Application of Stochastic Catastrophe Model to Tactile Organic Functions

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This paper describes the effects of localized vibration, which is one of the most important factors of measurement environment, on the discrimination of surface roughness by means of tactile sense. When the difference of surface roughness is judged through tactile sense of finger tip exposed to localized vibration, the judgement gets dispersive as the increase of acceleration amplitude. For the purpose of analysing this phenomenon, a stochastic catastrophs model is applied. As a result, it is confirmed that such a phenomenon can be explained qualitatively by this cusp catastrophe model.

1. Introduction

It has been widely done to judge the surface roughness on the finished surface after machining workpiece through tactile sense. In this case, there arises a problem that the judgement of the surface roughness gets dispersive due to the effects of measurement environment, mental state of inspectors, individual variations, and so on. There are little studies concerning the effects of the measurement environment such as illumination, temperature, vibration and noise on tactile sense.

This paper describes the effects of lovalized vibration on the ability of discriminating the difference of surface roughness through tactile sense by means of a stochastic catastrophe model[1,2].

2. Experiments

Two sheets of sandpapers with different roughness are combined as shown in Table 1, and these five kinds of combinations are used as experimental specimens. These sandpapers fit Japanese Industrial Standard (JIS).

Firstly, the interval scale value of these 5 kinds of specimens(A,B,C,D,E) is decided by dint of the method of paired comparison devised by Sceffe[3]. Subjects are 20 men and 20 women before and after 20 years old. The interval scale of specimen A with the least difference of roughness is fixed to 0, and the interval scale in which the specimen A corresponds to the criterion is obtained.

Secondly, the effects of localized vibration on tactile sense is investigated. Experimental specimens are fastened on the oscillator. With the localized vibration exposed to the forefinger tip, subject touches the experimental specimens by turns. The response to the difference for the surface roughness is evaluated by means of numerical values from 1 to 50. The frequency is fixed to 50 Hz, and the localized vibration shown in Table 2 is used. The acceleration amplitude is represented with rms

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Table 1. Interval scale for the difference of surface roughness

specimen	combination of sandpapers	interval scale value	interval scale
A	No.600–No.320	-1.4	0
B	No.320–No.240	-0.79	0.62
C	No.320–No.150	0.13	1.53
D	No.320–No.120	0.65	2.04
E	No.600–No.120	1.42	2.82

(root mean square) value. The localized vibration mentioned above is exposed at random to one subject, and each of the experimental specimens(A,B,C,D,E) is evaluated 5 times. Subjects are 6 men and 4 women before and after 20 years old. Moreover, these subjects are examined by YG characteristic test.

Table 2. Localized vibration

frequency [Hz]	acceleration amplitude [m/s^2]				
	0	1	2	3	4
50	5	6	7	8	9
	10	11	13	15	

3. Experimental results

As there are 5 evaluation values judged by one subject for each of the experimental specimens, geometric mean is calculated for each experimental specimen. And, the ratios of the geometric mean of another experimental specimen to that of the specimen A are calculated. As shown in Fig. 1, there is a very strong correlation between this ratio and the interval scale of Table 1. When the acceleration amplitude is 0 m/s^2, there are 5 geometric means for each specimen. These 5 geometric means for each specimen are averaged, and the ratios of the mean value of another specimen to that of the specimen A are calculated. In this way, the slope of straight line for 0 m/s^2 is obtained. By making use of this relation, the judgement for the difference of roughness will be evaluated. The ratio of the slope for each acceleration amplitude to that for 0 m/s^2 is considered as a criterion in case of judging the difference of roughness through tactile sense of finger tip exposed to localized vibration. This is called evaluation ratio here. In other words, this represents the ratio of the ability of the differentiation of finger tip exposed to localized vibration to that not exposed to localized vibration. The evaluation ratio of 0 m/s^2 is the ratio of the slope for each acceleration amplitude 0 m/s^2 to the mean value of slope of 5 acceleration amplitudes. From the data of 10

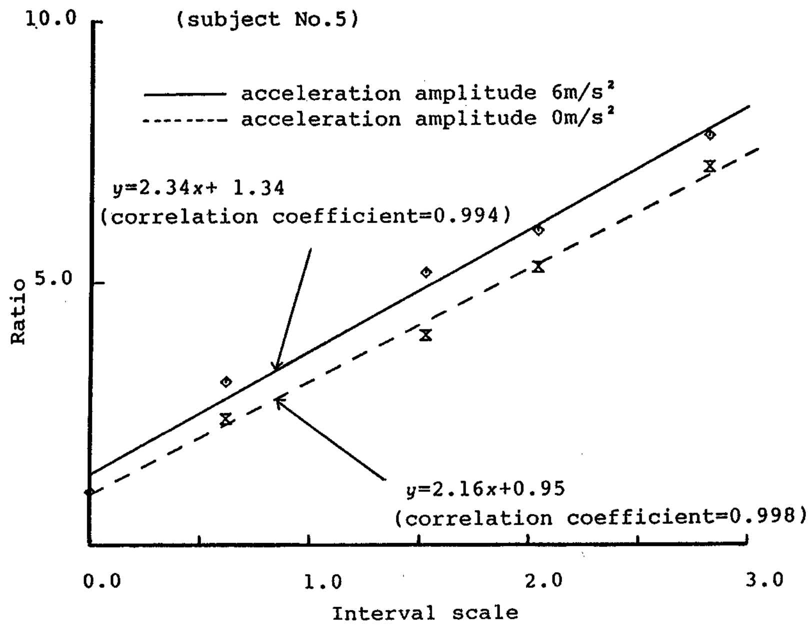

Fig. 1 Relation between interval scale and ratio

subjects, the evaluation ratio is calculated for each acceleration amplitude. The results are shown in the histogram from Fig. 2 to Fig. 7.

The results of YG characteristic test are shown in Table 3. The marks of 4 items (depressive state, change of mood, inferiority complex and nervousness), the total of these marks (i. e. the marks of emotional unstability) and the mean and variance of evaluation ratio are calculated for each subject. The maximum of these 4 marks is 100. The higher these marks are, the more remarkably the phenomena as mentioned above appear.

Table 3. Results of YG characteristic test and mean value and variance of evaluation ratio

subject	1	2	3	4	5	6	7	8	9	10
depressive state	18	20	6	2	81	9	30	96	97	17
change of mood	49	20	2	37	91	20	50	99	34	5
inferiority complex	24	57	4	12	67	19	62	95	78	66
nervousness	28	80	3	28	96	58	85	100	85	65
total (emotinal unstability)	119	177	15	79	335	106	227	390	294	153
mean value of evaluation ratio	0.67	0.67	0.62	0.86	1.17	0.62	1.00	0.85	0.77	1.00
variance of evaluation ratio	0.08	0.03	0.08	0.10	0.09	0.11	0.47	0.09	0.23	0.17

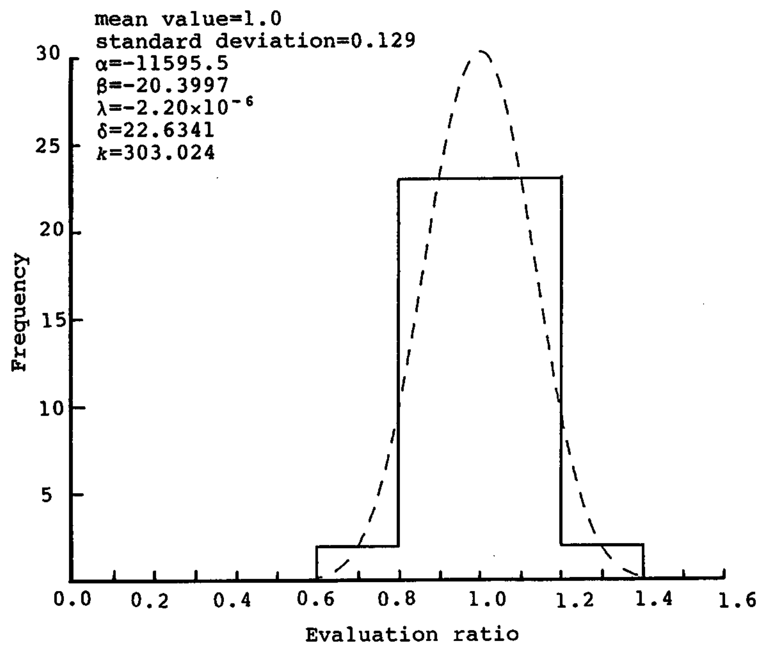

Fig. 2 Histogram of evaluation ratio and its estimated curve (0m/s^2)

4. Considerations

The effects of localized vibration on tactile sense of finger tip are considered by means of the eveluation ratio. When the evaluation ratio is large, it means that the difference of roughness is discriminated to be more than in case of not exposed to localized vibration. When the evaluation ratio is small, it means that the difference of roughness is discriminated to be less than in case of not exposed to localized vibration. When the evaluation ratio is nearly equal to 1.0, the judgement for the difference of roughness is regarded to be the same with the case when not undergoing localized vibration.

In case of 0 m/s^2 (Fig. 2), the evaluation ratios are distributed within a narrow range near 1.0. This means that the difference of roughness is normally judged. In case of 1 m/s^2 (Fig. 3) and 3 m/s^2 (Fig. 4), the evaluation ratios begin to be distributed within a wide range. In case of 7 m/s^2 (Fig. 5), 11 m/s^2 (Fig.6) and 13 m/s^2 (Fig. 7), the frequency distribution of the evaluation ratio spreads from near 1.0 to both sides. Namely, the frequency distribution of the evaluation ratio has bimodality. This shows that the difference of roughness is overevaluated or underevaluated as the increase of acceleration amplitude. In other words, the evaluation for the difference of roughness changes discontinuously between the overevaluation and the underevaluation, and the difference of roughness is not normally judged. In this way, the localized vibration affects the ability of discriminating the difference of roughness through tactile sense of finger tip.

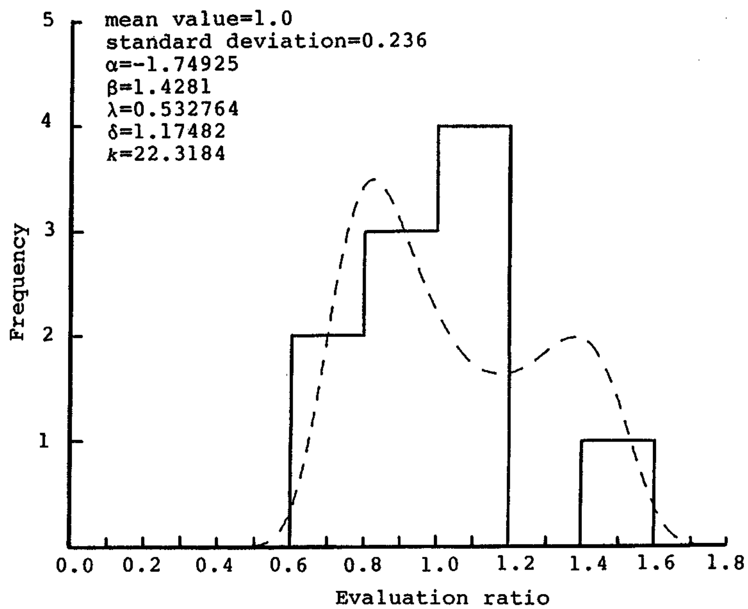

Fig. 3 Histogram of evaluation ratio and its estimated curve ($1m/s^2$)

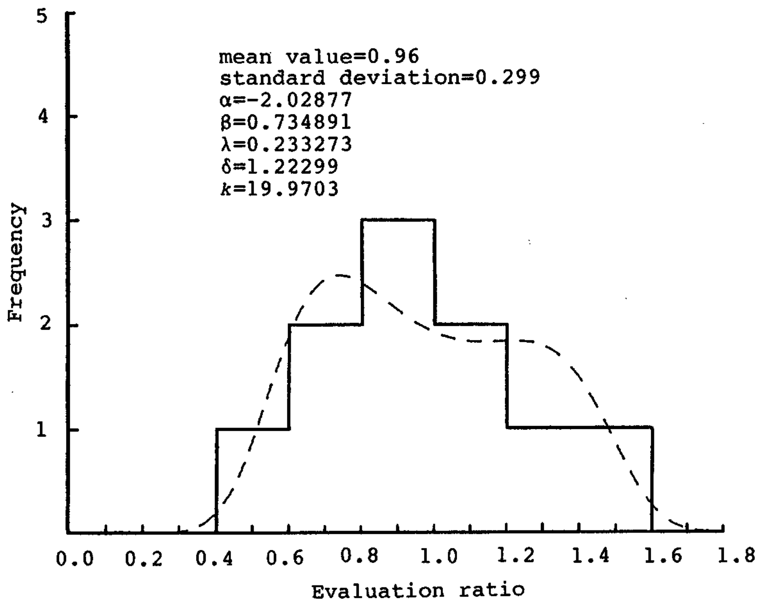

Fig. 4 Histogram of evaluation ratio and its estimated curve ($3m/s^2$)

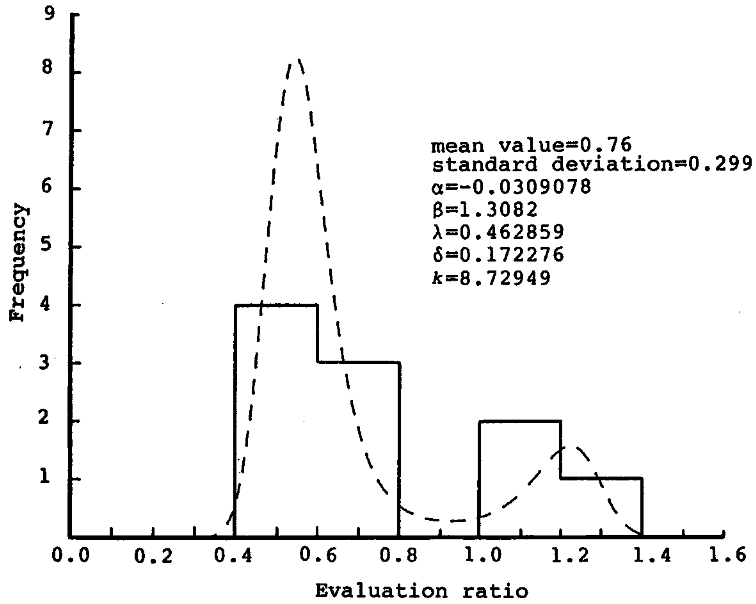

Fig. 5 Histogram of evaluation ratio and its estimated curve (7m/s^2)

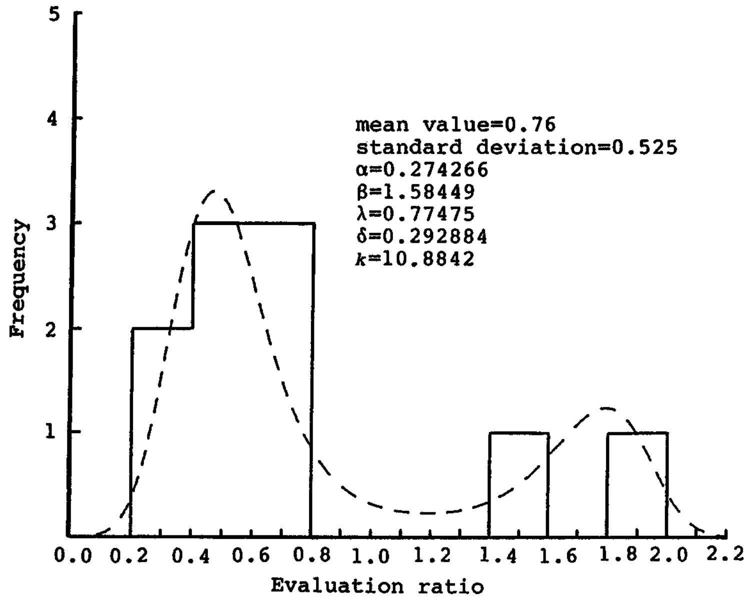

Fig. 6 Histogram of evaluation ratio and its estimated curve (11m/s^2)

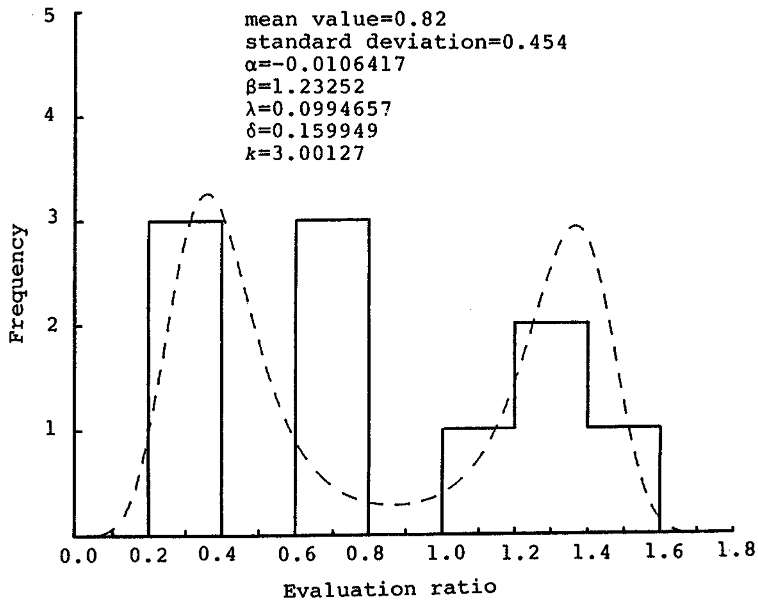

Fig. 7 Histogram of evaluation ratio and its estimated curve (13m/s^2)

The judgement of surface roughness through tactile sense is considered to be composed of the following process. The measurement object is observed through tactile sense of finger tip. The information obtained in such a way is dealt with in the cerebral center, and the surface roughness is judged. The statistical model, which pays attention to the statistical tendency of data, is taken up for the purpose of marking this mechanism clear. In this study, the difference of roughness is regarded to be judged through the process of Fig. 8. The experiment in this study relates to the characteristics of organic functions, and the data obtained is dispersive. The phenomenon that the judgement for the difference of roughness changes discontinuously and gets dispersive is analysed by means of the stochastic catastrophe model[1,2].

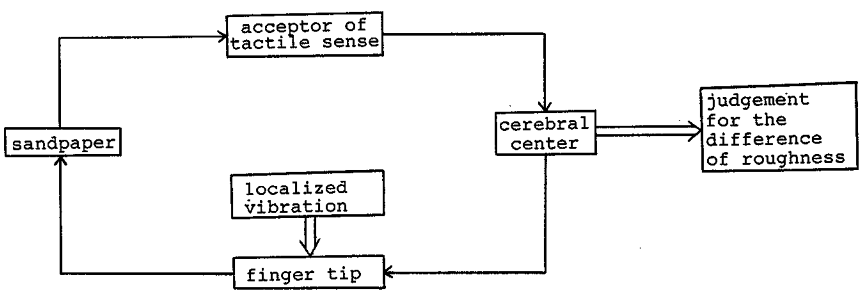

Fig. 8 Process of the judgement for the difference of roughness through tactile sense of finger tip exposed to localized vibration

The stochastic catastrophe model introduces probabilistic concepts to the deterministic catastrophe model and is capable of dealing with discontinuous phenomena involving randomness. In this model, the following cusp probability density function is essential.

$$f(x) = k\cdot\exp\left[\left\{\alpha(x-\lambda)+\frac{\beta}{2}(x-\lambda)^2-\frac{1}{4}(x-\lambda)^4\right\}/\delta\right] \quad (1)$$

x : random state variable

α : asymmetric parameter

β : bifurcation parameter

λ : scale parameter

δ : dispersion parameter

The constant k is chosen so that $f(x)$ satisfies the condition of probability density function. The parameters α, β are of special importance and correspond to normal factor and splitting factor of catastrophe model, respectively. In this paper, the evaluation ratio corresponds to the random state variable x. If the value of the following D is negative, the p.d.f $f(x)$ has bimodality.

$$D = 27\alpha^2 - 4\beta^3 \quad (2)$$

This corresponds to the bifurcation set of cusp catastrophe model. When the value of D is negative, the state variable x exists within the bifurcation set, and it is probable that x discontinuously changes. The p.d.f $f(x)$ has unimodality, if the value of D is more than 0. Based on the frequency distributions of the evaluation ratio, the dashed curves from Fir. 2 to Fig. 7 are estimated by means of the method of moment. In this method, the parameters of (1) are estimated as follows. The sample moments of the same number with the parameters estimated are taken up. The equations obtained by equalizing the sample moments and the population moments are solved with the parameters.

In case of 0 m/s^2 (Fig. 2), $f(x)$ is the unimodal function. In case of 1 m/s^2 (Fig. 3) and 3 m/s^2 (Fig. 4), $f(x)$ begins to change from the unimodal function to the bimodal function. In case of 7 m/s^2 (Fig. 5), 11 m/s^2 (Fig. 6) and 13 m/s^2 (Fig. 7), $f(x)$ is bimodal. The values of 4 parameters in (1) and D for each acceleration amplitude are shown in Table 4. Within the range of acceleration amplitude less than 5 m/s^2 (except for 2 m/s^2), the value of D is positive, and $f(x)$ begins to change from the unimodal function to the bimodal function. Above 4 m/s^2, the value of D is negative, and $f(x)$ has bimodality. In other words, the acceleration amplitude of localized vibration more than 4 m/s^2 affects the judgement for the difference of roughness. Within the range of this experiment, the judgement for the difference of roughness at the acceleration amplitude more than 4 m/s^2 is regarded to be inappropriate as work environment.

Table 4. Value of α, β, λ, δ and D

Acceleration amplitude (m/s^2)	α	β	λ	δ	D
0	-11595.5	-20.3997	-2.20×10^{-6}	22.6341	3.63×10^{9}
1	-1.74925	1.4281	0.532764	1.17482	70.96637
2	-0.018403	1.0555	-0.104769	0.463068	-3.78922
3	-2.02877	0.734891	0.233273	1.22299	109.54195
4	-0.346743	0.862165	0.130161	0.647044	0.68274
5	0.420504	1.97534	0.647995	0.198103	-26.05662
6	0.138371	1.2859	0.0402729	0.348114	-7.98819
7	-0.0309078	1.3082	0.462859	0.172276	-6.37606
8	1.31306	2.42045	0.818551	0.171193	-10.17017
9	-0.474931	1.133256	0.418728	0.659655	-3.37489
10	0.300129	1.92532	0.689862	0.547351	-26.11545
11	0.274266	1.58449	0.74475	0.292884	-13.88115
13	-0.0106417	1.23252	0.0994657	0.15994	-7.18355

Applying the stochastic cusp catastrophe model to the problem that the evaluation for the difference of roughness gets dispersive as the increase of acceleration amplitude, it is pointed out that the p.d.f $f(x)$ of the evaluation ratio has bimodality. From these discussions, it follows that such a phenomenon can be qualitatively explained by the cusp catastrophe manifold shown in Fig. 9. The acceleration amplitude and the emotional unstability of subjects are taken up as the factors affecting the judgement for the difference of roughness. Similarly, it is pointed out that the evaluation gets dispersive in case of relatively comparing and judging through organic functions under a stress(alcohl, noise, etc.)[4]. In order to consider such a problem, the approach of this study will be effective.

By the way, the emotional unstability is regarded to be related to the selectivity[4], which is one of the psychological factors. When the stimulus(localized vibration) is transmitted to the cerebral center, the selectivity functions, and the information for the judgement of the difference of roughness is restrained. Thus, the difference of roughness must be evaluated with less information. In other words, as the increase of the acceleration amplitude of localized vibration, the selectivity functions, and the evaluation of the difference of roughness gets unstable. The following matters are shown from Table 3. The subject 1, 2, 3, 6 are emotinally stable in moderation, and the mean values of evaluation ratio for them are low. The subject 5, 7, 8, 9 are emotionally unstable, and the mean values of evaluation ratio for them are high. However, the mean values of evaluation ratio for the subject 4, 10 are high in spite of their emotional stability. In the range of this study, the relation between the dispersion of evaluation ratio and the emotional unstability of subjects is not clarified. Comparing Table 5 and Table 6, it is seen for every subject that the judgement of the difference of roughness gets unstable when undergoing localized vibration. From Table 6, it is clear that the state of the dispersion of evaluation ratio differs between subjects. This is probably because the selectivity functions differently between subjects. Based on the considerations concerning the relation between the selectivity and the emotional unstability[4), the emotional unstability has been taken up as a factor here. The relation between the selectivity and the emotional unstability must be discussed in more detail.

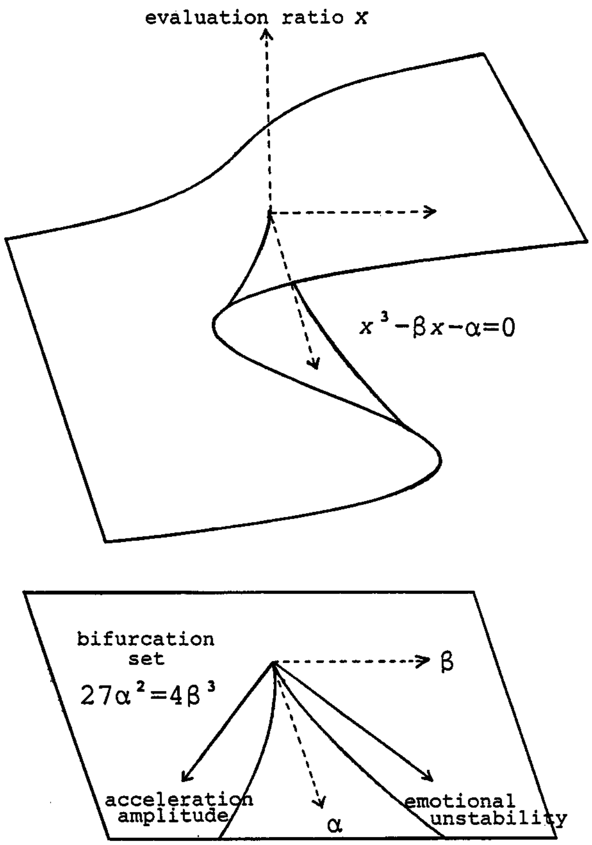

Fig. 9 Cusp catastrophe manifold

Table 5. Distribution of evaluation ratio (0 m/s^2)

evaluation ratio \ subject	1	2	3	4	5	6	7	8	9	10
0.4 – 0.6										
0.6 – 0.8		1				1				
0.8 – 1.0	3	2	3	1	2	1	4	2	3	2
1.0 – 1.2	2	2	2	4	3	3	1	2	2	3
1.2 – 1.4								1		

Table 6. Distribution of evaluation ratio (1 ~ 15m/s^2)

evaluation ratio \ subject	1	2	3	4	5	6	7	8	9	10
0.2 – 0.4	3		2			4			2	
0.4 – 0.6	4	5	3	3		3		3	4	1
0.6 – 0.8	2	6	4	5	1	4	4	5	2	4
0.8 – 1.0	2	1	2	1	3			1	3	3
1.0 – 1.2	1	1	2	2	4	1	7	2		
1.2 – 1.4	1			1	1		2	2		3
1.4 – 1.6				1	4	1			1	
1.6 – 1.8										1
1.8 – 2.0									1	1

5. Conclusions

The effects of localized vibration on the ability of discriminating the difference of surface roughness through tactile sense of finger tip has been considered. The conclusions can be summarized as follows.

1 : The localized vibration of the acceleration amplitude more than 4 m/s^2 affects the tactile sense of finger tip, and the judgement of the difference of roughness gets dispersive.

2 : Analysing such a problem by means of the stochastic cusp catastrophe model, it is clarified that the unstability of the judgement of the difference of roughness can be grasped by the probability density function for the evaluation ratio.

3 : Such a phenomenon can be qualitatively modeled by the cusp catastrophe model, in which the emotional unstability of subjects and the acceleration amplitude correspond to the conflicting factors.

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