Application of Position Angle Matrix to Calculation of Polyphase Line Currents

Application of Position Angle Matrix to Calculation of Polyphase Line Currents

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In this paper the line current along the combined dissymmetrical multi-conductor transmission system is calculated by making use of the position angle matrix. Detailed numerical examples are presented on calculation of the successive reflections of waves.

1. Introduction

In the previous paper[1)] we dealt with the boundary value problem in the cascade connected symmetrical multi-conductor transmission system. Then we gave analytically the line potential of any point in the compact form by using the position angle matrix and showed some numerical results for a practical example concerning such a problem. Problems of this type are of special engineering interest in connection with the design of ground wires and other protective schemes, and, in general, in the study of traveling waves.

In this paper our method is applied to the calculation of the line current along the combined dissymmetrical multi-conductor transmission system. The assumption is more generalized than that of the previous paper[1)], that is, different lines are connected in series and the arrangement of conductors at any stage is dissymmetrical. But there is the electric source at the sending point only and no initial value of voltage or current on the transmission system in this paper.

By introducing position angle matrices it has been able to describe most simply the line current even in such a more generalized system. Moreover it has been shown that this acquisition may be construed well reasonably. This does not lead only to carry out the computations with ease, but also this leads to get equations of currents of such a system from the diagram directly using the position angles. Thus the solution can be got without solving the boundary value problem of partial differential equations. As a result of this theory, a useful methodology has been established so that the combined dissymmetrical multi-conductor transmission system may be analyzed very simply and easily.

The numerical calculations in the case of the three stage cascade connected symmetrical two-conductor transmission system as a simple and practical example for the above method are given in this paper. They may carry out systematically programs for calculating by using the digital computer and repeating the same calculation very readily.

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2. General Solutions

By way of illustration of the general equations, consider the circuit of Fig. 1, which is the i-stage combined dissymmetrical n-conductor transmission system. It shows a pair of traveling voltage and current waves at an arbitrary point x_s on the s-th stage line, matrices $[e_s(x_s)]$ and $[i_s(x_s)]$ in operational form. Then we have the following well known equations[2].

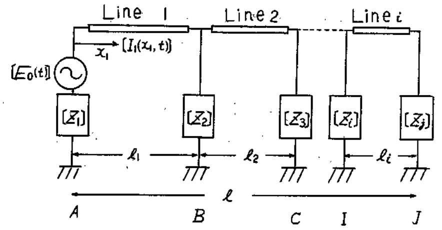

Fig. 1 Simplified equivalent circuit of many stage cascade connected multi-conductor transmission system.

$$-\frac{d}{dx_s}[e_s(x_s)] = [Z_s(p)]\,[i_s(x_s)] \tag{1}$$

$$-\frac{d}{dx_s}[i_s(x_s)] = [Y_s(p)]\,[e_s(x_s)] \tag{2}$$

where,

$$[Z_s(p)] = [L_s]p + [R_s] = [pL_s + R_s] \tag{3}$$

$$[Y_s(p)] = [C_s]p + [G_s] = [pC_s + G_s] \tag{4}$$

and $[R_s]$, $[L_s]$, $[C_s]$ and $[G_s]$ are $n \times n$ matrices representing the resistance, inductance, capacitance and leakance per unit length of transmission line. From Eqs. (1) and (2), we obtain

$$\frac{d^2}{dx_s^2}[i_s(x_s)] = [g_s]^2\,[i_s(x_s)] \tag{5}$$

$$[e_s(x_s)] = -[g_s^*]\frac{d}{dx_s}[i_s(x_s)] \tag{6}$$

where,

$$[g_s]^2 = [Y_s(p)]\,[Z_s(p)] = [pC_s + G_s]\,[pL_s + R_s] \tag{7}$$

$$[g_s^*] = [Y_s(p)]^{-1} = [pC_s + G_s]^{-1} \tag{8}$$

The general solution for Eq. (5) is given[3] by

$$[i_s(x_s)] = \sinh\,[g_s]\,x_s \cdot [\alpha_s] + \cosh\,[g_s]\,x_s \cdot [\beta_s] \tag{9}$$

where integral constants $[\alpha_s]$ and $[\beta_s]$ are determined so as to satisfy the boundary conditions.

Substituting Eq. (9) in (6), and solving for the potential, there results

$$[e_s(x_s)] = -[g_s^*]\,[g_s]\,(\cosh\,[g_s]\,x_s\cdot[\alpha_s] + \sinh\,[g_s]\,x_s\cdot[\beta_s]) \tag{10}$$

3. Current Waves

Consider first the circuit Fig. 2, a n-conductor transmission system having a length l_1. A group of voltage sources $[E_0(t)]$ and a group of impedances $[Z_1]$ are in series at the sending terminal and a group of impedances $[Z_2]$ is at the receiving terminal.

Then the boundary conditions may be expressed as

$$[E_1(0,t)] = [E_0(t)] - [Z_1]\,[I_1(0,t)] \tag{11}$$

$$[E_1(l_1,t)] = [Z_2]\,[I_1(l_1,t)] \tag{12}$$

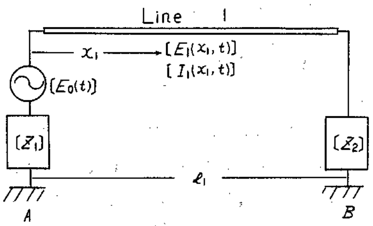

Fig. 2 Fundamental circuit of n-conductor transmission system.

It will be also convenient to introduce the notation

$$[e_1(x_1)] = \mathcal{L}\,[E_1(x_1,t)]$$

$$[i_1(x_1)] = \mathcal{L}\,[I_1(x_1,t)]$$

$$[e_0] = \mathcal{L}\,[E_0(t)]$$

Then Eqs. (11) and (12) become

$$[e_1(0)] = [e_0] - [\tilde{Z}_1]\,[i_1(0)] \tag{13}$$

$$[e_1(l_1)] = [\tilde{Z}_2]\,[i_1(l_1)] \tag{14}$$

in which the matrices $[\tilde{Z}_1]$ and $[\tilde{Z}_2]$ are the operational forms of $[Z_1]$ and $[Z_2]$. From Eqs. (9), (10) and (14), $[\alpha_1]$ and $[\beta_1]$ turn out to be

$$[\alpha_1] = -\mathrm{SH}([g_1]\,l_1 + [\delta_2^*])\,\left\{\mathrm{CH}([g_1]\,l_1 + [\delta_2^*])\right\}^{-1}\,[i_1(0)] \tag{15}$$

$$[\beta_1] = [i_1(0)] \tag{16}$$

where

$$\begin{aligned}\tanh[\delta^*] &= \sinh[\delta^*](\cosh[\delta^*])^{-1}\\ &= (\cosh[\delta^*])^{-1}\sinh[\delta^*]\\ &= [g]^{-1}[g^*]^{-1}[\tilde{Z}]\end{aligned} \tag{17}$$

Substituting Eqs. (15) and (16) in (9) there results

$$\begin{aligned}[i_1(x_1)] &= \mathrm{CH}\left\{[g_1](l_1 - x_1) + [\delta_2^*]\right\}\\ &\quad \times \left\{\mathrm{CH}([g_1]l_1 + [\delta_2^*])\right\}^{-1}[i_1(0)]\end{aligned} \tag{18}$$

It is evident that Eq. (18) is a type of natural multi-dimensional expansion when the current is represented using the position angle.
Equation (18) may be rewritten as

$$\begin{aligned}&[\mathrm{CH}\left\{[g_1](l_1 - x_1) + [\delta_2^*]\right\}]^{-1}[i_1(x_1)]\\ &= \left\{\mathrm{CH}([g_1]l_1 + [\delta_2^*])\right\}^{-1}[i_1(0)]\end{aligned} \tag{19}$$

The position angle matrices in the positive direction of x_1 from the point A and at the point A, when the point B is assumed to be the standard point, are written by $[g_1](l_1 - x_1) + [\delta_2^*]$ and $[g_1]l_1 + [\delta_2^*]$, respectively. Equation (19) shows that the matrix product at an arbitrary point of a line $[\mathrm{CH}\{[g_1](l_1 - x_1) + [\delta_2^*]\}]^{-1}[i_1(x_1)]$ has a constant value.

Now take the case, Fig. 3, of the two-stage transmission system having the same constants as in the previous case. Then the fundamental circuit equations are:

$$[e_1(x_1)] = -[g_1^*][g_1](\cosh[g_1]x_1 \cdot [\alpha_1] + \sinh[g_1]x_1 \cdot [\beta_1]) \tag{20}$$

$$[i_1(x_1)] = \sinh[g_1]x_1 \cdot [\alpha_1] + \cosh[g_1]x_1 \cdot [\beta_1] \tag{21}$$

$$[e_2(x_2)] = -[g_2^*][g_2](\cosh[g_2]x_2 \cdot [\alpha_2] + \sinh[g_2]x_2 \cdot [\beta_2]) \tag{22}$$

$$[i_2(x_2)] = \sinh[g_2]x_2 \cdot [\alpha_2] + \cosh[g_2]x_2 \cdot [\beta_2] \tag{23}$$

and the boundary conditions are

$$[e_1(0)] = [e_0] - [\tilde{Z}_1][i_1(0)] \tag{24}$$

$$[e_1(l_1)] = [e_2(0)] = [\tilde{Z}_2]([i_1(l_1)] - [i_2(0)]) \tag{25}$$

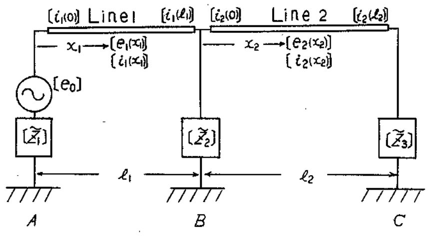

Fig. 3 An equivalent circuit of cascade connected polyphase transmission system.

$$[e_2(l_2)] = [\tilde{Z}_3]\,[i_2(l_2)] \tag{26}$$

There are only four integration constants to be determined from the terminal conditions. Referring to above equations, there is

$$[\alpha_1] = -\left([\Omega^*]\cosh[g_1]l_1 + \sinh[g_1]l_1\right)^{-1}\left([\Omega^*]\sinh[g_1]l_1 + \cosh[g_1]l_1\right)[i_1(0)] \tag{27}$$

$$[\beta_1] = [i_1(0)] \tag{28}$$

$$[\alpha_2] = -\mathbf{SH}\left([g_2]l_2 + [\delta_3^*]\right)\left\{\mathbf{CH}\left([g_2]l_2 + [\delta_3^*]\right)\right\}^{-1}[i_2(0)] \tag{29}$$

$$[\beta_2] = [i_2(0)] \tag{30}$$

$$[\Omega^*] = \left[[U] + \left\{\mathbf{TH}\left([g_2]l_2 + [\delta_3^*]\right)\right\}^{-1}\tanh[\delta_{2m}^*]\right] \times (\tanh[\delta_2^*])^{-1} \tag{31}$$

in which

$$\tanh[\delta_2^*] = [g_1]^{-1}[g_1^*]^{-1}[\tilde{Z}_2]$$

$$\tanh[\delta_{2m}^*] = [g_2]^{-1}[g_2^*]^{-1}[\tilde{Z}_2]$$

$$\tanh[\delta_3^*] = [g_2]^{-1}[g_2^*]^{-1}[\tilde{Z}_3]$$

The impedance matrix $[\tilde{z}_1]$ looking from the end terminal of line 1 toward line 2, and the impedance matrix $[\tilde{z}_2]$ looking from the start terminal of line 2 toward the end terminal are related as

$$[\tilde{z}_1]^{-1} = [\tilde{Z}_2]^{-1} + [\tilde{z}_2]^{-1}$$

Thus

$$[\tilde{z}_1]^{-1}\ [g_1^*]\cdot[g_1] = [\tilde{Z}_2]^{-1}\ [g_1^*]\ [g_1]$$
$$+\ [\tilde{z}_2]^{-1}\ [g_2^*]\ [g_2]\ [g_2]^{-1}\ [g_2^*]^{-1}\ [\tilde{Z}_2]\ [\tilde{Z}_2]^{-1}\ [g_1^*]\ [g_1] \tag{32}$$

Putting the position angle matrix looking from the end terminal of line 1 toward line 2 $[\phi_1^*]$, there is

$$\text{TH}\ [\phi_1^*] = [g_1]^{-1}\ [g_1^*]^{-1}\ [\tilde{z}_1]$$

and since $[g_2]\, l_2 + [\delta_3^*]$ is the position angle matrix looking from the start terminal of line 2 toward the end terminal, the next relation holds

$$\text{TH}\,([g_2]\, l_2 + [\delta_3^*]) = [g_2]^{-1}\ [g_2^*]^{-1}\ [\tilde{z}_2]$$

Therefore Eq. (32) becomes

$$\begin{aligned}(\text{TH}\ [\phi_1^*])^{-1} &= (\tanh\,[\delta_2^*])^{-1} + \left\{\text{TH}\,([g_2]\, l_2 + [\delta_3^*])\right\}^{-1} \\ &\quad \times \tanh\ [\delta_{2m}^*]\,(\tanh\ [\delta_2^*])^{-1} \\ &= [\Omega^*]\end{aligned}$$

and Eq. (27) becomes

$$[\alpha_1] = -\left\{\text{CH}\,([\tilde{\phi}_1^*] + [g_1]\, l_1)\right\}^{-1}\ \text{SH}\,([\tilde{\phi}_1^*] + [g_1]\, l_1)\ [i_1(0)] \tag{33}$$

where $[\tilde{\phi}_1]$ is the conjugate position angle matrix of $[\phi_1]$ which satisfies the next relation

$$\text{SH}\ [\phi]\ (\text{CH}\ [\phi])^{-1} = (\text{CH}\ [\tilde{\phi}])^{-1}\ \text{SH}\ [\tilde{\phi}]$$

Substituting Eqs. (28) ~ (30) and (37) in (21) and (23)

$$\begin{aligned}[i_1(x_1)] &= \text{CH}\left\{[g_1]\ (l_1 - x_1) + [\phi_1^*]\right\} \\ &\quad \times \left\{\text{CH}\,([g_1]\, l_1 + [\phi_1^*])\right\}^{-1}\ [i_1(0)]\end{aligned} \tag{34}$$

$$\begin{aligned}[i_2(x_2)] &= \text{CH}\left\{[g_2]\ (l_2 - x_2) + [\delta_3^*]\right\} \\ &\quad \times \left\{\text{CH}\,([g_2]\, l_2 + [\delta_3^*])\right\}^{-1}\ [i_2(0)]\end{aligned} \tag{35}$$

Comparison of Eqs. (34) and (35) with Fig. 3, it shows that the content of these equations is the same as that of Eq. (18). In other words, both Eqs. (34) and (35) show that the equation multiplied the current matrix by the inverse matrix of the pseudohyperbolic cosine of the position angle from the left side has constant value.

In the following examples the application of the general equations derived above is restricted to three stages. Increasing the number of stages involved merely magnifies the amount of calculation that must be done. In general, complete set of equations for the s-th stage may be written:

$$[i_s(x_s)] = \mathbf{CH} \left\{[g_s]\,(l_s - x_s) + [\phi_s{}^*]\right\} \times \left\{\mathbf{CH}\left([g_s]\,l_s + [\phi_s{}^*]\right)\right\}^{-1} [i_s(0)] \tag{36}$$

where $[\phi_s{}^*]$ is the position angle matrix looking from the end terminal of line a toward the receiving terminal of transmission system. The details of the solutions for this case will not be undertaken here.

Thus, the operation equations for the current at an arbitrary point on a transmission line is

$$[i_s(x_s)] = \mathbf{CH}\ [\theta_{sx}^*]\ (\mathbf{SH}\ [\theta_s{}^*])^{-1} \sinh [\delta_{sm}^*] \prod_{\sigma=S-1}^{1} \mathbf{CH}\ [\phi_\sigma{}^*] \times (\mathbf{SH}\ [\theta_\sigma^*])^{-1} \sinh [\delta_{\sigma m}^*] \cdot [\tilde{Z}_1]^{-1} [e_0] \tag{37}$$

where

$$[\theta_{sx}^*] = [g_s]\,(l_s - x_s) + [\phi_s{}^*]$$

$$[\theta_\sigma^*] = [\delta_{\sigma m}^*] + [g_\sigma]\,l_\sigma + [\phi_\sigma^*]$$

$$(\mathbf{TH}\ [\phi_{\sigma-1}^*])^{-1} = \left[\,[U] + \left\{\mathbf{TH}([g_\sigma]\,l_\sigma + [\phi_\sigma^*])\right\}^{-1} \tanh [\delta_{\sigma m}^*]\,\right] \times (\tanh [\delta_\sigma^*])^{-1}$$

$$\prod_{\sigma=0}^{1} \mathbf{CH}\ [\phi_\sigma^*] \cdot (\mathbf{SH}\ [\theta_\sigma^*])^{-1} \sinh [\delta_{\sigma m}^*] = [U]$$

$$\tanh\ [\delta_\sigma^*] = [g_{\sigma-1}]^{-1}\ [g_{\sigma-1}^*]^{-1}\ [\tilde{Z}_\sigma]$$

$$\tanh [\delta_{\sigma m}^*] = [g_\sigma]^{-1}\ [g_\sigma^*]^{-1}\ [\tilde{Z}_\sigma]$$

$$\mathbf{TH}\,[\phi_i{}^*] = \tanh [\delta_j{}^*] = [g_i]^{-1}\ [g_i{}^*]^{-1}\ [\tilde{Z}_j]$$

This is the general operational equation, whose solution, subject to the circuit conditions, yields the explicit equations of the transmission line transients. If the position angle matrix is used, the current equation can be obtained easily without solving the boundary value problem of partial differential equations.

Thus, if $[e]$ is given at the sending point as a function of time and all the parameters are known in operational form, then the voltages and currents along the lines are determined by solving the above equations. In particular, if $e_1, e_2, \ldots, e_n$ are sinusoidal waves, the solution may be obtained by operational calculus.

4. Numerical Examples

In this chapter there are given the numerical results for the circuit of 3-stage line

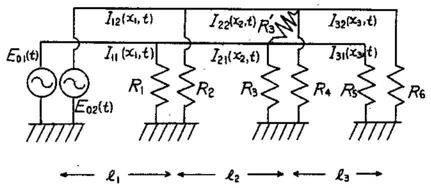

Fig. 4 Two-conductor system grounded through resistances.

system as Fig. 4. These very simple 2-conductor circuits adequately illustrate the method of analysis with minimum amount of algebraic exercise. Referring to Fig. 3, let

$$[L_1] = [L_2] = [L_3] = \begin{bmatrix} L & M \\ M & L \end{bmatrix}$$

$$[C_1] = [C_2] = [C_3] = \begin{bmatrix} C & C' \\ C' & C \end{bmatrix}$$

$$[R_1] = [R_2] = [R_3] = \begin{bmatrix} R+R' & R' \\ R' & R+R' \end{bmatrix}$$

$$[G_1] = [G_2] = [G_3] = [0]$$

for which:

$$L = 1.585 \text{ mH/km}, \quad M = 0.364 \text{ mH/km}$$

$$C = 0.00746\ \mu\text{F/km}, \quad C' = -0.00167\ \mu\text{F/km}$$

and the line lengths are

$$l_1 = 450 \text{ km}, \quad l_2 = 300 \text{ km}, \quad l_3 = 450 \text{ km}.$$

The line resistances, ground-return circuit resistances, grounding resistances etc., are taken as shown in Table 1.

Now, it is obviously $[\tilde{Z}_1] = [0]$ in Fig. 4, then Eq. (37), putting $[\delta_{1m}^*] = [0]$, simplify to

Table 1 Parameters

Conductor 1	Conductor 2	$E_{01}(t)$ (pu)	$E_{02}(t)$ (pu)	R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	R_4 (Ω)	R_3' (Ω)	R_5 (Ω)	R_6 (Ω)	Line resist. (Ω)	Ground-return circuit resist. (Ω)
○	●	1.0	0.5	100	1,000	100	1,000	10	20	30	0.02	1
□	■	1.0	0.5	1,000	100	100	1,000	100	20	30	0.01	1
△	▲	1.0	0.5	1,000	100	100	1,000	1,000	20	30	0.02	3
X	※	$\sin \omega t$	$\cos \omega t$	100	1,000	100	1,000	100	20	30	0.02	3

$\omega = 196.352$ (l/s)

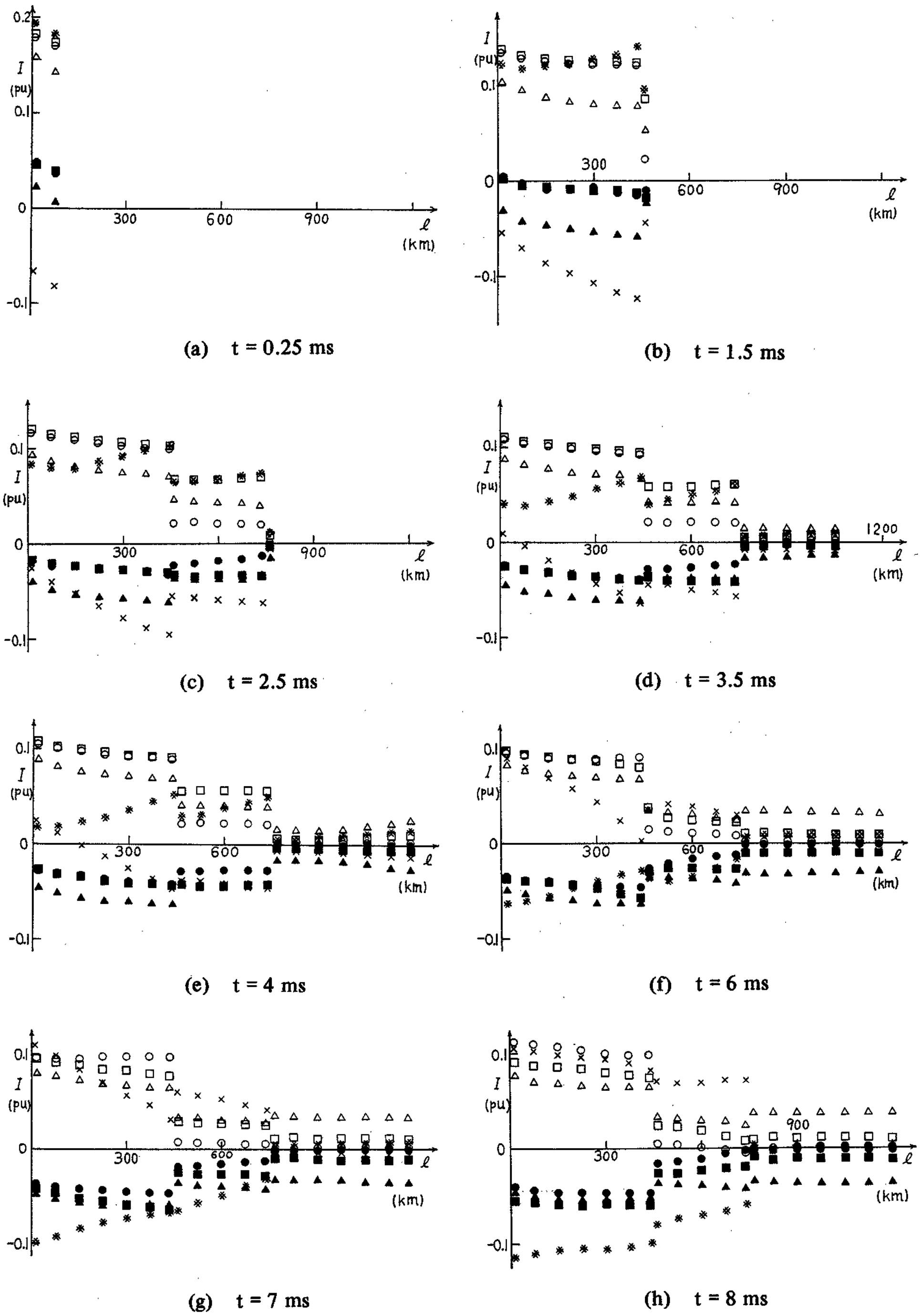

Fig. 5 Current distributions of lines when the sources at sending points are assumed.

$$[i_1(x_1)] = \mathrm{CH}\left\{[g_1](l_1 - x_1) + [\phi_1^*]\right\}\left\{\mathrm{SH}([g_1]l_1 + [\phi_1^*])\right\}^{-1}$$
$$\times [g_1]^{-1}[g_1^*]^{-1}[e_0]$$

$$[i_s(x_s)] = \mathrm{CH}\,[\theta_{sx}^*]\cdot(\mathrm{SH}[\theta_s^*])^{-1}\sinh[\delta_{sm}^*]$$
$$\times\prod_{\sigma=s-1}^{2}\mathrm{CH}\,[\phi_\sigma^*]\cdot(\mathrm{SH}[\theta_\sigma^*])^{-1}\sinh[\delta_{\sigma m}^*]\,\mathrm{CH}[\phi_1^*]$$
$$\times\left\{\mathrm{SH}([g_1]l_1 + [\phi_1^*])\right\}^{-1}[g_1]^{-1}[g_1^*]^{-1}[e_0]$$

where

$$s \geqslant 2, \quad \prod_{\sigma=1}^{2}\mathrm{CH}\,[\phi_\sigma^*]\cdot(\mathrm{SH}[\theta_\sigma^*])^{-1}\sinh[\delta_{\sigma m}^*] = [U]$$

Figures 5(a) to 5(h) show results of numerical calculation obtained. On these figures are recorded the current distributions along the lines.

5. Conclusions

In view of the foregoing discussion, we may conclude that introduction of the position angle matrix makes for simplification the representation of line currents in a multiple-stage, multiple-conductor transmission system. The line current can be found from the system circuit diagram without solving each tedious boundary-value problem with partial differential equations. If this is done a relatively simple method of calculation is available as will be shown. The results based upon this method show good agreement with those obtained by solving the differential equations.

Acknowledgement

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