Tendon Control System for Flexible Space Structures

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This paper is concerned with the design of active vibration control system for large space structures having very high flexibility and low damping. Capability of a tendon control system is studied for a beam-like truss structure. The closed-loop systems are designed for two cases of feedback sensors selected; one is colocated transverse displacement/velocity feedback system, and the other is colocated rotational displacement/velocity feedback system. The former system leads to an inherent instability considering infinite number of vibrational modes. Numerical simulation is given for an illustrative example.

1. Introduction

The progress of a space transportation system makes it possible to place very large satellites into orbit. With the advent of the capability of constructing large space structures (LSS) such as space stations, large earth-orbiting antennas and huge solar power satellites, new technical challenges in the field of control engineering have evolved[1]. LSS's involve a higher level of mechanical flexibility than has been experienced in space before, and their associated mission requirements, *e.g.*, extremely accurate pointing and shape determination, necessitate new design methodologies not encountered with classical spacecraft. Since they have many resonant low frequencies and very light natural damping, the structural vibration will easily be excited by the external and internal disturbances in space environment. Thus, the concept of an actively controlled LSS has received a great deal of recent attention, in which a variety of sensors and actuators are located in the structure and operated through on-board computer controllers to provide an artificial damping to the dynamics[2].

A large number of works have been made on LSS active control systems and a variety of ideas of hardware implementation have been proposed on control device design. Some of them use inertial or optical sensors and force actuators, such as CMG's and electrodynamic actuators, and they have already been tested by laboratory models. However, the use of tendon actuator, *i.e.*, a moment actuator with tensile wire linkages, has not been examined except that it has been adopted in the design of Hoop-Column model of earth-orbiting antenna[3]. This paper investigates the capabilities and limitations of a tendon control system when it is applied to the active control of a flexible beam structure.

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2. Tendon Control of a Beam-like Space Truss Structure

Many large space structures, although complex in detail, often are approximated as a beam, plate, or thin shell, *i.e.*, the dynamic behavior of these structure may be governed by proper beam, plate or shell equations.

For example, a beam-like truss structure with uniform elements may be approximated by an equivalent beam element as illustrated in Figs. 1(a) and (b)[4]. When the beam possesses a large transverse shear rigidity, then the governing equations of motion reduce to Euler-Bernoulli beam equations:

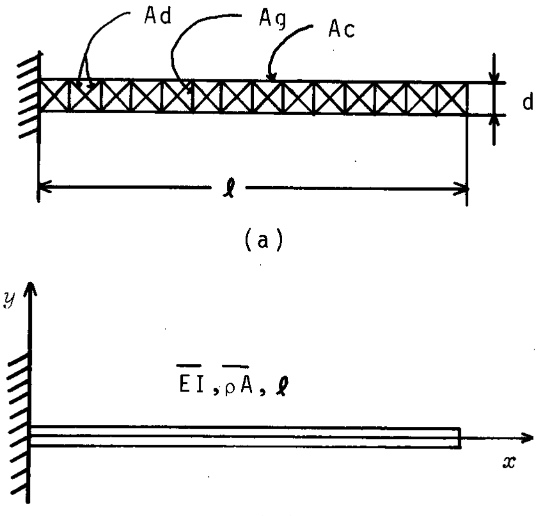

Fig. 1 Cantilever truss structure and equivalent beam with uniform mass distribution.

$$\overline{\rho A}\,\frac{\partial^2 y(x,t)}{\partial t^2} + \overline{EI}\,\frac{\partial^4 y(x,t)}{\partial x^4} = F(x,t) \tag{2.1}$$

$$F(x,t) = \frac{\partial M(x,t)}{\partial x} \tag{2.2}$$

where $y(x,t)$ represents instantaneous displacements of the beam off its equilibrium position, and $F(x,t)$ and $M(x,t)$ are respectively applied control force and moment distributions. The boundary conditions for the cantilever beam are:

$$\begin{aligned} & y(o,t) = \left.\frac{\partial y(x,t)}{\partial x}\right|_{x=0} = 0 \\ & \left.\frac{\partial^2 y(x,t)}{\partial x^2}\right|_{x=l} = \left.\frac{\partial^3 y(x,t)}{\partial x^3}\right|_{x=l} = 0 \end{aligned} \tag{2.3}$$

The displacement $y(x,t)$ can be represented by a linear combination of space-dependent eigen functions $Y_n(x)$ multiplied by time-dependent generalized coordinates $H_n(t)$.

$$y(x,t) = \sum_{n=1}^{\infty} Y_n(x) H_n(t) \tag{2.4}$$

Introducing Eq. (2.4) into Eq. (2.1) and applying orthogonality conditions of eigen functions, one can derive the mode equations for the generalized coordinates

$$\frac{d^2 H_m(t)}{dt^2} + \omega_m^2 H_m(t) = Q_m(t) \qquad (m = 1, 2, \ldots) \tag{2.5}$$

where

$$Q_m(t) = \int_0^l F(x,t) Y_m(x) dx \,/\, \overline{\rho A}\, l \qquad (m = 1, 2, \ldots) \tag{2.6}$$

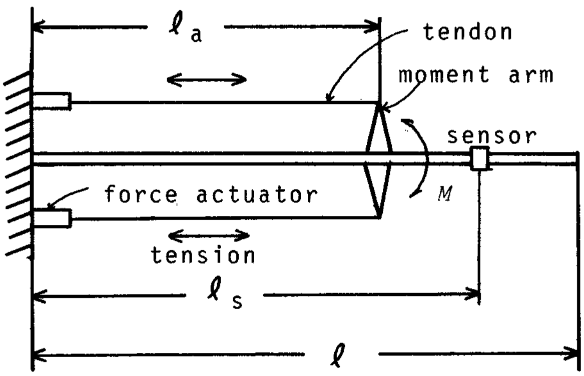

Fig. 2 Tendon control system for beam vibration suppression.

are generalized modal control forces.

Figure 2 illustrates a tendon control system designed for vibration suppression of a beam structure. The tendon actuator consists of a couple of force actuators linked with a moment arm through tensile wires and generates control moments at the arm point. Displacement and velocity sensors are placed on a proper position of the beam and the sensor signals are fedback to the tendon actuators. Placement of the sensors and actuators represents a substantial degree of freedom to the designer and is usually not a straightforward question. However, there exists a certain necessary condition for the relative position of actuators and sensors to guarantee the stability of the closed-loop system.

Placement of the actuator and sensor at the same position of structure is referred to "colocation" and is proved to provide a stable closed-loop system in the case of a proper velocity damping applied[5]. In the following we investigate in detail the effect of the selected feedback signals and control point on the stability of tendon control system through the analysis of characteristic equations and root locus plot.

Closed-loop systems of two different types are investigated here, *i.e.*:

1) transverse displacement/velocity feedback control;
2) rotational displacement/velocity feedback control.

In the first system, control moment $M(x,t)$ is determined from the transverse displacement and velocity at the sensor location as

$$M(x,t) = -\left\{K_1 y(l_s,t) + K_2 \frac{\partial y(l_s,t)}{\partial t}\right\} \delta(x - l_a) \tag{2.7}$$

where K_1 and K_2 are feedback gains; l_a and l_s are actuator and sensor positions, respectively, and $\delta(x)$ denotes Dirack delta function. Thus, generalized control force $Q_m(t)$ are written in the following form.

$$Q_m(t) = -\frac{Y'_m(l_a)}{\rho A l} \sum_{n=1}^{\infty} Y_n(l_s) \left\{K_1 H_n(t) + K_2 \dot{H}_n(t)\right\} \tag{2.8}$$

Similarly, $M(x,t)$ and $Q_m(t)$ for the second control system are given as follows

$$M(x,t) = -\left\{K_1 \frac{\partial y(l_s,t)}{\partial x} + K_2 \frac{\partial}{\partial t}\left(\frac{\partial y(l_s,t)}{\partial x}\right)\right\} \delta(x - l_a) \tag{2.9}$$

$$Q_m(t) = -\frac{Y'_m(l_a)}{\rho A l} \sum_{n=1}^{\infty} Y'_n(l_s) \left\{K_1 H_n(t) + K_2 \dot{H}_n(t)\right\}$$

$$(m = 1, 2, \ldots) \tag{2.10}$$

We first derive the characteristic equation of the closed-loop system when transverse displacement and velocity are fedback. To this end, let us substitute Eq. (2.8) into Eq. (2.5), and take Laplace transform, *i.e.*:

$$(s^2 + \omega_m^2)\bar{H}_m(s) = -\frac{1}{\rho A l} Y'_m(l_a)(K_1 + K_2 s) \sum_{n=1}^{\infty} Y_n(l_s)\bar{H}_n(s).$$

$$(m = 1, 2, \ldots) \tag{2.11}$$

Then, multiplying $Y_m(l_s) / (s^2 + \omega_m^2)$ to both sides of the equation and taking summation for $m = 1, 2, \ldots$, one obtains the following relation

$$\sum_{m=1}^{\infty} Y_m(l_s)\bar{H}_m(s) = -\frac{1}{\rho A l} \sum_{m=1}^{\infty} Y_m(l_s) Y'_m(l_a) \frac{K_1 + K_2 s}{s^2 + \omega_m^2} \sum_{n=1}^{\infty} Y_n(l_s)\bar{H}_n(s)$$

$$\tag{2.12}$$

or

$$1 + \frac{1}{\rho A l} \sum_{m=1}^{\infty} Y_m(l_s) Y'_m(l_s) \frac{K_1 + K_2 s}{s^2 + \omega_m^2} = 0 \tag{2.13}$$

which gives the characteristic equation of the closed-loop system.

If one takes the expansion terms to the Nth mode, the characteristic equation can be written in the following form.

$$s^{2N}+D_1 s^{2N-1}+\cdots+D_{2k}s^{2N-2k}+D_{2k+1}s^{2N-(2k+1)}+\cdots\cdots+D_{2N-1}s$$
$$+D_{2N}=0 \qquad (2.14)$$

where

$$D_1 = K_2\sum_{n=1}^{N} B_n$$
$$D_{2k} = (K_1/K_2)D_{2k-1}+\sum_{n_1=1}^{N}\sum_{n_2=n_1+1}^{N}\cdots\cdots\sum_{n_k=n_{k-1}+1}^{N}\omega_{n_1}^2\omega_{n_2}^2\cdots\cdots\omega_{n_k}^2$$
$$(n_1 \neq n_2 \neq \cdots \neq n_k,\ k=1,\cdots,N)$$
$$D_{2k+1} = K_2\sum_{n=1}^{N}\sum_{n_1=1}^{N}\sum_{n_2=n_1+1}^{N}\cdots\cdots\sum_{n_k=n_{k-1}+1}^{N}B_n\omega_{n_1}^2\omega_{n_2}^2\cdots\cdots\omega_{n_k}^2$$
$$(n \neq n_1 \neq n_2 \neq \cdots \neq n_k,\ k=1,\cdots,N-1)$$

and

$$B_n = Y_n(l_s)\,Y_n'(l_a)/\overline{\rho A}\,l. \qquad (2.15)$$

Similarly, one can derive characteristic equation of the rotational displacement/velocity feedback control system in the same form as Eq. (2.14) except that the coefficients B_n $(n = 1, ..., N)$ become

$$B_n = Y_n'(l_s)Y_n'(l_a)/\overline{\rho A}\,l. \qquad (2.16)$$

Through the well-known Routh-Hurwitz criterion the stability of the closed-loop system can be tested directly from the coefficients of the characteristic polynomial without explicitly evaluating the roots. The positivity of all the coefficients D_1, D_2, ..., D_{2N} represents a necessary condition for stability. In case of rotational displacement/velocity feedback system, if one takes the colocated placement of actuators and sensors, *i.e.*, $l_a = l_s = l_c$, then all the D_j's $(j = 1, ..., 2N)$ are assured to be positive because each B_n $(n = 1, ..., N)$ has a positive value:

$$B_n = \left\{Y_n'(l_c)\right\}^2/\overline{\rho A}\,l. \qquad (2.17)$$

Therefore, the closed-loop system at least satisfies the necessary condition for stability.

One the contrary, when transverse displacement/velocity feedback control is adopted, each B_n for colocation includes cross product $Y_n(l_c)Y_n'(l_c)$ as

$$B_n = Y_n(l_c)Y_n'(l_c)/\overline{\rho A}\,l. \qquad (2.18)$$

and changes its sign according to the mode number and coordinate of the colocation point. If D_j has a negative value for any j then the closed-loop system becomes unstable. Thus colocated sensor/actuator placement for this system does not assure a stable feedback control and requires more detailed analysis of mode shapes and their derivatives. Since the first mode has positive mode shape $Y_1(x)$ and derivative $Y_1'(x)$ $(= \partial Y_1(x)/$

Table 1 Sign of $Y_n(x)$, $\partial Y_n(x)/\partial x$, and $B_n(x)$.

(a) 2nd mode

x/l	0.00	..	0.471	..	0.774	..	1.000
$Y_2(x)$	0.00	−	−	−	0.000	+	+
$\frac{\partial}{\partial x}Y_2(x)$	0.00	−	0.000	+	+	+	+
$B_2(x)$	0.00	+	0.000	−	0.000	+	+

(b) 3rd mode

x/l	0.00		0.291		0.501		0.692		0.868		1.000
$Y_3(x)$	0.00	−	−	−	0.000	+	+	+	0.000	−	−
$\frac{\partial}{\partial x}Y_3(x)$	0.00	−	0.000	+	+	+	0.000	−	−	−	−
$B_3(x)$	0.00	+	0.000	−	0.000	+	0.000	−	0.000	+	+

(c) 4th mode

x/l	0.00	..	0.208	..	0.356	..	0.501	..	0.644	..	0.780	..	0.906	..	1.000
$Y_4(x)$	0.00	−	−	−	0.000	+	+	+	0.000	−	−	−	0.000	+	+
$\frac{\partial}{\partial x}Y_4(x)$	0.00	−	0.000	+	+	+	0.000	−	−	−	0.000	+	+	+	0.000
$B_4(x)$	0.00	+	0.000	−	0.000	+	0.000	−	0.000	+	0.000	−	0.000	+	0.000

∂x) for $0 < x < l$, the coefficient B_1 of the linear displacement/velocity feedback system is assured to be positive, *i.e.*, $B_1 > 0$ for $0 < x < l$. The signs of B_n, $n = 2, 3, 4$ are summarized in Table 1 (a) – (c). From these tables one can find that the regions of x where B_n, $n = 1, 2, 3, 4$ are all positive are $0 < x < 0.208l$ and $0.906l < x < l$. Taking the infinite number of modes into account, this region degenerates to the two points, *i.e.*, the root ($x = 0$) and edge ($x = l$). Therefore, in case of a transverse displacement/velocity feedback system delete the edge is the only one colocation point which satisfies both the observability and stability conditions considering infinite number of modes.

3. Numerical Example

The cantilever truss shown in Fig. 1(a) is taken to illustrate a numerical example. We design a feedback control system using a tendon actuator and colocated sensors. The dimension and properties of the truss members are given in Table 2, where A_c, A_g, and A_d indicate the member cross-sectional areas, E is the modulus of elasticity, and ρ is the mass density. The equivalent bending rigidity $\overline{EI}$ and mass per unit length $\overline{\rho A}$ for the beam model are obtained using the procedure suggested in Ref. 4, and listed in the same table. We show in Table 3 the mode frequencies of the equivalent beam model to the fourth mode. The modes with higher frequencies are truncated from the dynamic model.

We examine with these numerical data two types of closed-loop system employing different feedback sensors investigated in the preceeding section. One is a transverse displacement/velocity feedback system, and the other is a rotational displacement/velocity feedback system.

Taking colocation point as $l_a = l_s = 100$ m or 150 m, and constraining feedback

Table 2 Dimension and properties of the truss members and equivalent beam characteristics.

length, l	$7.5 \times 20 = 150$	[m]
depth, d	5	[m]
Young's modulus, E	71.7×10^9	[N/m^2]
mass density, ρ	2768	[kg/m^3]
cross sectional area, A_c	81×10^{-6}	[m^2]
A_g	60×10^{-6}	[m^2]
A_d	40×10^{-6}	[m^2]
equivalent bending rigidity, $\overline{EI}$	7.17×10^7	[Nm^2]
mass per unit length, $\overline{\rho A}$	0.875	[kg/m]
moment arm of tendon	20	[m]

Table 3 Natural frequencies for beam.

mode number	frequency ω_n (rad/sec)
$n = 1$	1.41359
$n = 2$	8.86272
$n = 3$	24.82514
$n = 4$	48.64171

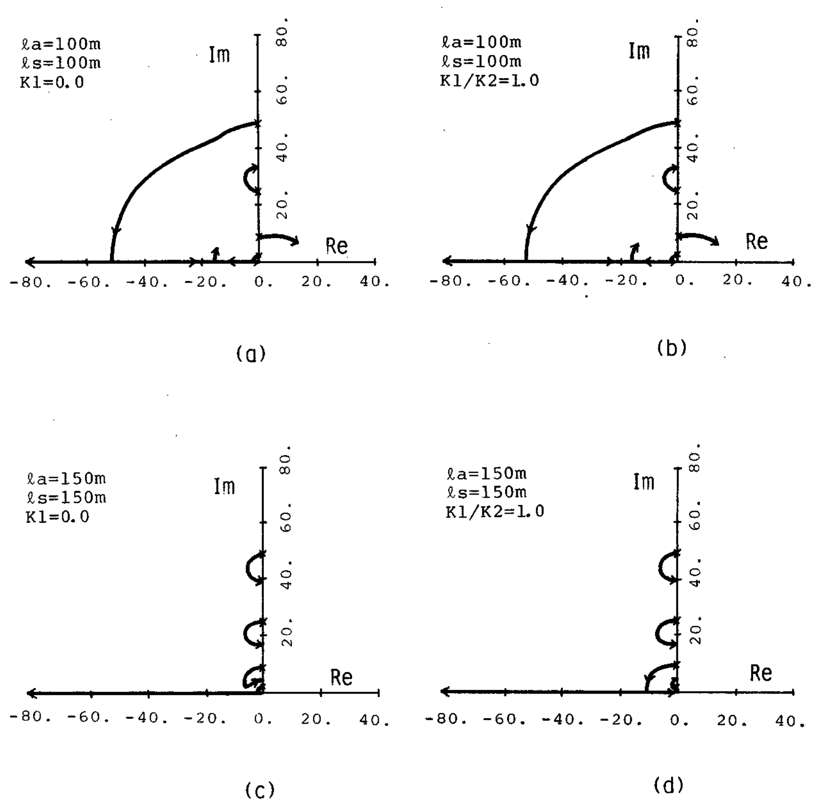

Fig. 3 Root locus plot of closed-loop systems using transverse displacement/velocity feedback.

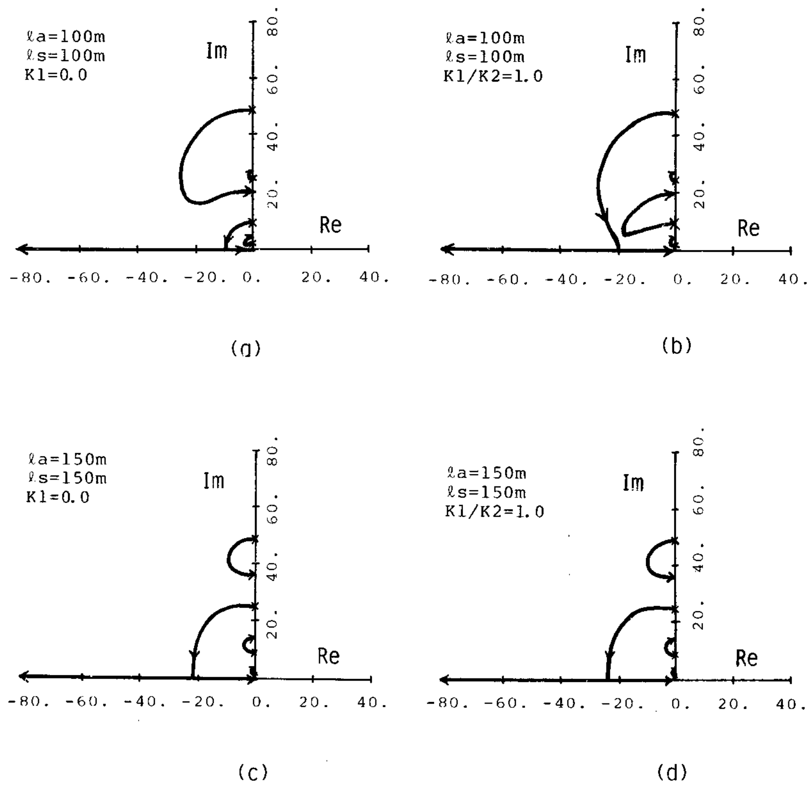

Fig. 4 Root locus plot of closed-loop systems using rotational displacement/velocity feedback.

Table 4 Determined feedback gains and modal damping parameters of the closed-loop system.

	$l_a = l_s$	K_1	K_2	1st mode	2nd mode	3rd mode	4th mode
tramsverse disp./veloc. feedback	100.0 m	0	431438	1.000	—	0.046	1.000
	100.0 m	431438	431438	1.000	—	0.046	1.000
	150.0 m	0	16833	1.000	0.512	0.262	0.135
	150.0 m	11221	11221	0.505	1.000	0.282	0.130
rotational disp./veloc. feedback	100.0 m	0	830776	0.829	1.000	0.029	0.780
	100.0 m	830776	830776	0.384	0.749	0.028	0.776
	150.0 m	0	431438	0.368	0.162	1.000	0.075
	150.0 m	431438	431438	0.188	0.160	1.000	0.075

gains as $K_1 = 0$ or $K_1 = K_2$, we can plot root loci of the closed-loop system. Root locus plots for these four different conditions are shown in Fig. 3(a)–(d) for the transverse displacement/velocity feedback system, and in Fig. 4(a)–(d) for the rotational displacement/velocity feedback system.

We determine a set of feedback gains so as to maximize the damping ratio of the first mode. The feedback gains thus determined and mode damping parameters of the closed-loop dynamics are summarized in Table 4. From the table and root locus plot

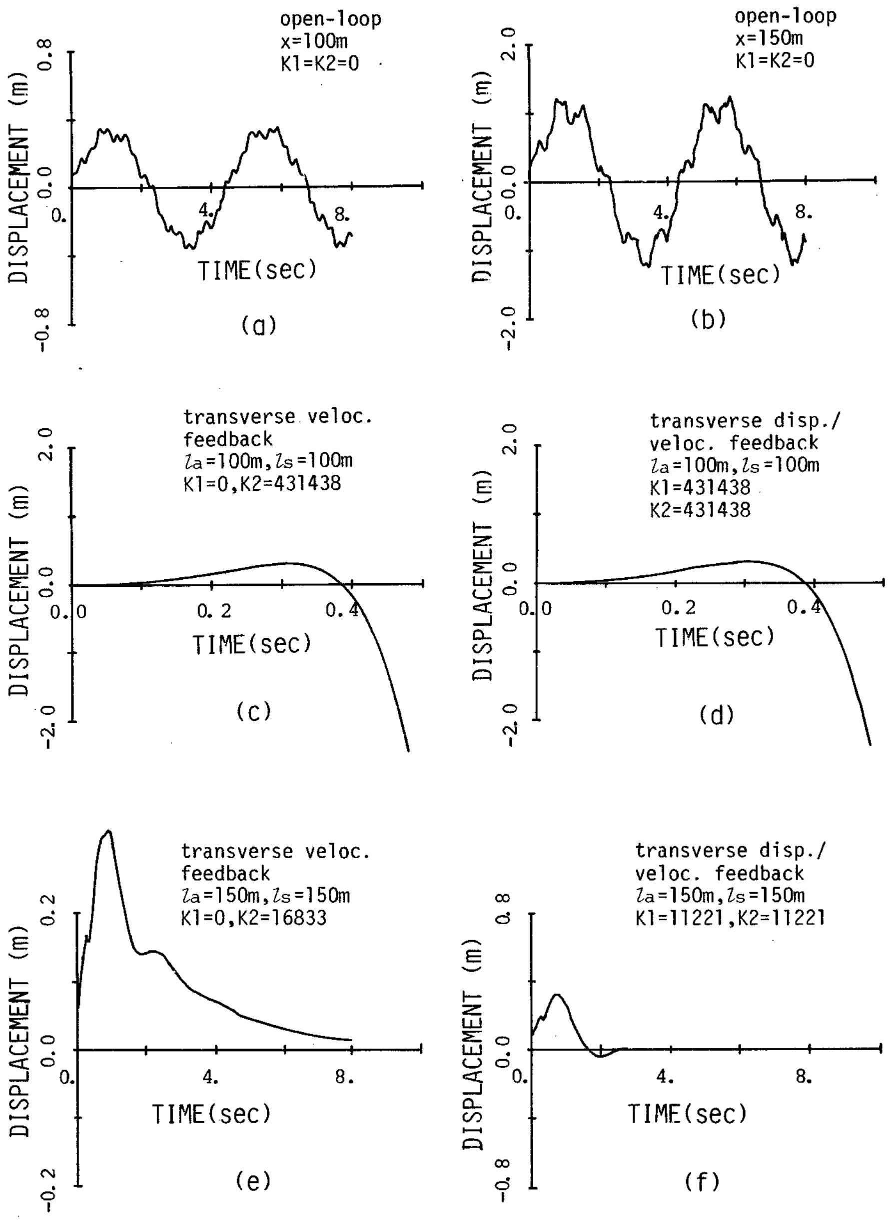

Fig. 5 Transient response of the beam to an impulsive force input applied at the edge.

one finds that the second mode of the transverse displacement/velocity feedback system becomes unstable when colocation point is selected as $(l_a, l_s) = (100, 100)$. The reason of unstability can be explained by the fact that the parameter B_2 is negative for this colocation placement, *i.e.*, $x/l = 0.667$.

Response of the beam element to an impulsive force applied on the edge at time $t = 0$ is shown in Figs. 5(a)–(j), where transverse displacement at the colocation point is computed with the open-loop or designed closed-loop dynamics. Among the three stable closed-loop systems, the rotational velocity feedback system with colocation point (l_a, l_s) = (100, 100) and gain constraint as K_1 = 0 seems to be optimal. However the closed-loop dynamics of this system has very low damping in the 3rd mode, so that the control moment and tension of the tendon become highly oscillatory as shown in Figs. 5(k) and (l).

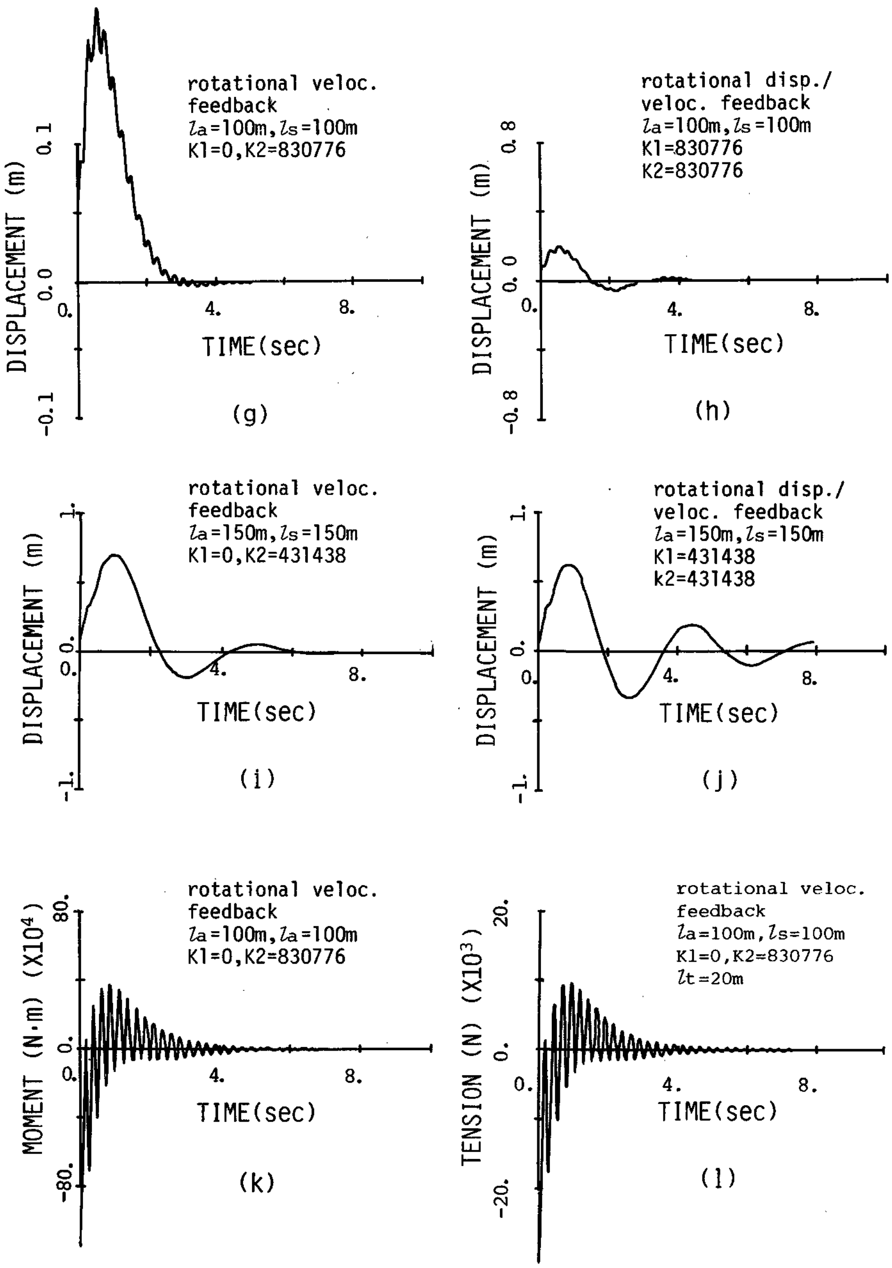

Fig. 5 (Continued.)

4. Conclusions

In this study we have shown the inherent instability of a tendon control system for a cantilever beam using colocated sensors and actuators. If a linear (transverse) displacement/velocity feedback is adopted, it is shown that the colocation point must be the edge, or otherwise the closed-loop system would be unstable considering infinite number of flexible modes. We also have shown that one can employ rotational displacement/velocity sensors for which the closed-loop system at least satisfies a necessary condition for stability.

The discussions described above are rather obscure and more definite relation between colocation point and the stability of the closed-loop system should be derived. Further studies are also required for optimizing the colocation point and feedback gains through a parameter optimization technique or LQ regulator theory. Experimental studies are also needed to verify the feasibility of a tendon control system for flexible space structures.

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