Analysis of Off-Design Point Performance of Gas Turbine

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The literature on the gas turbine already contains much information concerning the performance of various engine schemes both in simpler and in more complicated forms. Such information enables the design performance to be estimated, but there appears to have been very little works on the optimum performance under off-design conditions. In this paper, we attempt to include in a comparison of off-design point performance characteristics of stationary gas turbine and to optimize them. The multiplier method is used as an optimization procedure. As a result of this work, it is possible to indicate the optimization technique by which the comparable off-design point performance of different gas turbine schemes may be assessed.

1. Introduction

So far the optimum performance of gas turbine at design conditions has been calculated by the so-called cycle calculation procedure with trial and error method. Therefore, the increase of the number of components requires more elaborated calculations to determine an optimum working cycle for the complicated engine schemes. In this relation, authors have regarded the gas turbine as a system which consists of a number of subsystems (components), and have determined the design point working conditions of stationary gas turbines and aircraft engines by optimization procedure with multiplier method[1]. While the off-design point (part load) performance has also been obtained by trial and error method for many types of loads under specified operating conditions such as constant maximum temperature, constant engine speed and constant air-fuel ratio. In this study, therefore, such parameters as engine speed, mass flow rate, air-fuel ratio which give the maximum thermal efficiency or dimensionless specific work are specified, then the optimum operating line in part load can be obtained. If the parameters matched for the method of reducing output indivudually and they can be controlled actively, they become the desired values for feedback control. The gas turbine can also be operated with maximum thermal efficiency. The calculation is carried out for the following schemes as 1/C, 1/C/E, 1/LP and 1/LP/E.

2. Performance Characteristics of Components

The conventional approximation with simple equations to express the off-design point characteristics of the subsystems (components) is adopted. These characteristics

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are represented by some kind of parameters. In this study, the corrected values have been chosen, that is, the corrected mass flow rate and corrected rotational speed, which are given by

$$\bar{G} = \frac{G\sqrt{\theta}/\gamma}{(G\sqrt{\theta}/\gamma)_d} \tag{1}$$

$$\bar{N} = \frac{N/\sqrt{\theta}}{(N/\sqrt{\theta})_d} \tag{2}$$

where θ and γ show the temperature ratio and pressure ratio nondimensionalized by the atmospheric temperature T_0 and pressure P_0, and suffix d refers to the values at design point.

2.1 Compressor

By the dimensional analysis, pressure ratio γ_C and adiabatic efficiency of the compressor η_C is expressed by the following relations

$$\gamma_C = f_{C_1}(\bar{G}_C, \bar{N}_C), \quad \eta_C = f_{C_2}(\bar{G}_C, \bar{N}_C)$$

For convenience, the performance map is divided into many grids, then the characteristic values on the grids are approximated by some polynomial equations[2] as

$$\gamma_C = (\gamma_C)_d \left\{ A_1 (\bar{G}_C - A_2)^2 + A_3 \right\} \tag{3}$$

$$A_i = \sum_{j=1}^{5} a_{ij} \bar{N}_C^{(j-1)}$$

$$\eta_C = (\eta_C)_d \left\{ B_1 (\bar{G}_C - B_2)^2 + B_3 \right\} \tag{4}$$

$$B_i = \sum_{j=1}^{3} b_{ij} \bar{N}_C^{(j-1)}$$

where a_{ij}, b_{ij} (i = 1, 2, 3) show the coefficients. For the surge line, the following equation is given

$$(\gamma_C)_s = C_1 (\bar{G}_C) + C_2 \tag{5}$$

where suffix s refers to the surging. The model adopted for the analysis is a high pressure compressor of twin-spool turbofan engine. The characteristics is obtained by the method of least square and computational results are depicted by the solid lines in Fig. 1.

2.2 Combustor

The combustion efficiency varies as the air-fuel ratio. As pointed out by Matsumoto[3], however, for tubular combustor it changes very slightly within the range of 60 to 100 of the air-fuel ratio. In addition, these effects are too small, so that it is regarded as constant.

Since pressure loss is caused by the aerodynamic drag, it is assumed that pressure

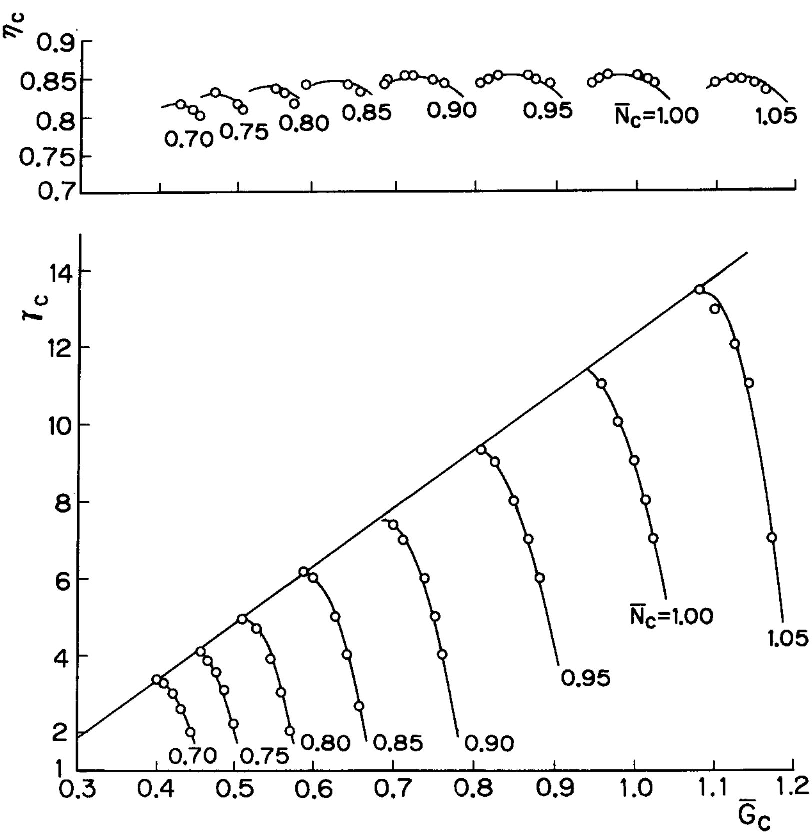

Fig. 1 Characteristics of compressor.

loss coefficient ϵ_B is proportional to the dynamic pressure at the combustor inlet, so that

$$\epsilon_B = (\epsilon_B)_d \, \bar{G}_B^2 \tag{6}$$

The same equation can be derived by taking account of the momentum change due to the friction and heat release in the combustor.

2.3 Turbine

In the turbine, as shown in Fig. 2 the conventional ellipse law has been established between the corrected mass flow $\bar{G}_T$ and the pressure ratio γ_T as

$$\gamma_T = 1/\sqrt{1 - \{1-(\gamma_T)_d^{-2}\} \, \bar{G}_T^2} \tag{7}$$

The adiabatic efficiency of the turbine η_T is generally obtained by a function of u/c_0, where u shows the peripheral velocity and c_0 is an equivalent velocity of the enthalpy drop, $\sqrt{2\Delta h_T}$. It is verified that these relations could be depicted by a parabola in Fig. 3[4]. Since the peripheral velocity u is proportional to the rotational speed, the following equation is given

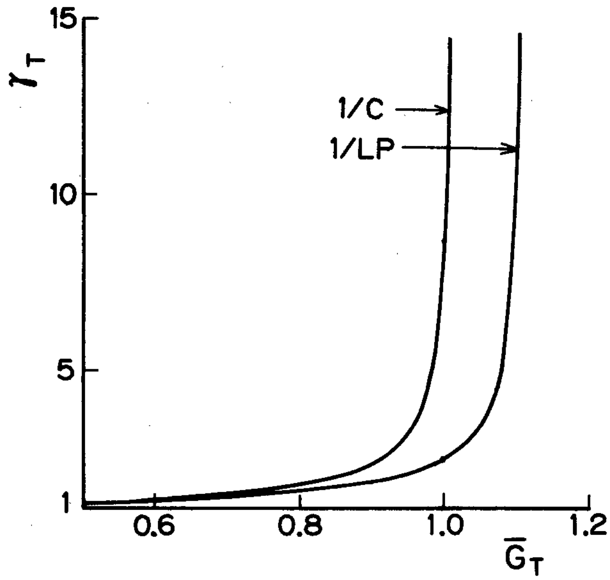

Fig. 2 Turbine characteristics (pressure ratio).

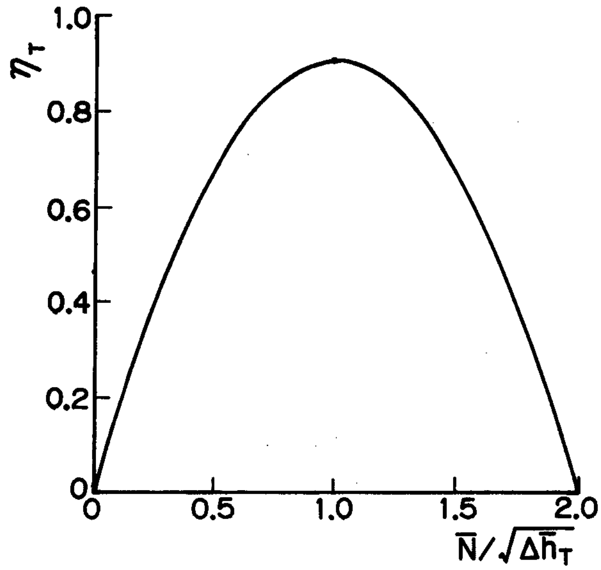

Fig. 3 Turbine characteristics (adiabatic efficiency).

$$\eta_T = (\eta_T)_d \left\{1-(N_T/\sqrt{\Delta\overline{h}_T}-1)^2\right\} \tag{8}$$

where $\Delta\overline{h}_T$ is the relative enthalpy drop and is shown by the following equation with temperature ratio θ_T

$$\Delta\overline{h}_T = (1-\theta_T)/\left\{1-(\theta_T)_d\right\} \tag{9}$$

On the other hand, by the definition of the adiabatic efficiency, there must be the relation between η_T and γ_T, so that

$$\eta_T = (1-\theta_T)/(1-\gamma_T^{-m}) \tag{10}$$

where $m = (\kappa - 1)/\kappa$.

3. Application of Optimization Procedure

The arrangement and block diagram of gas turbine without regenerator is depicted in Fig. 4. In scheme 1/LP, the relation between work output of power turbine and rotational speed does not affect the operation of the total system, so that the power turbine can be included in power unit. In the end, since only gas generator should be considered, the same model can be applied to both schemes 1/C and 1/LP.

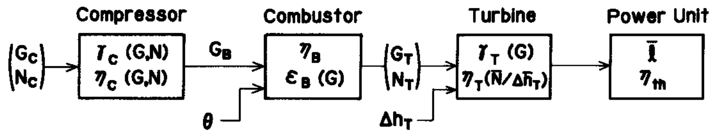

Fig. 4 Arrangement and block diagram of schemes 1/C and 1/LP.

There can be many variables considered in solving the optimization problem. In this study, the corrected inlet mass flow of compressor, corrected rotational speed of compressor, temperature ratio and relative adiabatic enthalpy drop are chosen as variables, then be written vectorially as

$$\mathbf{X} = (\bar{G}_C, \bar{N}_C, \theta, \Delta\bar{h}_T) \tag{11}$$

Consequently, the relative values for other components are expressed by

$$\bar{G}_B = \bar{G}_C \frac{\sqrt{1+(\gamma_C^m-1)/\eta_C}\,/\gamma_C}{(\sqrt{\theta_C}\,/\gamma_C)_d} \tag{12}$$

$$\bar{G}_T = \bar{G}_C \frac{\sqrt{\theta}\,/\ \{\gamma_C(1-\epsilon_B)\}}{(\sqrt{\theta}\,/\ \{\gamma_C(1-\epsilon_B)\}\,)_d} \tag{13}$$

$$\bar{N}_T = \bar{N}_C\,(\sqrt{\theta})_d/\sqrt{\theta} \tag{14}$$

In scheme 1/C, two objective functions, that is, thermal efficiency η_{th} and dimensionless specific output $\bar{l}$ are given by

$$f_1(\mathbf{X}) = \eta_{th} = \eta_B\,\bar{l}\,/\,[\theta - \left\{1+(\gamma_C^m-1)/\eta_C\right\}] \tag{15}$$

$$f_2(\mathbf{X}) = \bar{l} = \theta(1-\gamma_T^{-m})\eta_T - (\gamma_C^m-1)/\eta_C \tag{16}$$

The equality constraint can be obtained due to load factor, ϕ, Eq. (10) and pressure balance between inlet and outlet of gas turbine, and these are expressed by

$$h_1(\mathbf{X}) = \bar{G}_C\,\bar{l}\,/(\bar{l})_d - \phi = 0 \tag{17.1}$$

$$h_2(\mathbf{X}) = \eta_T - (1-\theta_T)/(1-\gamma_T^{-m}) = 0 \tag{17.2}$$

$$h_3(X) = \gamma_C(1-\epsilon_B)/\gamma_T - 1 = 0 \tag{17.3}$$

The rotational speed, maximum temperature ratio and pressure ratios of compressor and turbine are constrained by the following inequalities

$$g_1(X) = \bar{N}_C - 1 \leqslant 0 \tag{18.1}$$

$$g_2(X) = \theta - (\theta)_d \leqslant 0 \tag{18.2}$$

$$g_3(X) = \gamma_C - (\gamma_C)_s \leqslant 0 \tag{18.3}$$

$$g_4(X) = \left\{1 - (\gamma_T)_d^{-2}\right\} \bar{G}_T - 1 \leqslant 0 \tag{18.4}$$

$$g_5(X) = A_2 - \bar{G}_C \leqslant 0 \tag{18.5}$$

In above equations, Eq. (18.5) is introduced for mathematical treating, and it means that the compressor is operable in right half area indicated by Eq. (3). Though these equations are implicit functions of X for simplicity, the generality of the problem can not be lost.

In scheme 1/LP, work output is generated by expansion of gases from outlet of compressor-turbine to atmosphere. The equilibrium of work between compressor and turbine is introduced in place of Eq. (17.3). The following two equations are modified as

$$\bar{l} = \theta \left\{1 - (1 - \gamma_{HT}^{-m})\eta_{HT}\right\} (1 - \gamma_{LT}^{-m})\eta_{LT} \tag{19}$$

$$h_3(X) = \theta(1 - \gamma_C^{-m})\eta_C - (\gamma_{HT}^{m} - 1)/\eta_{HT} = 0 \tag{20}$$

where $\gamma_{LT} = \gamma_C(1-\epsilon_B)/\gamma_{HT}$, and suffix LT and HT show low pressure and high pressure turbines, respectively.

In the case with regenerator such as 1/C/E and 1/LP/E, the same optimization procedure is applicable except for some aspects. By splitting the regenerator into hot and cold side, the optimization model is converted to series system from that with recycling. In both sides of regenerator, there has been constraints by the effectiveness.

The number of independent variable is given by subtracting the number of equality constraints from those of parameters. The present case is reduced to a linear search problem. It results in more complicated equations. While, as the number of parameters and equality constraints is augmented, the more search time is required. If another constraints added in this study, the variables can be determined regardless of the objective function. After all, if such operating conditions as constant engine speed for power generation or propeller law for marine application are specified, the highest off-design point performance suitable for individual load can be obtained.

4. Optimum Off-Design Point Performance

The design point data is shown in Table 1. The model compressor is the high pressure compressor of twin-spool turbofan engine. The expansion ratios of turbine at design point, therefore, are different according to load type. The adiabatic efficiency of

Table 1 Design point data.

Pressure Ratio of Compressor	$\gamma_C = 8.91$
Adiabatic Efficiency of Compressor	$\eta_C = 0.846$
Adiabatic Efficiency of Turbine	$\eta_T = \eta_{HT} = 0.90$, $\eta_{LT} = 0.88$
Pressure Loss Coefficient	$\epsilon_B = \epsilon_{EX} = 0.03$
Atmospheric Temperature	$T_0 = 288.15$ K
Atmospheric Pressure	$p_0 = 0.1013$ MPa
Specific Heat	$Cp = 1.114$ kJ/kg K

power turbine and the effectiveness of regenerator cold side are assumed to be constant.

The thermal efficiency-load factor relations are shown in Fig. 5 for schemes 1/C and 1/C/E, and Fig. 6 for 1/LP and 1/LP/E, respectively. The maximum temperature corresponding to these conditions are also shown in Figs. 7 and 8. Within higher load range, a trend of the decrease of thermal efficiency for each method of reducing output have no difference as pointed out by Hamajima[5]. While, within lower load range, the differences are enlarged, especially for 1/C. In the case with regenerator, the high efficiency is attained for the constant temperature operation. At lower load range, however, surging occurs at about 40 percent load in the example chosen. If one's criterion is the maintenance of high thermal efficiency over the wider possible load range, the mixed operation with constant maximum temperature for higher load range and that along surging line for smaller load factor should be selected. To avoid surging, we should set the surging margin as indicated by inequality constraint. As reported earlier[6], constant engine speed operation with regenerator should be excluded.

Compressor operating lines are shown in Fig. 9 (1/C and 1/C/E) and Fig. 10 (1/LP

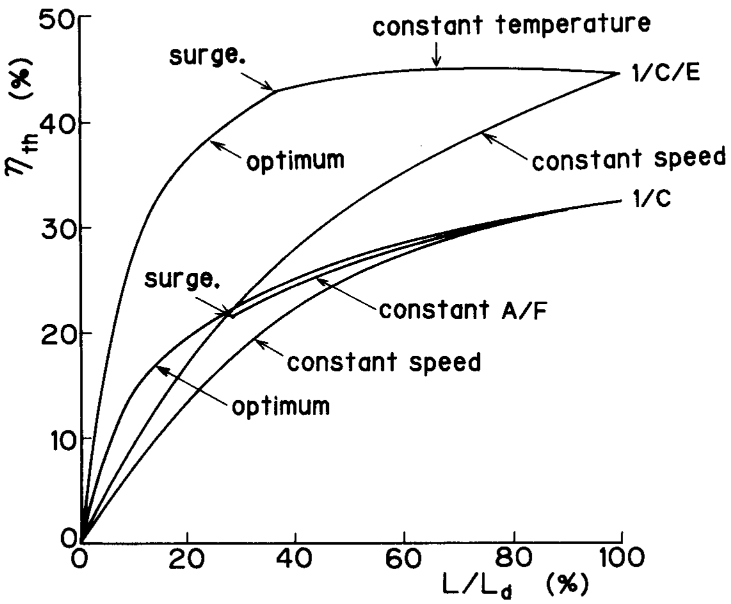

Fig. 5 Off-design point performance of schemes 1/C and 1/C/E.

and 1/LP/E.) For turbo-shaft type engines, as the operating method (load type) has direct influence on the efficiency, it is too difficult to maintain high efficiency over a wide load range. In the scheme with free turbine, choking at power turbine restrains the load characteristics. To maintain the optimum operation, therefore, the flow velocity

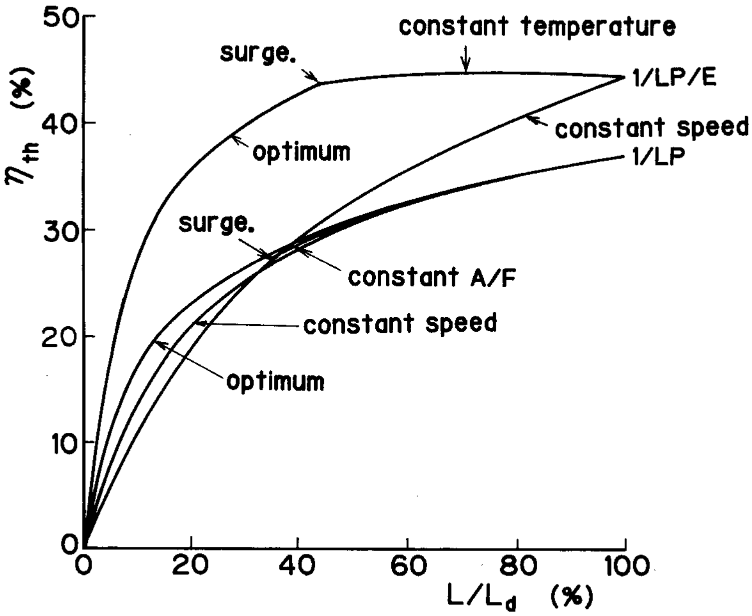

Fig. 6 Off-design point performance of schemes 1/LP and 1/LP/E.

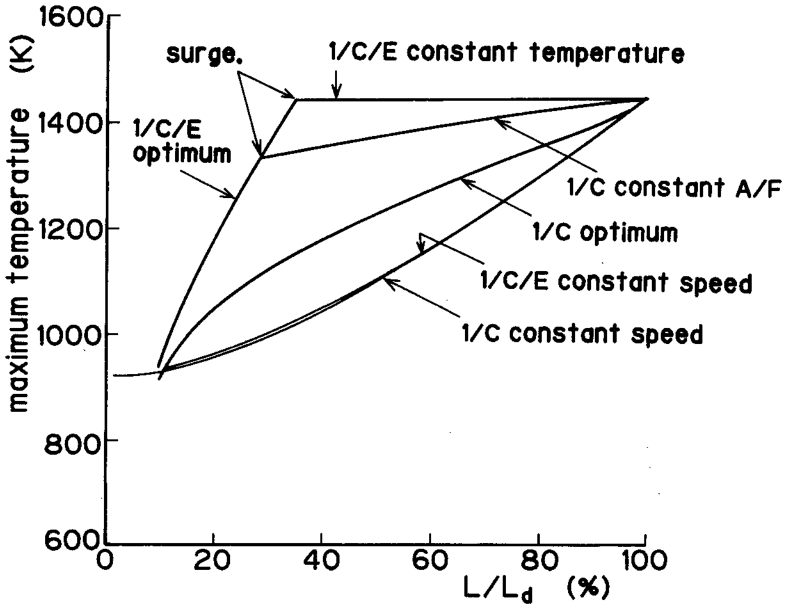

Fig. 7 Variation of turbine inlet temperature with output of schemes 1/C and 1/C/E.

should be regulated between compressor-turbine and power turbine. If the variable nozzle or variable area duct are available, these parameters can be used as the desired values for feedback control.

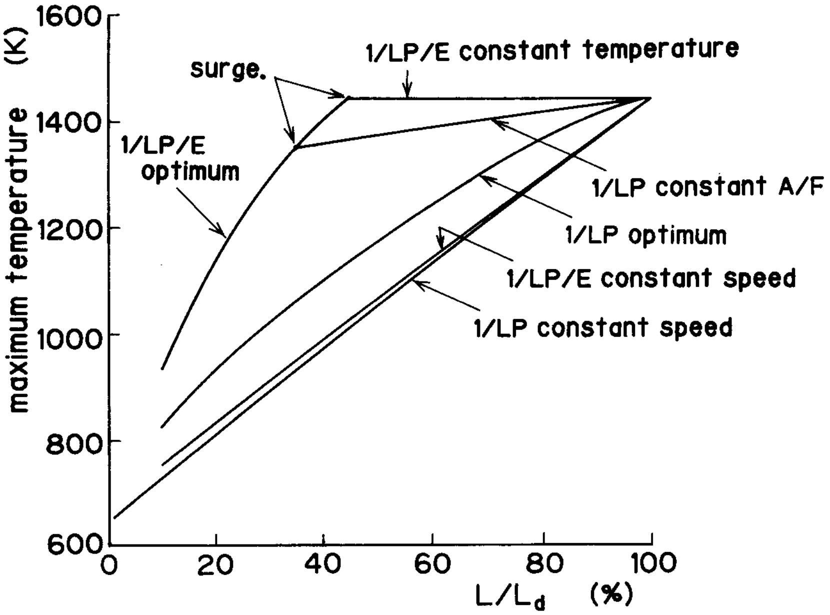

Fig. 8 Variation of turbine inlet temperature with output of schemes 1/LP and 1/LP/E.

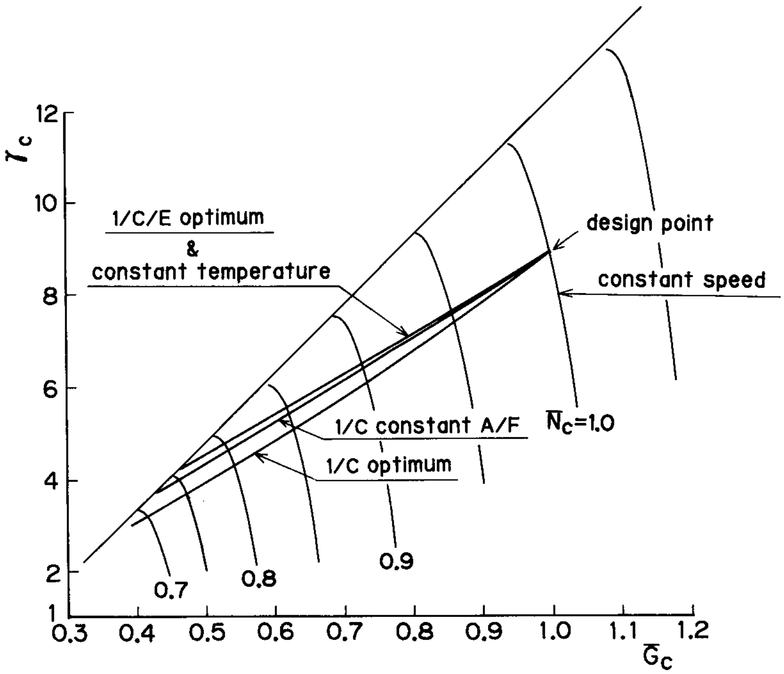

Fig. 9 Compressor operating lines of schemes 1/C and 1/C/E.

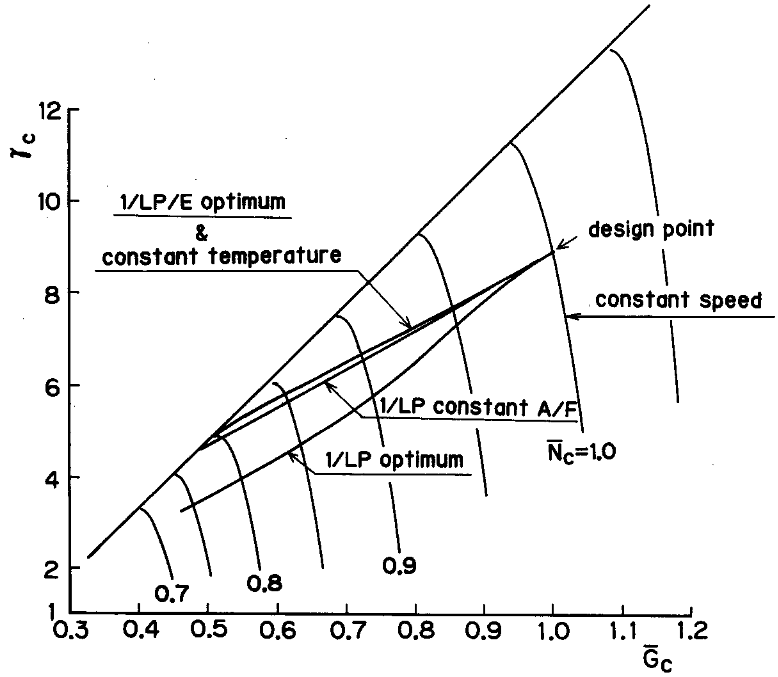

Fig. 10 Compressor operating lines of schemes 1/LP and 1/LP/E.

5. Conclusions

The off-design point performance has been presented by the optimization procedure with multiplier method. The following conclusions have been drawn.

(1) In schemes 1/C and 1/LP, the optimum off-design point performance has shown a same trend as any other operations such as constant rotational speed, constant air-fuel ratio except for the small load range.

(2) In schemes with regenerator, 1/C/E and 1/LP/E, the constant maximum temperature operation brings about an excellent efficiency at higher load range. While at small load factor, the operation along the surge line with appropriate margin is preferable. Since constant engine speed operation where no surge occurs, has the insufficient performance, it should be excluded.

In this study, since simple approximation with equations is used to present characteristics of components, the validity of optimized characteristics of gas turbine system should be confirmed by comparing the experimental results.

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