-1}([P^*_{m,i}] - [\tilde{Q}_i(x_i, l_i)])\end{aligned} \qquad (12)$$

where

$$[Q_i(0, x_i)] = \int_0^{x_i} \sinh\,[k_i]\,\xi_i \cdot [k_i]^{-1}\,[Q_i(\xi_i)]\,d\xi_i$$

$$[\tilde{Q}_i(x_i, l_i)] = \int_{x_i}^{l_i} \mathrm{SH}\left\{[\tilde{Q}_{i+1}] + [k_i]\,(l_i - \xi_i)\right\}\,[k_i]^{-1}\,[Q_i(\xi_i)]\,d\xi_i$$

Here the matrix $[P^*_{m,i}]$ is as follows:

for the case of the one-stage transmission system ($m = 1$)

$$[P^*_{1,1}] = \mathrm{SH}\,[\tilde{\theta}_2]\cdot[P_{1,l}]$$

for the case of the two-stage transmission system ($m = 2$)

$$\begin{aligned}[P^*_{2,1}] = &\mathrm{SH}\,[\tilde{\theta}_2]\cdot[P_{1,l}] - [\gamma_1]\,\mathrm{SH}\,([\tilde{\theta}_3] + [k_2]\,l_2)\,[P_{2,0}] \\ &+ [\gamma_1]\,(\mathrm{SH}\,[\tilde{\theta}_3]\cdot[P_{2,l}] - [T_2])\end{aligned}$$

$$[P^*_{2,2}] = \mathrm{SH}\,[\tilde{\theta}_3]\cdot[P_{2,l}]$$

for the case of the three-stage transmission system ($m = 3$)

$$\begin{aligned}[P^*_{3,1}] = &\mathrm{SH}\,[\tilde{\theta}_2]\cdot[P_{1,l}] - [\gamma_1]\,\mathrm{SH}\,([\tilde{\theta}_3] + [k_2]\,l_2)\cdot[P_{2,0}] \\ &- [\gamma_1]\,[\gamma_2]\,\mathrm{SH}\,([\tilde{\theta}_4] + [k_3]\,l_3)\cdot[P_{3,0}] \\ &+ [\gamma_1]\,(\mathrm{SH}\,[\tilde{\theta}_3]\cdot[P_{2,l}] - [T_2]) \\ &+ [\gamma_1]\,[\gamma_2]\,(\mathrm{SH}\,[\tilde{\theta}_4]\cdot[P_{3,l}] - [T_3])\end{aligned}$$

$$\begin{aligned}[P^*_{3,2}] = &\mathrm{SH}\,[\tilde{\theta}_3]\cdot[P_{2,l}] - [\gamma_2]\,\mathrm{SH}\,([\tilde{\theta}_4] + [k_3]\,l_3)\cdot[P_{3,0}] \\ &+ [\gamma_2]\,(\mathrm{SH}\,[\tilde{\theta}_4]\cdot[P_{3,l}] - [T_3])\end{aligned}$$

$$[P^*_{3,3}] = \mathrm{SH}\,[\tilde{\theta}_4]\cdot[P_{3,l}]$$

where

$$[\gamma_j] = \mathrm{SH}\,([\tilde{\theta}_{j+1}] + [k_j]\, l_j + [\bar{\theta}'_j])\left\{\mathrm{SH}\,([k_j]\, l_j + [\bar{\theta}'_j])\right\}^{-1}$$
$$\times\ \mathrm{SH}\,[\bar{\theta}'_{j+1}]\left\{\mathrm{SH}\,([\tilde{\theta}_{j+2}] + [k_{j+1}]\, l_{j+1} + [\bar{\theta}'_{j+1}])\right\}^{-1}$$

$$[T_j] = \int_0^{l_j} \mathrm{SH}\left\{[\tilde{\theta}_{j+1}] + [k_j]\,(l_j - \xi_j)\right\}[k_j]^{-1}\,[Q_j(\xi_j)]\, d\xi_j$$

$$[P_{i,0}] = [k_i]^{-1}\, p\,[L_i]\;[I_i(0,0)]$$

$$[P_{i,l}] = [k_i]^{-1}\, p\,[L_i]\;[I_i(l_i,0)]$$

Thus the general form of the matrix $[P^*_{m,i}]$ turns out to be

$$[P^*_{m,i}] = \mathrm{SH}\,[\tilde{\theta}_{i+1}]\cdot[P_{i,l}] - \sum_{\lambda=i}^{m-1}\Big(\prod_{j=i}^{\lambda}[\gamma_j]\Big)\,\mathrm{SH}\,([\tilde{\theta}_{\lambda+2}] + [k_{\lambda+1}]\, l_{\lambda+1})\,[P_{\lambda+1,0}]$$
$$+\sum_{\lambda=i}^{m-1}\Big(\prod_{j=i}^{\lambda}[\gamma_j]\Big)\,(\mathrm{SH}\,[\tilde{\theta}_{\lambda+2}]\cdot[P_{\lambda+1,l}] - [T_{\lambda+1}]) \qquad (13)$$

where

$$(\mathrm{SH}\,[\tilde{\theta}])^{-1}\,\mathrm{CH}\,[\tilde{\theta}] = \mathrm{CH}\,[\theta]\,(\mathrm{SH}\,[\theta])^{-1}$$

$$\tanh\,[\delta'_{i+1}] = [z_{i+1}]\;[k^*_{i+1}]^{-1}\;[k_{i+1}]$$

$$\tanh\,[\delta_{i+1}] = [z_{i+1}]\;[k^*_i]^{-1}\;[k_i]$$

and $\sum_{\lambda=r}^{s} = 0$ when $s < r$. Exactly the following relationships hold for the coefficients $[\delta]$, $[\delta']$, $[\theta]$ and $[\theta']$.

$$\tanh\,[\delta_i]\,(\mathrm{TH}\,[\theta_i])^{-1} = [U] + \tanh\,[\delta'_i]\left\{\mathrm{TH}\,([k_i]\, l_i + [\theta_{i+1}])\right\}^{-1}$$

$$\tanh\,[\delta'_i]\,(\mathrm{TH}\,[\bar{\theta}'_i])^{-1} = [U] + \tanh\,[\delta_i]\left\{\mathrm{TH}\,([k_{i-1}]\, l_{i-1} + [\bar{\theta}'_{i-1}])\right\}^{-1}$$

The equation of the line current may be obtained by the same procedure as the case of the line potential, that is

$$[i_i(x_i)] = [k^*_i]^{-1}\;[k_i]\,(\mathrm{CH}\left\{[k_i]\,(l_i - x_i) + [\theta_{i+1}]\right\}$$
$$\times\left\{\mathrm{SH}\,([k_i]\, l_i + [\theta_{i+1}])\right\}^{-1}([e_i(0)] - [Q_i(0,x_i)])$$
$$+ [k_i]^{-1}\, p\,[L_i]\;[I_i(x_i,0)] - \cosh\,[k_i]\, x_i\left\{\mathrm{SH}\,([\tilde{\theta}_{i+1}] + [k_i]\, l_i)\right\}^{-1}$$
$$\times\,([P^*_{m,i}] - [\tilde{Q}_i\,(x_i, l_i)]\,)) \qquad (14)$$

Equations (12) and (14) give all the line potentials and currents for the case of $(m = 1 : i = 1)$, $(m = 2 : i = 1, i = 2)$ and $(m = 3 : i = 1, i = 2, i = 3)$.

We now return to the consideration of the general case. We lead here the line potential in the case of $(m = m, i = i + 1)$ assuming that Eqs. (12) and (14) hold in the case of $(m = m, i = i)$.

The general solutions on the $(i + 1)$-th stage for the line equations are:

$$[e_{i+1}(x_{i+1})] = \sinh [k_{i+1}]\, x_{i+1} \cdot [\alpha_{i+1}] + \cosh [k_{i+1}]\, x_{i+1} \cdot [\beta_{i+1}]$$
$$+ [k_{i+1}]^{-1} \int_0^{x_{i+1}} \sinh [k_{i+1}] (x_{i+1} - \xi_{i+1}) [Q_{i+1}(\xi_{i+1})]\, d\xi_{i+1} \tag{15}$$

$$[i_{i+1}(x_{i+1})] = -[k^*_{i+1}]^{-1} [k_{i+1}] (\cosh [k_{i+1}]\, x_{i+1} \cdot [\alpha_{i+1}]$$
$$+ \sinh [k_{i+1}]\, x_{i+1} \cdot [\beta_{i+1}]) + [k^*_{i+1}]^{-1} \Big\{ p[L_{i+1}] [I_{i+1}(x_{i+1}, 0)]$$
$$- \int_0^{x_{i+1}} \cosh [k_{i+1}] (x_{i+1} - \xi_{i+1}) [Q_{i+1}(\xi_{i+1})]\, d\xi_{i+1} \Big\} \tag{16}$$

From Fig. 2 the boundary conditions may be expressed as

$$[e_i(l_i)] = [e_{i+1}(0)] = [z_{i+1}] ([i_i(l_i)] - [i_{i+1}(0)]) \tag{17}$$

From Eqs. (12), (15) and (17)

$$[e_{i+1}(0)] = [\beta_{i+1}]$$
$$= \mathrm{SH}\, [\theta_{i+1}] \Big\{ \mathrm{SH}([k_i]\, l_i + [\theta_{i+1}]) \Big\}^{-1} ([e_i(0)] - [Q_i(0, l_i)])$$
$$+ \sinh [k_i]\, l_i \Big\{ \mathrm{SH}([\tilde{\theta}_{i+1}] + [k_i]\, l_i) \Big\}^{-1} [P^*_{m,l}] \tag{18}$$

From Eqs. (14), (16) and (17)

$$[e_{i+1}(0)] = [z_{i+1}] ([k^*_i]^{-1} [k_i] (\mathrm{CH}\, [\theta_{i+1}] \Big\{ \mathrm{SH}([k_i]\, l_i + [\theta_{i+1}]) \Big\}^{-1}$$
$$\times ([e_i(0)] - [Q_i(0, l_i)]) + [k_i]^{-1} p\, [L_i]\, [I_i(l_i, 0)]$$
$$- \cosh [k_i]\, l_i \Big\{ \mathrm{SH}([\tilde{\theta}_{i+1}] + [k_i]\, l_i) \Big\}^{-1} [P^*_{m,l}])$$
$$+ [k^*_{i+1}]^{-1} [k_{i+1}]\, [\alpha_{i+1}] - [k^*_{i+1}]^{-1} p\, [L_{i+1}]\, [I_{i+1}(0, 0)])$$
$$= \tanh [\delta_{i+1}] (\mathrm{CH}\, [\theta_{i+1}] \Big\{ \mathrm{SH}([k_i]\, l_i + [\theta_{i+1}]) \Big\}^{-1}$$
$$\times ([e_i(0)] - [Q_i(0, l_i)]) + [P_{i,l}]$$
$$- \cosh [k_i]\, l_i \Big\{ \mathrm{SH}([\tilde{\theta}_{i+1}] + [k_i]\, l_i) \Big\}^{-1} [P^*_{m,l}])$$
$$+ \tanh [\delta'_{i+1}] ([\alpha_{i+1}] - [P_{i+1,0}]) \tag{19}$$

Eliminating the term $\Big\{ \mathrm{SH}([k_i]\, l_i + [\theta_{i+1}]) \Big\}^{-1} ([e_i(0)] - [Q_i(0, l_i)])$ from Eqs. (18) and (19) and rewriting them, we get

$$\Big\{ [U] - \tanh [\delta_{i+1}] \cdot (\mathrm{TH}\, [\theta_{i+1}])^{-1} \Big\} [e_{i+1}(0)] + \tanh [\delta_{i+1}]$$
$$\times (\Big\{ (\mathrm{TH}\, [\theta_{i+1}])^{-1} \sinh [k_i]\, l_i + \cosh [k_i]\, l_i \Big\} \Big\{ \mathrm{SH}([\tilde{\theta}_{i+1}] + [k_i]\, l_i) \Big\}^{-1} [P^*_{m,l}]$$
$$- [P_{i,l}]) = \tanh [\delta'_{i+1}] ([\alpha_{i+1}] - [P_{i+1,0}]) \tag{20}$$

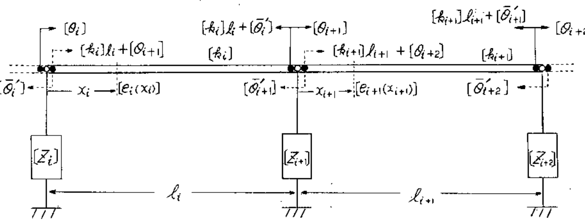

Fig. 2 An equivalent circuit of multiple-stage cascade connected polyphase transmission system.

Consider the circuit of Fig. 2, which is a general case of Fig. 1, where the impedance matrix $[z_{i+1}^*]$ looking from the end terminal of line i toward line $(i+1)$, and the impedance matrix $[z_{i+1}^{*\prime}]$ looking from the start terminal of line $(i+1)$ toward the end terminal are related as

$$[z_{i+1}^*]^{-1} = [z_{i+1}]^{-1} + [z_{i+1}^{*\prime}]^{-1}$$

Hence

$$[k_i]^{-1}[k_i^*][z_{i+1}^*]^{-1} = [k_i]^{-1}[k_i^*][z_{i+1}]^{-1} + [k_i]^{-1}[k_i^*][z_{i+1}]^{-1} \times [z_{i+1}][k_{i+1}^*]^{-1}[k_{i+1}][k_{i+1}]^{-1}[k_{i+1}^*][z_{i+1}^{*\prime}]^{-1}$$

and

$$(\mathrm{TH}\,[\theta_{i+1}])^{-1} = (\tanh\,[\delta_{i+1}])^{-1} + (\tanh\,[\delta_{i+1}])^{-1}\tanh[\delta_{i+1}'] \times \left\{\mathrm{TH}\,([k_{i+1}]\,l_{i+1} + [\theta_{i+2}])\right\}^{-1}$$

Therefore

$$\tanh\,[\delta_{i+1}]\,(\mathrm{TH}\,[\theta_{i+1}])^{-1} = [U] + \tanh\,[\delta_{i+1}'] \times \left\{\mathrm{TH}\,([k_{i+1}]\,l_{i+1} + [\theta_{i+2}])\right\}^{-1} \quad (21)$$

From Eqs. (20) and (21) we may obtain $[\alpha_{i+1}]$ as follows:

$$[\alpha_{i+1}] = -\left\{\mathrm{TH}\,([k_{i+1}]\,l_{i+1} + [\theta_{i+2}])\right\}^{-1}[e_{i+1}(0)] + (\tanh\,[\delta_{i+1}'])^{-1}\tanh\,[\delta_{i+1}]\left\{(\mathrm{SH}\,[\tilde{\theta}_{i+1}])^{-1}[P_{m,i}^*] - [P_{i,l}]\right\} + [P_{i+1,0}]$$

The impedance matrix $[w_{i+1}']$ looking from the end terminal of line i toward the start terminal, and the impedance matrix $[w_{i+1}]$ looking from the start terminal of line $(i+1)$ toward line i are related as

$$[w_{i+1}]^{-1} = [z_{i+1}]^{-1} + [w_{i+1}']^{-1}$$

thus

$$\tanh[\delta'_{i+1}]\cdot(\mathrm{TH}[\bar{\theta}'_{i+1}])^{-1}=[U]+\tanh[\delta_{i+1}]$$
$$\times\left\{\mathrm{TH}([k_i]\,l_i+[\bar{\theta}'_i])\right\}^{-1} \qquad (22)$$

From Eqs. (21) and (22)

$$\begin{aligned}(\tanh[\delta'_{i+1}])^{-1}\tanh[\delta_{i+1}]&=\left\{\mathrm{SH}([\tilde{\theta}_{i+2}]+[k_{i+1}]\,l_{i+1})\right\}^{-1}\\&\quad\times\mathrm{SH}([\tilde{\theta}_{i+2}]+[k_{i+1}]\,l_{i+1}+[\bar{\theta}'_{i+1}])\\&\quad\times(\mathrm{SH}[\bar{\theta}'_{i+1}])^{-1}\,\mathrm{SH}([k_i]\,l_i+[\bar{\theta}'_i])\\&\quad\times\left\{\mathrm{SH}([\tilde{\theta}_{i+1}]+[k_i]\,l_i+[\bar{\theta}'_i])\right\}^{-1}\mathrm{SH}[\tilde{\theta}_{i+1}]\\&=\left\{\mathrm{SH}([\tilde{\theta}_{i+2}]+[k_{i+1}]\,l_{i+1})\right\}^{-1}[\gamma_i]^{-1}\,\mathrm{SH}[\tilde{\theta}_{i+1}]\end{aligned}$$

then

$$\begin{aligned}[\alpha_{i+1}]&=-\left\{\mathrm{TH}([k_{i+1}]\,l_{i+1}+[\theta_{i+2}])\right\}^{-1}[e_{i+1}(0)]\\&\quad+\left\{\mathrm{SH}([\tilde{\theta}_{i+2}]+[k_{i+1}]\,l_{i+1})\right\}^{-1}[\gamma_i]^{-1}([P^*_{m,i}]\\&\quad-\mathrm{SH}[\tilde{\theta}_{i+1}]\cdot[P_{i,l}])+[P_{i+1,0}]\end{aligned} \qquad (23)$$

Substitution of $[\beta_{i+1}]$ in Eq. (18) and Eq. (23) into Eq. (15) results

$$\begin{aligned}[e_{i+1}(x_{i+1})]&=\mathrm{SH}\left\{[k_{i+1}](l_{i+1}-x_{i+1})+[\theta_{i+2}]\right\}\\&\quad\times\left\{\mathrm{SH}([k_{i+1}]\,l_{i+1}+[\theta_{i+2}])\right\}^{-1}([e_{i+1}(0)]\\&\quad-[Q_{i+1}(0,x_{i+1})])+\sinh[k_{i+1}]\,x_{i+1}\\&\quad\times\left\{\mathrm{SH}([\tilde{\theta}_{i+2}]+[k_{i+1}]\,l_{i+1})\right\}^{-1}([P^*_{m,i+1}]-[\tilde{Q}_{i+1}(x_{i+1},l_{i+1})])\end{aligned}$$

This equation is equal rightly to what the stage numbers have been exchanged from i to $i+1$ in Eq. (12). Furthermore, substituting $[\beta_{i+1}]=[e_{i+1}(0)]$ and Eq. (23) in Eq. (16) and rewriting it, we can get the equation that the stage numbers have been exchanged from i to $i+1$ in Eq. (14).

4. Conclusions

Owing to the foregoing analysis it must be emphasized that the position angle matrix is sufficiently useful for the multiple-stage combined polyphase transmission system although the system has the arbitrary initial values of voltages and currents on lines. In other words, the equation of the potential or the current at any point on the transmission line may be expressed simply by using the position angle matrix in the case of the boundary-value problem with initial values for the combined polyphase transmission system. Furthermore, we can easily find the line voltage or current from the system circuit diagram by means of the position angle matrix without solving the tedious initial and boundary value problem of simultaneous partial differential equations. In this paper we have not shown any numerical examples because all numerical calculations may be carried out systematically and straightforwardly by a digital computer.

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