An Approach to Optimum Design of Output Feedback Control system for Flexible Space Structure

An Approach to Optimum Design of Output Feedback Control System for Flexible Space Structures

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A design method for approximate pole placement is developed by using nonlinear programming. The approach is especially useful in the design of output feedback control system for large space structures, where the total number of actuators and sensors is small compared to the number of critical vibrational modes and exact pole placement is not possible. Two types of objective functions are proposed: one is the weighted sum of the pole location errors in the complex plane and the other is the function of the mode damping ratios. Both of these objective functions provide reasonable design parameters when they are applied to a numerical example of a tendon control system for a flexible beam structure.

1. Introduction

The dimensions of space structures being considered for future application are quite large. That of a solar power satellite, for instance, would be on the order of several kilometers. These large space structures (LSS) could be carried into orbit and deployed or assembled there, and they would be extremely mechanically flexible due to the size of these structures and lightweight construction materials. Further, they have very low rigidity and light damping, which may require active control technology to damp out vibrations caused by periodic or random disturbances in space environment.

A variety of ideas of actively controlled LSS has been proposed with their hardware implementations. The authors presented in the previous work an idea of tendon control system for a beam-like truss structure and investigated the stability of the closed-loop system using direct output feedback.[1] LSS has a large number of vibrational modes, and thus exact pole location of all these critical modes by output feedback is not possible when the number of available control devices is limited.

In this paper, a general approach to optimum design of output feedback control is proposed via a nonlinear programming technique. This approach is quite useful for approximate pole placement and it also provides a systematic design procedure considering the modal contribution to the control performance. Numerical examples of a tendon control system are given to demonstrate the capability of the proposed approach.

2. Output Feedback Control of Flexible Structures

In control problems, large complex structures in space are approximated as a more simple structural element such as a beam, plate, or thin shell.[2] Let us consider here a

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beam-like truss structure with uniform elements and approximate it as an equivalent beam element. If the transverse shear rigidity is large, the dynamic behavior of the structure is governed by Euler-Bernoulli beam equation:

$$\overline{\rho A}\frac{\partial^2 y(x,t)}{\partial t^2}+\overline{EI}\frac{\partial^4 y(x,t)}{\partial x^4}=F(x,t) \tag{2.1}$$

$$F(x,t)=-\frac{\partial M(x,t)}{\partial x} \tag{2.2}$$

$\overline{\rho A}$: mass per unit length

$\overline{EI}$: equivallent bending rigidity

where $y(x,t)$ represents instantaneous displacements of the beam off its equilibrium position, and $F(x,t)$ and $M(x,t)$ are respectively applied control force and moment distributions.

The displacement $y(x,t)$ can be represented by a linear combination of space-dependent eigen functions multiplied by time-dependent generalized coordinates, $H_n(t)$:

$$y(x,t)=\sum_{n=1}^{\infty} Y_n(x)H_n(t) \tag{2.3}$$

Introducing Eq.(2.3) into Eq.(2.1) and applying orthogonality conditions of eigen functions, one can derive the mode equations for the generalized coordinates:

$$\frac{d^2H_m(t)}{dt^2}+\omega^2{}_mH_m(t)=Q_m(t) \quad (m=1,2,\ldots) \tag{2.4}$$

$$Q_m(t)=\int_0^l F(x,t)Y_m(x)dx/\overline{\rho A}\,l \tag{2.5}$$

where $Q_m(t)$ are so-called generalized modal control forces.

The design objective of direct output feedback control using sensors and actuators located on the structural element is addressed here. The sensor outputs are multiplied by appropriate gain factors directly to generate the actuator commands. No state estimation is involved in this approach and, consequently the on-board computer requirement is reduced. The designer substantially can choose the type of the sensors (e.g. linear or angular displacement/velocity sensors) and actuators (e.g. thrusters or torquers). However, the maximum number of available control devices and their possible placement are usually limited.

Here we consider a direct output feedback control using one actuator and a couple of displacement/velocity sensors. If, for instance, angular displacement/velocity sensors and a moment actuator are adopted for the system, the control moment $M(x,t)$ and generalized force $Q_m(t)$ are given as follows

$$M(x,t)=-\left\{K_1\frac{\partial y(l_s,t)}{\partial x}+K_2\frac{\partial}{\partial t}\left(\frac{\partial y(l_s,t)}{\partial x}\right)\right\}\delta(x-l_a) \tag{2.6}$$

$$Q_m(+) = -\frac{Y_m'(l_a)}{\rho A l} \sum_{n=1}^{\infty} Y_n'(l_s) \left\{K_1 H_n(t) + K_2 \dot{H}_n(t)\right\} \tag{2.7}$$

where K_1 and K_2 are feedback gains, l_a and l_s are actuator and sensor positions, respectively, and $\delta(x)$ denotes Dirac delta function.

Thus, characteristic equation of the closed-loop system can be derived in case of lower N modes considered:

$$\begin{aligned} f_c(s) = s^{2N} + D_1 s^{2N-1} + \cdots + D_{2k} s^{2N-2k} + D_{2k+1} s^{2N-(2k+1)} + \cdots \\ + D_{2N-1} s + D_{2N} = 0 \end{aligned} \tag{2.8}$$

where

$$\left.\begin{aligned} D_1 &= K_2 \sum_{n=1}^{N} B_n \\ D_{2k} &= (K_1/K_2) D_{2k-1} + \sum_{n_1=1}^{N} \sum_{n_2=n_1+1}^{N} \cdots\cdots\cdots \sum_{n_k=n_{k-1}+1} \omega_{n_1}^2 \omega_{n_2}^2 \cdots \omega_{n_k}^2 \\ & \qquad (n_1 \neq n_2 \neq \cdots \neq n_k,\ k = 1, 2, \cdots, N) \\ D_{2k+1} &= K_2 \sum_{n=1}^{N} \sum_{n_1=1}^{N} \sum_{n_2=n_1+1}^{N} \cdots\cdots\cdots \sum_{n_k=n_{k-1}+1}^{N} B_n \omega_{n_1}^2 \omega_{n_2}^2 \cdots \omega_{n_k}^2 \\ & \qquad (n \neq n_1 \neq n_2 \neq \cdots \neq n_k,\ k = 1, 2, \cdots, N) \end{aligned}\right\} \tag{2.9}$$

The factors B_n are given in Table 1, according to the type of applied actuator and sensors.

Table 1 Value of the factor B_n in Eq.(2.9).

SENSOR	ACTUATOR	B_n
LINEAR DISP./VELOC.	THRUSTER	$Y_n(l_s) Y_n(l_a)/\rho A l$
	TORQUER	$Y_n(l_s) Y_n'(l_a)/\rho A l$
ANGULAR DISP./VELOC.	THRUSTER	$Y_n'(l_s) Y_n(l_a)/\rho A l$
	TORQUER	$Y_n'(l_s) Y_n'(l_a)/\rho A l$

3. Optimum Design via Nonlinear Programming

The basic objective of the structural control system is pole placement of the critical modes. In output feedback control, exact pole placement is possible only when the control devices (i.e. sensors and actuators) satisfy the observability and controllability conditions, and their total number exceeds the twice the number of controlled modes.[3] If these requirements cannot be satisfied, as usual in LSS control, exact pole placement is not possible. However an approximate pole placement algorithm could be employed to relocate the poles into some prescribed region of the complex plane or to minimize the location errors. A general approach to this problem is proposed in this section.

Since fundamental purpose of the approximate pole placement can be regarded as to relocate the closed-loop poles to the desired locations in the complex plane as close as possible, this problem generally could be solved by using an optimization technique.

The total pole location errors in the complex plane would be a reasonable candidate for the objective function to be minimized in this approach:

$$J_1(K_1, K_2; l_a, l_s) = \sum_{i=1}^{2N} a_i \left| P_i(K_1, K_2; l_a, l_s) - P_{d_i} \right|^2 \tag{3.1}$$

where $P_i(K_1, K_2; l_a, l_s)$ and $P_{d_i}(i = 1, 2, \cdots, 2N)$ are the $2N$ characteristic roots of the closed-loop system and their desired locations, and $a_i(i = 1, 2, \cdots, 2N)$ are associated weighting factors.

Another index of performance could be defined, considering the desired damping ratios of N critical modes, $\zeta_{d_j}(j = 1, 2, \cdots, N)$:

$$J_2(K_1, K_2; l_a, l_s) = \sum_{j}^{N} a_j \left\{ \zeta_j(K_1, K_2; l_a, l_s) - \zeta_{d_j} \right\}^2 \tag{3.2}$$

where

$$\zeta_j = \frac{|\alpha_j|}{\sqrt{\alpha_j^2 + \beta_j^2}}, P_j = \alpha_j + \sqrt{-1}\,\beta_j$$

Thus, approximate poles location problem reduces to the determination of the optimum gains (K_1 and K_2) and control device placements (l_a and l_s) which minimize the objective function defined as above. The problem constraints are that $P_i(K_1, K_2, l_a, l_s)$ $(i = 1, 2, \cdots, 2N)$ satisfy the characteristic equation and the values of design parameters (i.e. K_1, K_2, l_a, and l_s) are within some prescribed boundary: i.e.

$$f_c(P_i(K_1, K_2; l_a, l_s)) = 0 \qquad (i = 1, 2, \cdots, 2N) \tag{3.3}$$

$$g_j(K_1, K_2; l_a, l_s) \leqslant 0 \qquad (j = 1, 2, \cdots, n_c) \tag{3.4}$$

The optimization problem with equality and inequality constraints generally can be solved by using multiplier method.[4] However, straightforward application of this technique to the equality constraint, eq.(3.3), is difficult because the characteristic roots P_i are implicit functions of the design parameters. Henceforth, the proposed optimization algorithm explicitly solves the updated characteristic equation for the roots to recompute the objective function and its gradients. Then, the augmented Lagrangean function is defined by

$$L_t(x, \boldsymbol{\lambda}) = J(x) + \sum_{j=1}^{n_c} \frac{1}{2t_j} [\max\left\{0, \lambda_j + t_j g_j(x)\right\}^2 - \lambda_j^2] \tag{3.5}$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_{n_c})^T$ and $\boldsymbol{t} = (t_1, t_2, \cdots, t_{n_c})^T$ are penalty parameters.

In the optimization scheme the following steps are carried out:

Step 0: Given initial design parameters $\boldsymbol{x}^0 = (K_1, K_2; l_a, l_s)$, $0 \leqslant \boldsymbol{\lambda}^6 \in R^{n_c}$, $0 < \boldsymbol{t}^0 \in R^{n_c}$, $\epsilon > 0, k = 0$.

Step 1: Solve the characteristic equation $f_c(x^k \pm \delta x) = 0$ and obtain $2N$ roots $P_1, P_2, \cdots, P_{2N}$.

Step 2: Compute objective function $J(x^k \pm \delta x)$ and augmented Lagrangean function $L_{t^k}(x^k \pm \delta x, \boldsymbol{\lambda}^k)$.

Step 3: Approximate gradients $\partial L_{t^k}/\partial x$ by central differences and improve design parameters iteratively for unconstrained minimization by use of a conjugate gradient method.[5] Repeat Steps 1 and 2 for updated design parameters. If optimal x^k is obtained then go to Step 4.

Step 4: Update penalty parameters λ^k and t^k according to ordinary multiplier method.[4] If

$$\max_i \left| \max\left\{ g_i(x^k), -\frac{\lambda_i^k}{t_i^k} \right\} \right| \leqslant \epsilon$$

then complete the iteration, and otherwise return to Step 1 with $k = k + 1$.

4. Numerical Example

A numerical example is given in this section to illustrate the application of the proposed design procedure. The problem analysed here is taken from the tendon control system for a beamlike space structure formerly studied by the authors.[1] The flexible truss structure deployed from the rigid spacecraft is modeled as a cantilever beam with an equivallent bending rigidity $\overline{EI} = 7.17 \times 10^7$ $[Nm]$, mass per unit length $\overline{\rho A} = 0.875$ $[Kg/m]$ and length $l = 150$ $[m]$ (See Ref. 1 for more detailed data). In Table 2 the mode frequencies are given up to the fourth mode. The modes with higher frequencies are truncated from the dynamic model.

As shown in Fig. 1, the tendon actuator is a type of torquer consisting of a couple of force actuators with linkages of tensile wires and generates control moment at the arm point. In the following we design direct output feedback control using angular dis-

Table 2 Natural frequencies of a beam.

MODE NUMBER	FREQUENCY ω_n (rad/sec)
$n = 1$	1.41359
$n = 2$	8.86272
$n = 3$	24.82514
$n = 4$	48.64171

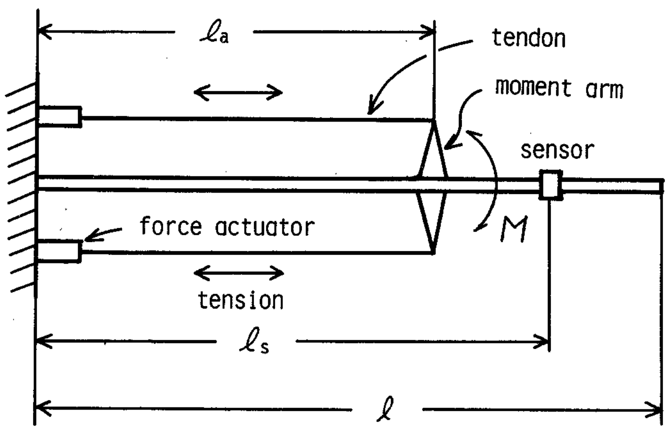

Fig. 1 Tendon control system for beam vibration suppression.

placement/velocity sensors located at the same position, i.e. $l_a = l_s = l_c$. Such a placement of the actuator and sensor at the same position of the structure is refered to "collocation", and has advantages in that it assures stable closed-loop pole locations in the case of a proper velocity damping and the spillover terms due to unmodeled modes do not cause instability.

The design parameters of this control system are the feedback gains K_1 and K_2 for the angular displacement and angular velocity, respectively, and the collocation position l_c. Closed-loop poles of the system moves in the complex plane according to these three parameters, as shown, for instance, in Fig. 2.

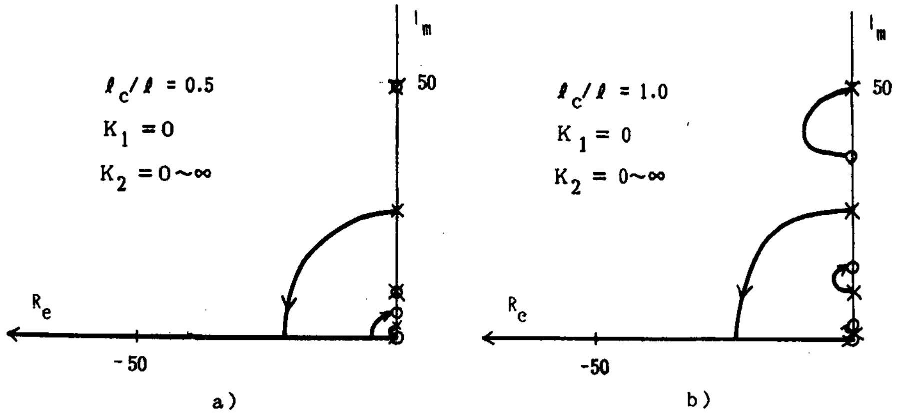

Fig. 2 Root locus plot of closed-loop system using angula velocity feedback.

Numerical analysis of optimization, using the two proposed objective functions, is described in the following.

Examine first the objective function J_2 and optimize the damping ratios for the critical modes. The proposed optimization algorithm is applied, taking the feedback gains K_1 and K_2 as design parameters whereas the collocation position l_c is preassigned and fixed in the optimization. In the optimization, the desired damping ratios are assumed to be $\zeta_{d_j} = 0.5$ and the weighting factors are set $a_j = 1$ for all j. Inequality constraints are introduced as

$$\mathrm{Re}\left\{ P_i(K_1, K_2; l_a, l_s) \right\} < 0 \quad (i = 1, 2, \cdots, 2N)$$

where $\mathrm{Re}\{\cdot\}$ means the real part of $\{\cdot\}$.

The optimum gains K_1^* and K_2^* are sought for a number of different collocation positions and the resulting optimum objective function $J_2(K_1^*, K_2^*; l_c)$ is plotted in Fig. 3 as a function of l_c/l. The optimum gains, achieved damping ratios and pole locations are listed in Table 3 for three collocation positions, (i.e. $l_c/l = 0.083$, 0.6 and 1.0) where the objective function exhibits distinct local minima.

Fig. 4 illustrates the variation of the objective function in the (K_1, K_2) plane. No local minimum is observed in this example. The results of optimization for different values of the desired (target) damping ratios are shown in Table 4. The collocation position is fixed at the beam edge and the optimum gains are sought for three values of ζ_{d_j}(i.e. ζ_{d_j}= 0.2, 0.4 and 0.6).

The objective function J_2 is useful as long as the closed-loop poles have complex

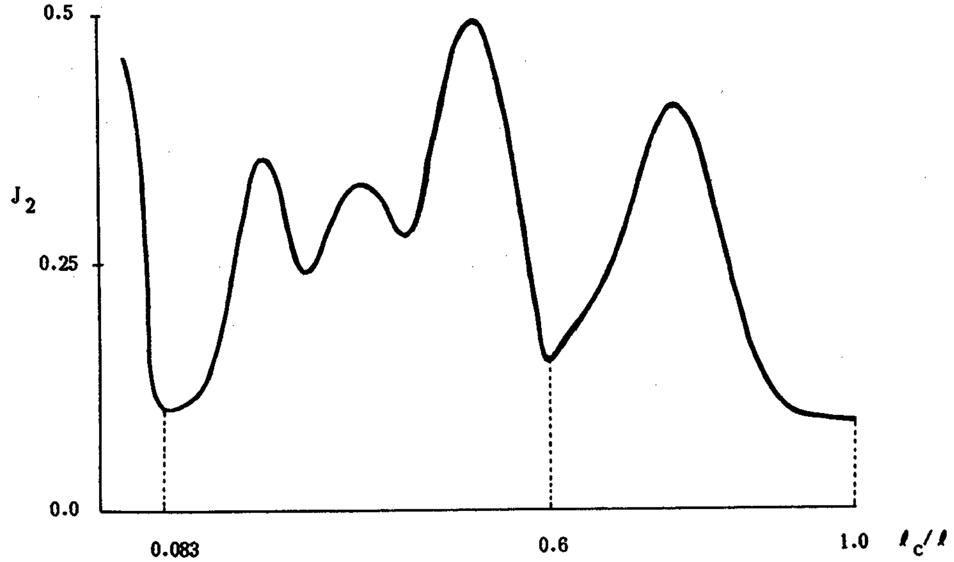

Fig. 3 Variation of minimum objective function as a function of collocation position.

Table 3 Results of optimization for three different collocation positions. (l_c/l = 0.083, 0.6, 1.0)

(a) l_c/l = 0.083

FEEDBACK GAINS		OPTIMUM J_2	MODAL DAMPING RATIOS			
K_1	K_2		ζ_1	ζ_2	ζ_3	ζ_4
-7.187×10^6	7.704×10^5	0.1026	0.5037	0.3666	0.5393	0.2114
POLE LOCATIONS		RE. PART	−0.2414	−2.906	−12.18	−7.946
		IM. PART	0.4139	7.374	19.02	36.73

(b) l_c/l = 0.6

FEEDBACK GAINS		OPTIMUM J_2	MODAL DAMPING RATIOS			
K_1	K_2		ζ_1	ζ_2	ζ_3	ζ_4
-1.178×10^5	6.100×10^5	0.1535	0.5356	0.2047	0.2453	0.4846
POLE LOCATIONS		RE. PART	−0.7523	−1.897	−7.391	−17.32
		IM. PART	1.186	9.068	29.21	31.26

(c) l_c/l = 1.0

FEEDBACK GAINS		OPTIMUM J_2	MODAL DAMPING RATIOS			
K_1	K_2		ζ_1	ζ_2	ζ_3	ζ_4
-4.017×10^5	1.731×10^5	0.09147	0.5044	0.4544	0.4965	0.2011
POLE LOCATIONS		RE. PART	−0.4284	−4.325	−11.60	−7.952
		IM. PART	0.7334	8.477	20.29	38.74

roots. However, in many cases, when the feedback gains are increased, some of the poles likely to move to the real axis and the use of performance index based on damping ratio becomes ineffective.

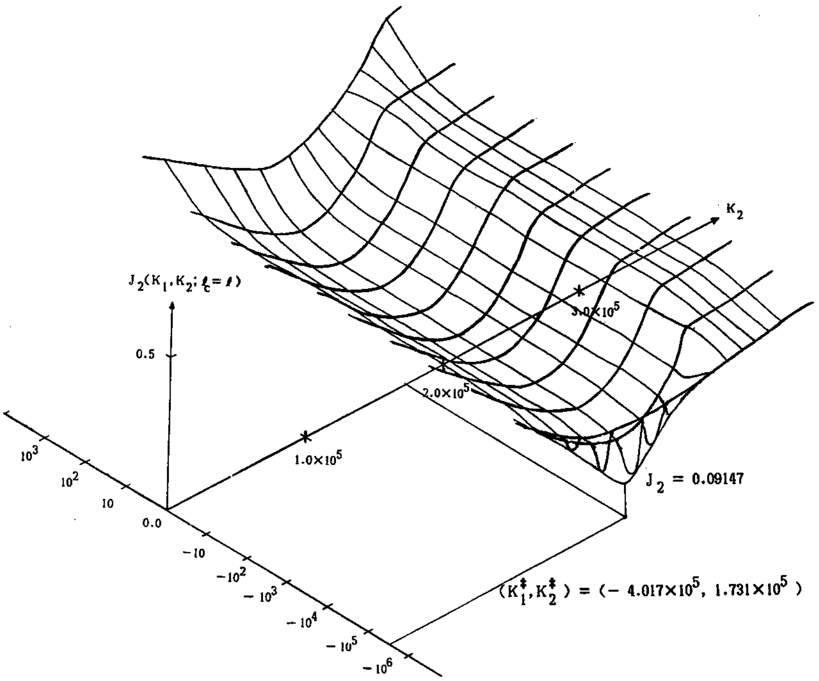

Fig. 4 Shape of objective function.

Table 4 Optimum gains and damping ratios for different target damping ratios.

OBJECTIVE FUNCTION	OPTIMUM GAINS		OPTIMUM J_2	MODAL DAMPING RATIOS			
	K_1	K_2		ζ_1	ζ_2	ζ_3	ζ_4
$\sum_{i=1}^{4}(\zeta_i-0.2)^2$	-2.988×10^5	1.031×10^5	0.0020872	0.1924	0.2243	0.2037	0.1623
$\sum_{i=1}^{4}(\zeta_i-0.4)^2$	-3.681×10^5	1.586×10^5	0.03732	0.3906	0.3909	0.4079	0.2074
$\sum_{i=1}^{4}(\zeta_i-0.6)^2$	-4.239×10^5	1.856×10^5	0.1750	0.6187	0.5157	0.5900	0.1908

The use of the objective function J_1 seems to be more reasonable and has a general capability for approximate pole placement. In this example, the placement of the target pole location is determined for providing the same damping ratios in each mode (ζ_{d_j} = 0.447, j = 1, 2, 3, 4) as shown in Fig. 5. The weighting factor $a_j = 1/\omega_j^2$ is used in the objective function to expect the uniform improvement in each mode. The result of gain optimization for the case of collocation position $l_c/l = 1.0$ is summarized in Table 5.

5. Conclusion

A design method to determine the optimum feedback gains and sensor/actuator collocation position for LSS control systems is proposed. This approach employs a nonlinear programming technique which determines the optimum design parameters so as to minimize the pole location errors in the complex plane. A numerical example shows

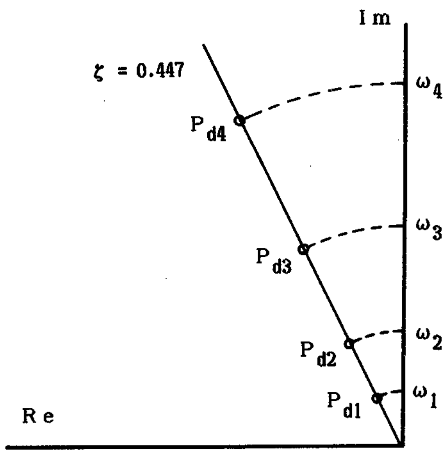

Fig. 5 Placement of desired poles in the complex plane with equal damping ratio.

Table 5 Optimum gains obtained by using the objective function J_1.

OPTIMUM GAINS		OPTIMUM J_1	POLE PLACEMENT			
K_1	K_2		P_1	P_2	P_3	P_4
-2.508×10^5	1.677×10^5	0.3011	-0.3165 $\pm 1.113\,i$ ζ=0.2735	-3.617 $\pm 8.913\,i$ ζ=0.3760	-11.15 $\pm 22.03\,i$ ζ=0.4515	-8.463 $\pm 39.07\,i$ ζ=0.2117
TARGET POLE LOCATIONS			-0.6299 $\pm 1.252\,i$	-3.980 $\pm 7.960\,i$	-10.73 $\pm 21.46\,i$	-21.46 $\pm 42.93\,i$
$\dfrac{\lvert P_{di}-P_i\rvert}{\omega_i}$			0.2425	0.1151	0.02851	0.2787

that this optimization approach is capable of designing output feedback control system for flexible structures with a small number of control devices.

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