A consideration on Machine Parameters of Induction Motors Driven by Adjustable Frequency Converter

A Consideration on Machine Parameters of Induction Motors Driven by Adjustable Frequency Converter

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A method for improving the performance of the induction motor, whose speed is controlled with variable frequency converters, by means of the appropriate selection of machine parameters is suggested. The optimal machine parameters are derived from the investigation about the slip frequencies, which yield the maximum output power, torque, power factor, and efficiency of the induction motor. Attentions are largely concentrated on the improvement in efficiency and power factor.

1. Introduction

With the developments in power electronics technics, signal processing and power conversion have become easier and convenient. Therefore, the robust and inexpensive induction motor has become a means of adjustable speed drive with high performance. By virtue of these merit, about 80 percent of all motors, applied in industrial and domestic loads, are induction motors. Thus, it is very important to improve the efficiency and the power factor of the induction motor for energy saving. And some authors have reported papers about the method for improving the efficiency of the adjustable speed induction motor by means of the control techniques[1),2)].

In general, the standard induction motor is designed to maintain high efficiency when it is driven in the region of 75 percent to the full load with fixed stator voltage and frequency. However, the motors are not always driven under these circumstances. Hence, the standard induction motor may not be appropriate for the efficient variable speed operation with any load conditions.

This paper deals with the machine parameters which yield the efficient and high power factor operation of the induction motor driven by adjustable frequency converters. The optimal slip frequencies corresponding to the maximum output power, torque, power factor, and efficiency on any operating conditions will be derived by the linear analysis of the induction motor. These slip frequencies are expressed by the functions of machine parameters. Consequently, it may be possible to operate the induction motor with high efficiency and power factor by sellecting the machine parameters which produce two of the optimal slip frequencies in closer value over the given speed range. One of the remarks derived by the proposed strategy is that the induction motors driven by adjustable frequency converters should have substantially smaller rotor resistance and slightly more leakage inductance than the standard induction motors. The results evaluated in this paper are utilized to study the influence of machine parameter on the optimal performance of the induction motor.

2. Performances at Constant Volts/Hz

The steady state performance of the induction motor is readily analysed by means

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of the fundamental equivalent circuit of Fig. 1. Although most static frequency converters generate an output voltage waveform with a significant harmonic content, it has been known that the presence of harmonics in the supply voltage usually has only a minor influence on the motor performance[3]. The present analysis, therefore, assumes balanced sinusoidal supply voltages. And, in order to specify the influences of the variation of the supply frequency, the motor equivalent circuit parameters are represented by inductances rather than conventional reactances.

For effective utilization, the air gap flux of the induction motor must be sustained

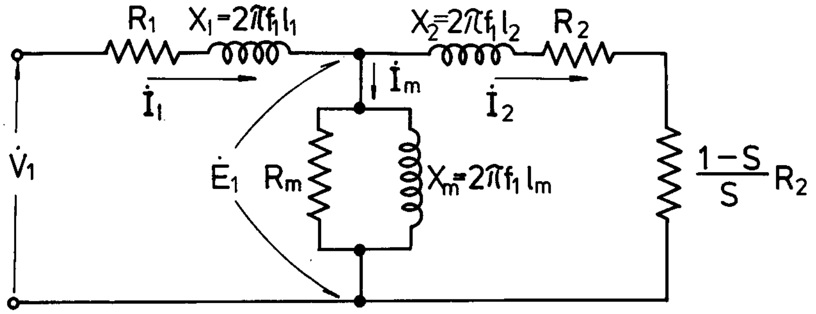

Fig. 1 Conventional equivalent circuit for one phase of induction motor.

with rated level at all frequencies. A constant air-gap flux is obtained when the ratio E_1/f_1 is constant, but if the stator leakage impedance is small, V_1 and E_1 are apploximately equal. Consequently, in a conventional drive system, the induction motor is driven under the constant V_1/f_1. Basic performances of the induction motor controlled by the constant V_1/f_1 and the constant E_1/f_1 are derived.

(1) Stator current

From the equivalent circuit of Fig. 1, the stator current is obtained in phasor formula as,

$$\dot{I}_1 = \left|\frac{\dot{V}_1}{f_1}\right| \frac{a+jb}{c+jd}, \tag{1}$$

where,

$$\left.\begin{aligned}
a &= l_{11}/l_m + (f_1/R_m)(R_2/f_2) \\
b &= 2\pi l\,(f_1/R_m) - (R_2/f_2)/2\pi l_m \\
c &= (R_1/f_1) - 2\pi l\left\{2\pi l\,(f_1/R_m) - (R_1/f_1)/2\pi l_m\right\} \\
&\quad + (R_2/f_2)(l_{11}/l_m + R_1/R_m) \\
d &= 2\pi l\left\{(l_{11}+l_m)/l_m + R_1/R_m\right\} \\
&\quad + (R_2/f_2)\left\{2\pi l\,(f_1/R_m) - (R_1/f_1)/2\pi l_m\right\} \\
l &= l_1 = l_2, \text{ and } l_{11} = l_1 + l_m.
\end{aligned}\right\} \tag{2}$$

By definition, the rotor frequency or slip frequency is expressed by,

$$f_2 = sf_1, \tag{3}$$

where f_1 is the variable stator frequency, and s is the corresponding rotor slip.

In those expressions it may be noticeable that the stator and the rotor side resistance components are represented by the ratios of the frequencies on each side respectively. Because, the resistance of the conductor varies with the function of frequency by the skin effect, and the equivalent core loss resistance is approximately inversely proportional to the supply frequency since the core losses almost consist of the hysteresis losses which are proportional to the frequency.

On the operation of constant E_1/f_1, stator current is given by,

$$\dot{I}_1' = \left|\frac{\dot{E}_1}{f_1}\right| \left\{ \frac{f_1}{R_m} + \frac{R_2/f_2}{(R_2/f_2)^2 + (2\pi l_2)^2} - j\left(\frac{1}{2\pi l_m} + \frac{2\pi l_2}{(R_2/f_2)^2 + (2\pi l_2)^2}\right)\right\}. \tag{4}$$

(2) Rotor current

The rotor current $\dot{I}_2$, based on the equivalent circuit of Fig. 1, is obtained as,

$$\dot{I}_2 = \left|\dot{V}_1/f_1\right| / (c + jd). \tag{5}$$

Similarly, for the constant E_1/f_1 operation, the rotor current is expressed by,

$$\dot{I}_2' = \left|\dot{E}_1/f_1\right| / (R_2/f_2 + j2\pi l_2). \tag{6}$$

(3) Output power

The output power of the m-phase induction motor is given as,

$$P_0 = m\left|\dot{I}_2\right|^2 R_2(1-s)/s = m\left|\dot{I}_2\right|^2 (R_2/f_2)(f_1 - f_2). \tag{7}$$

Thus, the output of the induction motor is proportional to the supply frequency when the motor is operated under the constant E_1/f_1 and the constant slip frequency.

(4) Torque

The torque of m-phase, p-pole pairs of the induction motor is derived by dividing Eq. (7) by motor angular speed. The resulting expression is,

$$T = \frac{P_0}{(2\pi f_1/p)(1-s)} = \frac{pm}{2\pi f_1}\left|\dot{I}_2\right|^2 \frac{R_2}{s} = \frac{pm}{2\pi}\left|\dot{I}_2\right|^2 \frac{R_2}{f_2}. \tag{8}$$

And for the constant E_1/f_1 operation, substituting Eq. (6) in Eq. (8), the torque is obtained as,

$$T' = \frac{pm}{2\pi}\left|\frac{\dot{E}_1}{f_1}\right|^2 \frac{f_2 R_2}{R_2^2 + (2\pi f_2 l_2)^2}. \tag{9}$$

Since the air-gap flux is proportional to E_1/f_1, the electromagnetic torque is proportional to the square of the air-gap flux at a given slip frequency f_2. Consequently, if the air-gap flux is maintained constant over all the operating conditions, the induction motor torque is determined solely by the absolute slip frequency f_2 and is independent of the supply frequency f_1.

(5) Power factor

The power factor of the induction motor is obtained from the stator current expression of Eq. (1) in which the supply voltage is selected as the reference phasor. Thus,

$$\cos\varphi = (\mathrm{Real}\,\dot{I}_1)\,/\,|\dot{I}_1| = \frac{ac + bd}{\sqrt{(a^2 + b^2)(c^2 + d^2)}} \,. \tag{10}$$

The power factors under the constant V_1/f_1 and E_1/f_1 operation are roughly equal.

(6) Efficiency

The efficiency of an induction motor is given by,

$$\eta = \left\{\frac{R_2/f_2(1 - f_2/f_1)}{ac + bd} - P_m \frac{f_1}{mV_1^2} \frac{c^2 + d^2}{ac + bd}\right\} \times 100\,(\%)\,, \tag{11}$$

where P_m is the mechanical losses of the induction motor. The efficiencies under the constant V_1/f_1 and E_1/f_1 operation are quite equal.

Fig. 2 shows the performance curves of the standard 3-phase induction motor. The

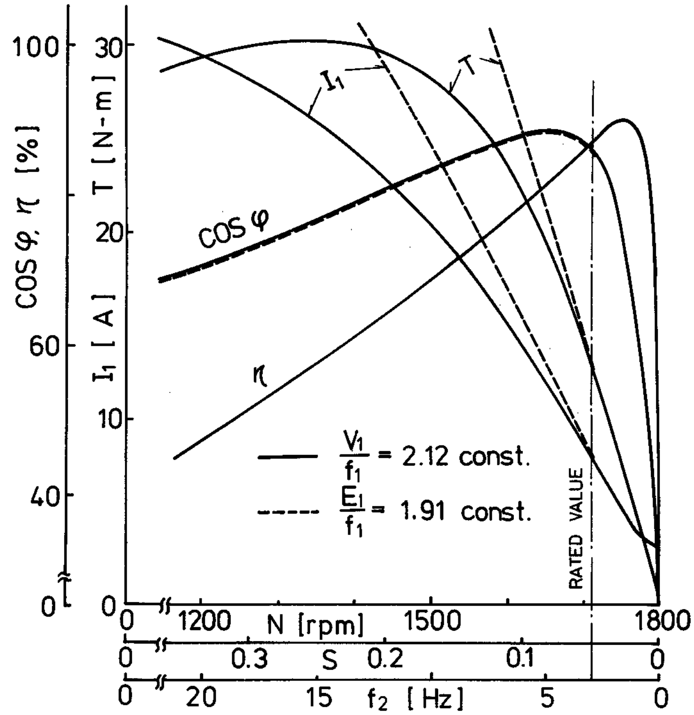

Fig. 2 Performance curves of standard poly-phase induction motor.

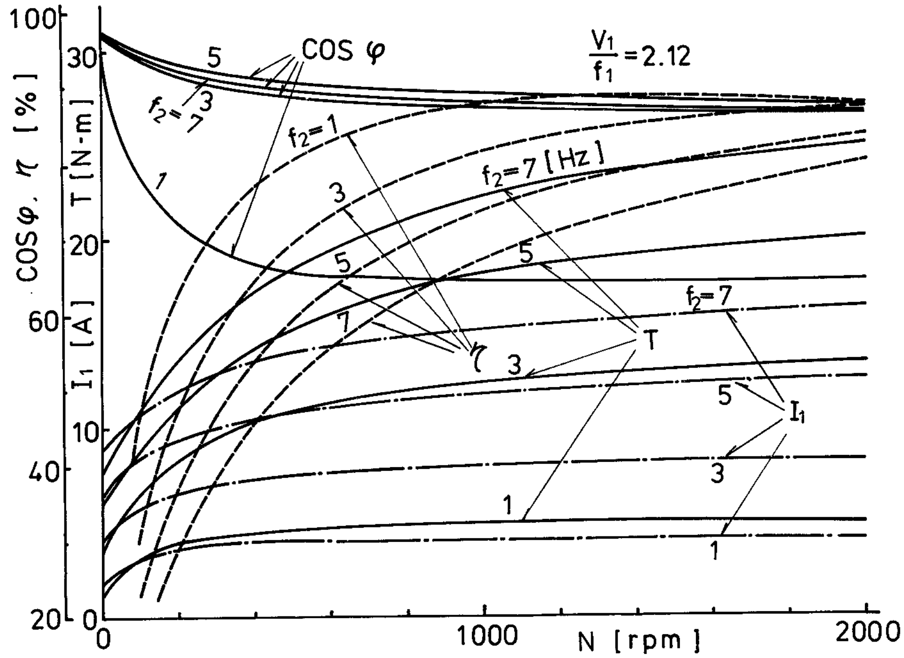

Fig. 3 Characteristic curves of stator current, torque, power factor, and efficiency with speed variation.

induction motor used in this work is 4-pole, 2.2 Kw, 220 V, 8.2 A, 60 Hz squirrel cage motor whose equivalent circuit parameters per phase are $R_1 = 0.895\Omega$, $R_2 = 0.775\Omega$, $X_1 = X_2 = 2\pi f_1 l = 0.0252 f_1 \Omega$, $X_m = 2\pi f_1 l_m = 0.641 f_1 \Omega$, $R_m = 4.62 \times 10^4 / f_1 \Omega$. The rated slip is 0.05, V_1/f_1 = 2.12 V/Hz, and E_1/f_1 = 1.91 V/Hz.

As shown in Fig. 2, in concerning with the standard induction motor, the maximum torque occurs at the considerably lower speed than the rated speed, and the power factor has its maximum value slightly below the rated speed. And the efficiency maintains high value when the motor is running in the region of 75 percent to the full load. The shapes of the characteristic curve near these maximum values have more sharp peakes in higher speed range. Therefore the selection of the slip frequency which yields the maximum value becomes severer with smaller slip operation of the induction motor.

Fig. 3 shows the variations of the stator current, torque, power factor, and efficiency of the induction motor whose supply frequency is adjusted by the frequency converter with rated V_1/f_1, and with various constant slip frequencies. In lower speed range, decrease in efficiency is remarkable while the power factor is slightly improved.

3. Optimal Slip Frequency

Many variable speed drives require a constant torque together with high efficiency and power factor. As discribed in the previous section, on the operation of constant E_1/f_1, the magnetic torque of the induction motor is determined solely by the slip frequency. Moreover, it may be possible to coincide the slip frequency, which yields the maximum efficiency, with one which produces the maximum power factor by the appropriate selection of the machine parameters.

(1) Optimal output power slip frequency

The optimal slip frequency f_{2mp} which yields the maximum output power can be calculated by solving the equation $dP_0/df_2=0$. Substituting Eq. (5) in Eq. (7), and differentiating Eq. (7), the optimal slip frequency is obtained as,

$$f_{2mp} = \frac{f_1}{1 \pm \sqrt{(1 + R_1/R_2)^2 + (2\pi f_1)^2 (2l_2/R_2)^2}} , \tag{12}$$

where the positive sign applies to the motor operation and negative sign corresponds to the generator operation.

And for the constant E_1/f_1 operation, using Eq. (6) and Eq. (7), the optimal frequency f'_{2mp} is derived as,

$$f'_{2mp} = \frac{f_1}{1 \pm \sqrt{1 + (2\pi f_1)^2 (l_2/R_2)^2}} . \tag{13}$$

(2) Breakdown slip frequency

Differentiating Eq. (8) with respect to slip frequency and equating to zero, yields the breakdown frequency f_{2mT} with constant V_1/f_1 operation as,

$$f_{2mT} = \pm \frac{f_1}{\sqrt{(R_1/R_2)^2 + (2\pi f_1)^2 (2l_2/R_2)^2}} . \tag{14}$$

On substituting Eq. (14) in Eq. (8), the breakdown torque T_m is given by,

$$T_m = \frac{pm}{2\pi} \left| \frac{\dot{V}_1}{f_1} \right|^2 \frac{1}{2\left\{R_1/f_1 + \sqrt{(R_1/f_1)^2 + (2\pi)^2 (2l_1)^2}\right\}} . \tag{15}$$

Similarly, applying Eq. (9), the breakdown slip frequency f'_{2mT} and the corresponding breakdown torque T'_m with respect to the constant E_1/f_1 operation are obtained as follows,

$$f'_{2mT} = \pm \frac{R_2}{2\pi l_2} , \tag{16}$$

and

$$T'_m = \frac{pm}{2\pi} \left| \frac{\dot{E}_1}{f_1} \right|^2 \frac{1}{4\pi l_2} . \tag{17}$$

By inspection of Eqs. (12) through (17), expressions for the constant E_1/f_1 are equivalent to those for the constant V_1/f_1 operation on which stator parameters are reduced to zero.

(3) Optimal power factor slip frequency

On the same procedure as above, the optimal slip frequency $f_{2m\varphi}$ which yields the maximum power factor is obtained by using Eq. (10). With respect to the constant V_1/f_1 operation,

$$f_{2m\varphi} = \frac{f_1}{2\pi f_1(2l_2/R_2)\left\{2\pi f_1 l_m/R_m \pm\sqrt{(l_{11}+l_2)/2l_2+(2\pi f_1 l_m/R_m)^2}\right\}-R_1/R_2}, \quad (18)$$

and to the constant E_1/f_1 operation,

$$f'_{2m\varphi} = \frac{f_1}{2\pi f_1(2l_2/R_2)\{2\pi f_1 l_m/R_m \pm\sqrt{(2\pi f_1 l_m/R_m - R_1/4\pi f_1 l_1)^2+(l_{11}+l_m)/4l_1}\}-R_1/R_2}. \quad (19)$$

As mentioned in the previous section, Eq. (18) and Eq. (19) are numerically almost equal.

(4) Optimal efficiency slip frequency

The optimal slip frequencies which produce the maximum efficiency with constant V_1/f_1 and E_1/f_1 operation are quite equal. Differentiating Eq. (11) by slip frequency, and equating to zero, the optimal slip frequency is given by,

$$f_{2m\eta} = f'_{2m\eta} = \frac{f_1}{1\pm\sqrt{1+\left\{1+(R_1/R_2)(l_{11}/l_m)^2+(2\pi f_1)^2(R_2/R_m)(l_2/R_2)^2\right\}/\left\{R_2/R_m+(1/2\pi f_1)^2(R_1/R_2)(R_2/l_m)^2\right\}}}. \quad (20)$$

As shown in above equations, optimal slip frequencies are represented with the ratios of machine parameters. The ratios including the rotor resistance R_2 that is comparatively easy to modulate at the motor design process, have significant influence on both the optimal slip frequency and the motor performance. Therefore, on the selection of the optimal machine parameters, it may be reasonable and efficient to identify the parameter ratios which consist of the rotor resistance.

Fig.4 shows the variations of optimal slip frequencies with supply frequency under the constant E_1/f_1 operation. In this figure, only the rotor resistance is reduced. The optimal slip frequencies have smaller values with reduced rotor resistance, and the optimal power factor slip frequency approaches to the optimal efficiency slip frequency. The influences on the optimal slip frequencies of the ratios R_1/R_2 and l_2/R_2 are illustrated in Fig.5. In this case, the leakage inductance is selected so as to maintain the ratio l_2/R_2 to be constant with change of rotor resistance. This procedure result in the independence of the optimal slip frequencies of the torque and output power from the values of the rotor resistance. These figures suggest the presence of machine parameters which enable the efficient wide spread speed operation of an induction motor with optimal power factor and with constant load torque.

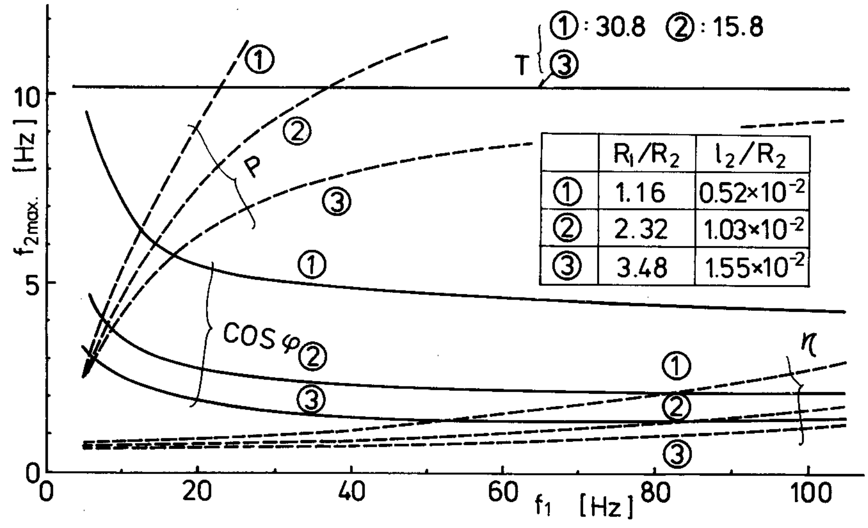

Fig. 4 Variations of optimal slip frequency with supply frequency (I).

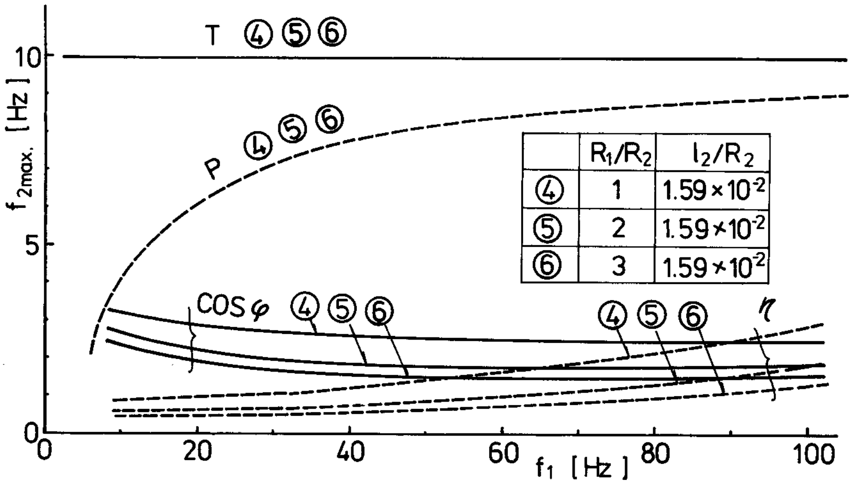

Fig. 5 Variations of optimal slip frequency with supply frequency (II).

Performance curves of induction motors with the machine parameters corresponding to ① and ③ in Fig.4, and to ⑤ and ⑥ in Fig.5 are illustrated together in Fig.6. Characteristics of the standard induction motor are drawn in solid lines. The slip frequencies are adopted to produce the same torque as the rated value and to maintain the rated stator current of the standard induction motor.

Fig.6 demonstrates that the efficiencies of the induction motors with lower rotor resistance and with slightly more rotor leakage inductance, that is better suited for the voltage type inverter drive[4], are improved without sacrificing the power factor. The increase in efficiency is comparatively small in the higher speed region but it is remarkable in the lower speed range. By the same strategies optimal machine parameters of the induction motor for any speed range and load characteristic may be adopted.

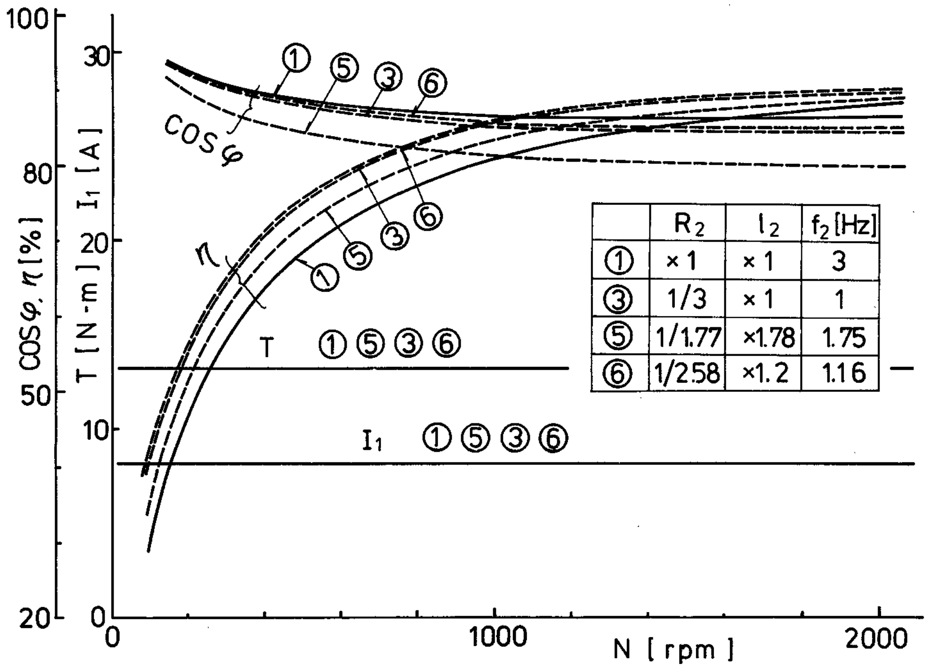

Fig. 6 Performance curves for several machine parameters.

4. Conclusions

The resultant expressions of the optimal slip frequencies are given by the ratios of machine parameters. The ratio of the rotor leakage inductance by the rotor resistance has considerable influences on the slip frequencies as well as the ratio produces important functions on the vector control and on the transient performance of the induction motor. Therefore, attention is largely concentrated on the rotor side parameters.

The improvement in efficiency, for example, is more or less 10 percent with the 50 percent rotor resistance and 120 percent leakage inductance of the standard motor. And the increase in efficiency is comparatively remarkable in the lower speed range.

The present analysis is carried out on the assumption of the balanced sinusoidal supply voltages, and on neglecting the parameter variations with the temperature, frequency, and current amplitude. The influences of the presence of harmonics in the supply voltage and the parameter variations on the optimal slip frequencies will be presented in another paper.

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