Induction Motor Parameters for Efficient Variable Speed Operation with Stator Voltage Control

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Energy efficiency improvement for variable speed operation of induction motors by adjusting the machine parameters can be effectively examined under some appropriate assumptions. With respect to a three phase induction motor, the optimum rotor resistance, which yields the maximum energy efficiency with a given speed range and load characteristic, and to a single phase condenser motor, the optimum condenser are obtained respectively. The results evaluated by the proposed method are verified experimentally with measurements on the actual induction motors.

1. Introduction

A cage rotor induction motor is inherently inflexible in speed when operated on a constant voltage and frequency a.c. supply. And the standard induction motor parameters are designed so as to produce maximum efficiency on the load less than the rated value.

Nowadays, power electronics technics have been developed and consequently the variable speed induction motor drive is being increasingly applied in industrial and domestic loads. Efficient wide range speed control of the cage rotor induction motor is possible when a variable voltage and frequency a.c. supply is available. While, a certain degree of speed variation can be achieved by reduction of the stator voltage that can be controlled easily by means of the series connected thyristors in each stator phase. Because of the simple circuit construction, the stator voltage control is widely applied for a fractional horsepower motor. However, on this speed control method, the energy efficiency of the motor is very much sacrificed.

Some papers are represented on the improvement in efficiency of induction motors[1),2)], but they are restricted to the energy savings in light load conditions. This paper deals with the optimum induction motor parameters, which yield the maximum energy efficiency, for energy efficient wide range variable speed operation with stator voltage control. Although improvements in energy efficiency achieved by the proposed optimum parameters are more or less 10 percent, since the fractional horsepower induction motors constitute a significant percentage of total electric motor capacity on use, very substantial energy savings may be possible.

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2. Basic Concept of Energy Efficiency

The induction motor speed control method by variation of the stator voltage is essentially a slip control process. If ω_o denotes the synchronous angular speed and ω is the rotational angular speed, the fractional slip is defined by $s = (\omega_0 - \omega) / \omega_0$. The gross mechanicall power output $P_0 = \tau\omega$, where τ is the internal motor torque. The total power input to the rotor, across the air gap from the stator, is $\tau\omega_0$. The difference between the power input to the rotor and the mechanical power output is dissipated as heat losses in the rotor resistance, thus the rotor copper loss is given by $s\tau\omega_0$.

Speed control using an adjustable stator voltage is inherently inefficient, since the rotor copper loss increases linearly with slip for a constant motor torque. Therefore, on the operation of wide spread speed regulation, it may be appropriate to estimate the motor efficiency or ratings by the energy efficiency that is the fraction of the total electrical energy input to the motor and the total mechanical energy output of the motor shaft.

Fig. 1 shows a schematic operating pattern of a motor duty. When a motor operates in the speed range between ω_1 to ω_2, the output energy of the motor in a given duty cycle is obtained by evaluating the product of torque, angular speed, and time for each of the duty cycle. That is,

$$W_0 = \int_{w_1}^{w_2} \tau(w) \cdot T(w) dw \qquad (1)$$

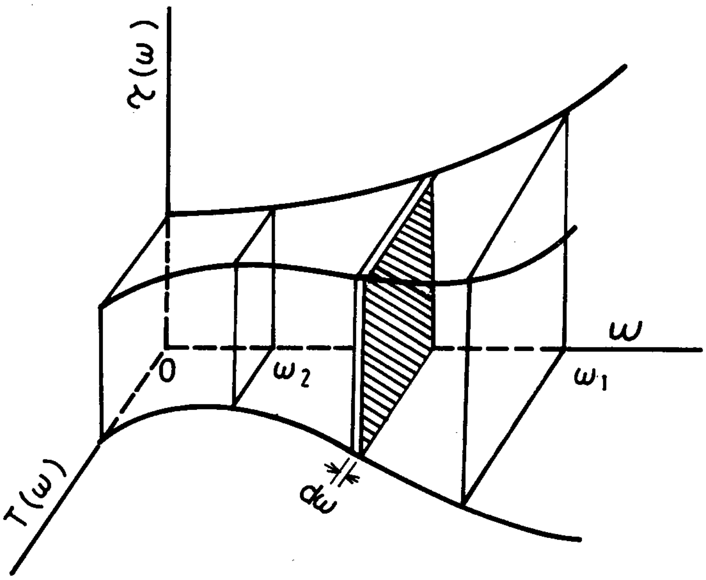

Fig. 1 Schematic operating pattern.

Also, the input energy to the motor in the same duty cycle is given by

$$W_i = \int_{w_1}^{w_2} \{ \tau(w) \cdot T(w) / \eta(w) \} dw \tag{2}$$

where, $\eta(\omega)$ is the motor efficiency at speed ω.
Energy efficiency, corresponds to the speed range from ω_1 to ω_2, is defined as

$$[\eta]_{w_1}^{w_2} = W_0 / W_i \times 100 \qquad (\%) \tag{3}$$

Eqs. (1) and (2) include motor torque that is a function of the speed. Thus, the energy efficiency is influenced significantly by the speed vs. torque characteristics of the load. Torque characteristic of a load is generally given as a function of speed, and expressed by

$$\tau_L(w) = \tau_N (w / w_N)^m \tag{4}$$

where, τ_N is rated torque and ω_N is rated speed of a motor respectively, and an exponent m denotes as follows : $m = -1$; constant power load, $m = 0$; constant torque load, $m = 1$; speed proportional load, $m = 2$; fan or pump load, and so on.

3. Optimum Machine Parameters

On the adjustable speed operation of induction motors with stator voltage control, when the speed range of operation and the load characteristic are given, energy efficiency may be evaluated by means of Eqs. (1), (2), (3), and (4). These equations include induction motor parameters, therefore, there might be a optimum machine parameter which yields maximum energy efficiency for any duty cycle of the induction motor.

3.1 Three phase induction motor

3.1.1 Basic equations

The nature of the energy efficiency improvement which is attainable can be examined by considering an idealized situation where motor parameter variations caused by saturation and temperature rise are neglected, and voltage controller is assumed to produce sinusoidal excitation at all voltage levels. Under these assumptions the conventional induction machine equivalent circuit of Fig. 2 is applied and basic motor equations are derived. The internal torque of a three phase induction motor is expressed as

$$\tau \simeq \frac{3 r_2}{w_0} \frac{V_1^2}{s\left[\left\{r_1 + \frac{r_2^2}{s}\left(1 + \frac{x_1}{x_m}\right) + \frac{r_1 x_2}{x_m}\right\}^2 + \left\{x_1 + x_2 + \frac{1}{x_m}\left(x_1 x_2 - \frac{r_1 r_2}{s}\right)\right\}^2\right]} , \tag{5}$$

and the motor efficiency is given by

$$\eta = \frac{s(1-s) r_2}{s^2\left\{r_1\left(1+\frac{x_2}{x_m}\right)^2 + \frac{x_2^2}{r_m}\left(1+\frac{r_1}{r_m}\right)\right\} + s r_2\left(1+\frac{2 r_1}{r_m}\right) + \frac{r_1 r_2^2}{x_m^2} + \frac{r_2^2}{r_m}\left(1+\frac{r_1}{r_m}\right)} . \tag{6}$$

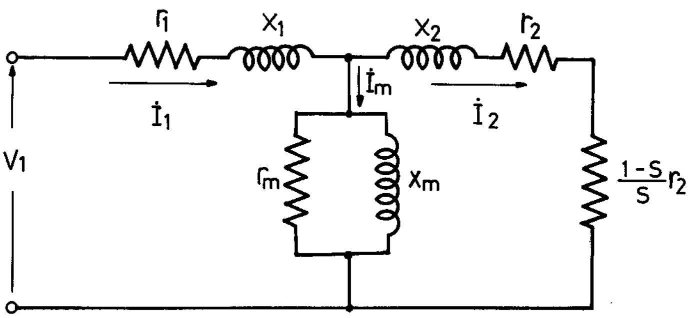

Fig. 2 Conventional equivalent circuit for one phase of induction motor.

For simplicity, assume the operating time for each speed in the duty cycle to be equal. On combining Eqs. (1), (2), (4), (5), and (6), the gross input energy and the output energy are expressed in terms of motor parameters and slips. The resulting expression for output energy is

$$W_0 = T\frac{\tau_N\, w_0}{(1-s_N)^m}\frac{1}{m+1}\{(1-s_1)^{m+1}-(1-s_2)^{m+1}\}\ , \tag{7}$$

also, input energy is

$$W_i = T\frac{\tau_N\, w_0}{(1-s_N)^m}\frac{1}{r_2}\int_{S_1}^{S_2} -(1-s)^{m-1}\Big[s\Big\{r_1\Big(1+\frac{x_2}{x_m}\Big)^2+\frac{x_2^2}{r_m}\Big(1+\frac{r_1}{r_m}\Big)\Big\} + r_2\Big(1+\frac{2r_1}{r_m}\Big)+\frac{r_2^2}{s}\Big\{\frac{r_1}{x_m^2}+\frac{1}{r_m}\Big(1+\frac{r_1}{r_m}\Big)\Big\}\Big]ds \tag{8}$$

where, T is total operating time, s_1 and s_2 are slips correspond to the upper speed ω_1 and lower speed ω_2 of the operating range respectively, and s_N is the rated slip. (Hereafter speed may be expressed as corresponding slip).

Substituting Eqs. (7) and (8) in the earlier energy efficiency expression Eq. (3), the following energy efficiency equation is obtained,

$$[\eta]_{S_1}^{S_2} = \frac{r_2\{(1-s_1)^{m+1}-(1-s_2)^{m+1}\}}{(m+1)\int_{S_1}^{S_2}-(1-s)^{m-1}\Big[s\Big\{r_1\Big(1+\frac{x_2}{x_m}\Big)^2+\frac{x_2^2}{r_m}\Big(1+\frac{r_1}{r_m}\Big)\Big\}+r_2\Big(1+\frac{2r_1}{r_m}\Big) + \frac{r_2^2}{s}\Big\{\frac{r_1}{x_m^2}+\frac{1}{r_m}\Big(1+\frac{r_1}{r_m}\Big)\Big\}\Big]ds} \tag{9}$$

On the assumption that the magnetizing circuit parameters r_m and x_m are two or more order larger than the other ones, and $r_m \gg x_m$, the relationship between break down slip s_T, which yields the maximum torque, and the motor parameters is expressed as

$$x_2 = \frac{1}{2}\Big(\frac{r_2}{s_T}\Big)^2 - r_1^2 \tag{10}$$

Generally, an induction motor operates stably in the region of $s_T > s \geq 0$, then the break down slip s_T defines the lower limit of a speed range on the fixed frequency operation.

3.1.2 Optimum rotor resistance

Since the characteristics of a three phase induction motor are more influenced by the rotor resistance than the other motor parameters, the optimum rotor resistance that maximize the energy efficiency, expressed by Eq. (9), is investigated in this section.

In order to generalized the following conclusions, motor parameters are represented in the normalized form that are given by the ratios of the magnetizing reactance x_m, e.g. r_1/x_m, x_1/x_m, and etc. Physically, r_1/x_m gives the approximate firures of a motor size, and x_1/x_m denotes the leakage factor.

On the operation of the upper speed $s_1 = 0.05$ and the lower speed $s_2 = s_T = 1$, the influences of a rotor resistance on energy efficiency are illustrated in Fig. 3, which is evaluated by Eq. (9). Referring to Fig. 3, the following interesting informations are obtained :

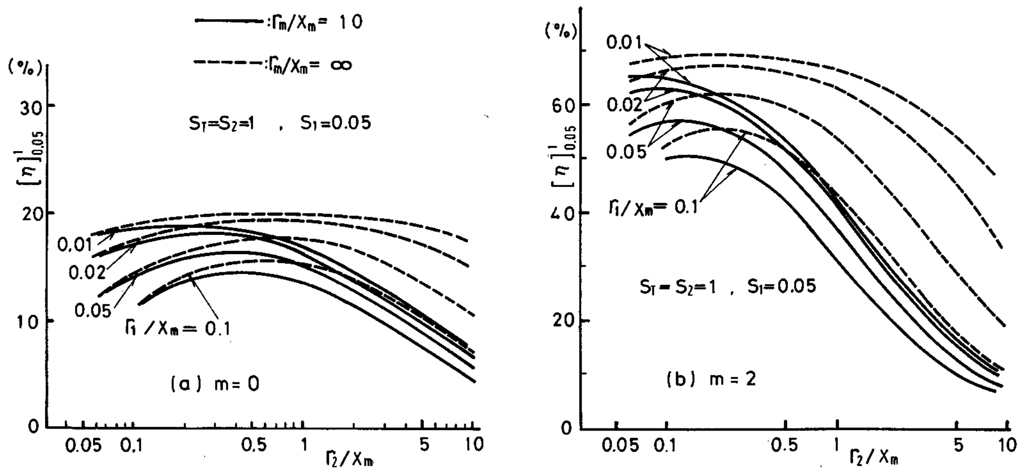

Fig.3 Variation in energy efficiency with rotor resistance.

(1). there is an optimum rotor resistance which yields maximum energy efficiency,
(2). the optimum rotor resistance is almost constant regardless of the motor size when the speed range and the load characteristics are given, and core losses are neglected,
(3). the energy efficiency with large number of r_1/x_m is more influenced by the rotor resistance,
(4). and the energy efficiency is considerably related to the charactenstics of the load.

In Fig. 4, the relations between optimum rotor resistance and speed range are plotted for a constant torque load and for a fan load respectively. These characteristics show that the optimum rotor resistance $[r_2/x_m]_{opt}$ is approximately obtained as the function of the lower speed range s_2 alomost regardless with motor size (r_1/x_m). They are as follows :

for constant torque load ($m=0$) ; $[r_2/x_m]_{\text{opt}} \cong 0.51 s_2 - 0.01$ (11)

and for fan load ($m = 2$) ; $[r_2/x_m]_{\text{opt}} \simeq 0.26 s_2^{0.5}$, $s_2 \leqq 0.7$
$\simeq 0.22$, $0.7 < s_2 < 1$ (12)

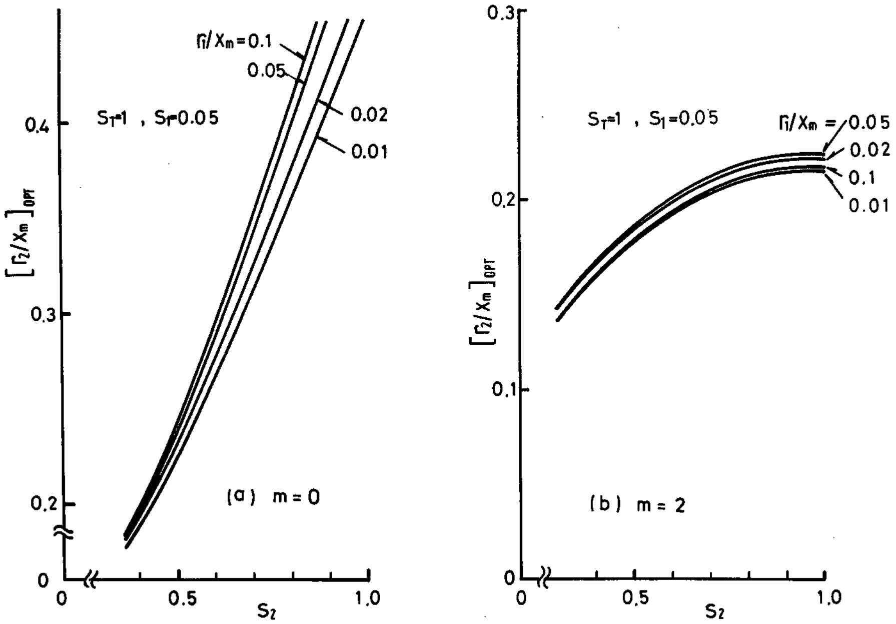

Fig. 4 Relations of optimum rotor resistance and speed range.

The influences of the core losses on energy efficiency are illustrated in Fig. 3 by dashed curves which are drawn for a relatively large amount of core losses ; for a typical induction motor, $r_m/x_m \simeq 10 \sim 10^2$. These curves show that the core losses are results in a reduced optimum rotor resistance, but the decrease in energy efficiency for smaller values of rotor resistance is little. Therefore, the foregoing results, which obtained by neglecting core losses, might be applied for standard three phase induction motors.

3.2 Single phase condenser run induction morot

3.2.1 Basic equations

By means of the same assumptions presented in the previous section and the conventional equivalent circuit for unsymmetrical two phase machine shown in Fig. 5, the following basic motor equations are derived. Stator winding impedances are

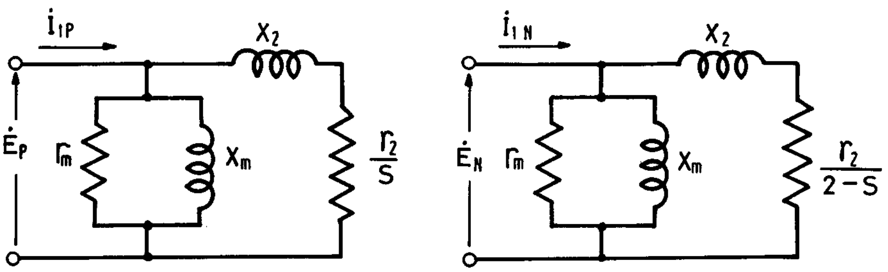

Fig. 5 Conventional rotor equivalent circuit for unsymmetrical two phase induction motor.

$$z_{M1} = r_1 + jx_1, \text{ and} \qquad z_{A1} = r_a + j(x_a - x_a) , \tag{13}$$

where, z_{M1} is main winding impedance and z_{A1} is auxiliary winding impedance, and x_c is the reactance of condenser at the supply frequency. All auxiliary winding quantities are refered to main winding by the effective turns ratio $\alpha = N_A / N_M$.

Induced torque of a condenser motor is expressed as

$$\tau = \frac{r_2}{w_0}\left\{\frac{|\dot{I}_{2P}|^2}{s} - \frac{|\dot{I}_{2N}|^2}{2-s}\right\} \cong \frac{V_1^2\, r_2}{w_0} \frac{\frac{1}{s} - \frac{r_2}{2-s}}{4x_2^2 + \left\{\frac{r_2}{s(1-s)} + r_1\right\}^2} , \tag{14}$$

and efficiency is evaluated by

$$\eta = P_0 / (P_{iM} + P_{iA}) \times 100 \quad (\%) \tag{15}$$

where, the output power of a motor $P_0 = \tau\omega$, P_{iM} and P_{iA} are the input powers of main and auxiliary windings respectively. Relations between the break down slip and the equivalent motor circuit parameters are obtained as

$$r_2 = \frac{s_T(2-s_T)\{r_1 s_T + 2\sqrt{r_1^2(s_T-1)^2 - (2-s_T)(3s_T-2)x_2^2}\}}{2-3s_T} . \tag{16}$$

3.2.2 Optimum condenser

The capacity of a condenser, which is connected in series with the auxiliary winding, influences on the characteristics of a condenser motor singnificantly, and also the adjustment of the capacity is easier than the other motor parameters. Therefore, in this section, the optimum reactance of a condenser at the supply frequency which yields the maximum energy efficiency is examined.

On the operation of the speed range between the upper speed $s_1 = 0.1$ and the lower speed $s_2 \leqq s_T = 0.5$, the influences of a reactance on energy efficiency that is evaluated by substituting Eqs. (14) and (15) to Eq. (3), are illustrated in Fig. 6. Also, the energy efficiency corresponds to the reactance x_{cb} that yields balanced operation of a condenser motor[3] is plotted by ○ sign. By inspection of Fig. 6, the following results with respect to the optimum reactance are confirmed :

(1). the optimum reactance can be adjusted when the speed range, the turns ratio, the leakage factor, and the motor size are given,

(2). influences of the load characteristics and core losses on the optimum reactance are negligible,

(3). the optimum reactance may be approximated by the reactance corresponds to the balanced operation of the condenser motor.

In Fig. 7, the relations between optimum reactance and speed range are plotted for turns ratio $\alpha = 1.2$ and for $\alpha = 1.6$ respectively. These characteristics show that the

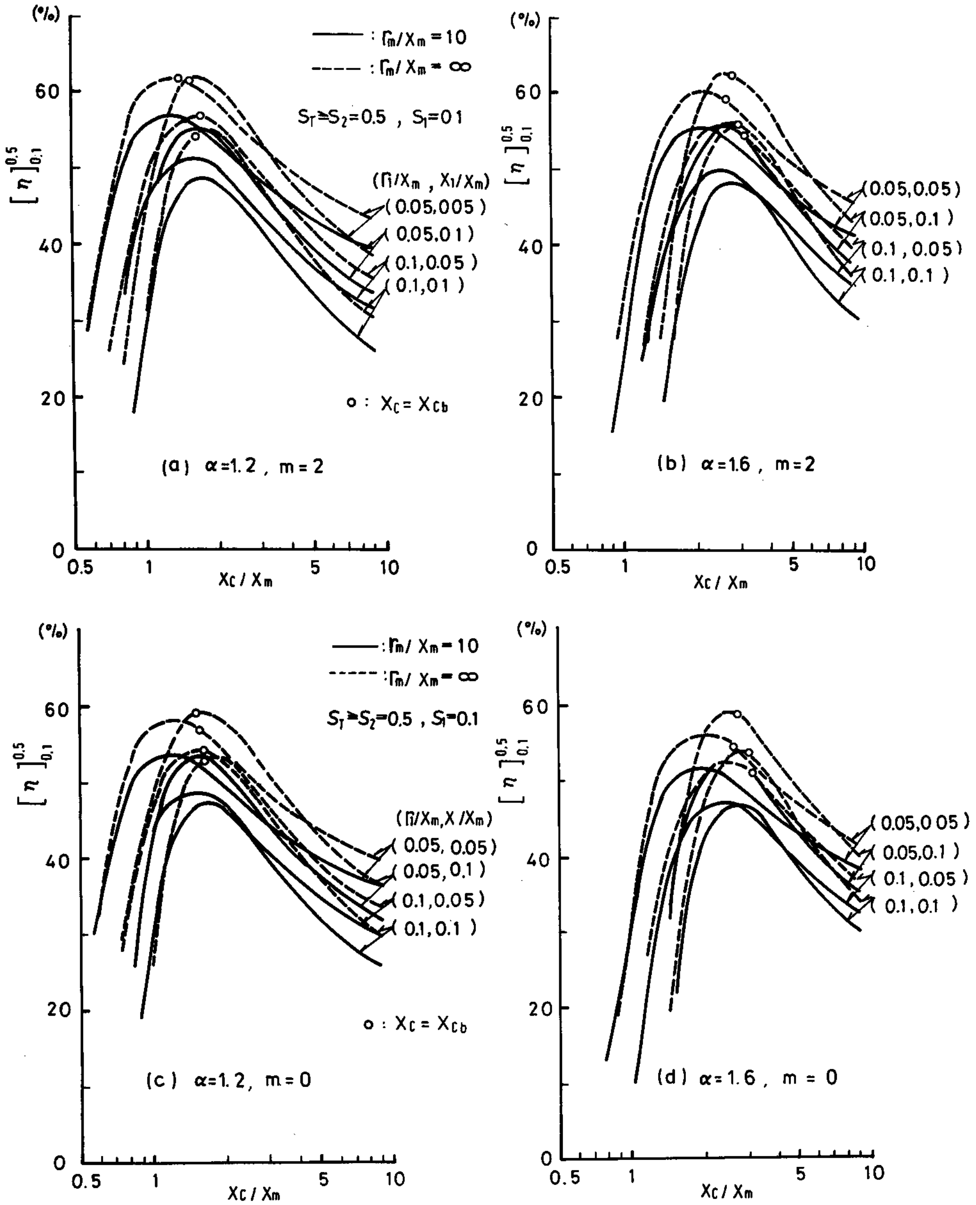

Fig. 6 Variation in energy efficiency with reactace of condenser.

optimum reactance $[x_c/x_m]_{opt}$ decreases inversely proportional to the lower speed range s_2, and the ratio of decrease depends almost only on the turns ratio. They are roughly expressed as follows,

$$\text{for } \alpha = 1.2\,;\, [x_c/x_m]_{\text{opt}} \simeq -1.03 s_2\ , \tag{17}$$

$$\text{and for } \alpha = 1.6\,;\, [x_c/x_m]_{\text{opt}} \simeq -1.77 s_2\ . \tag{18}$$

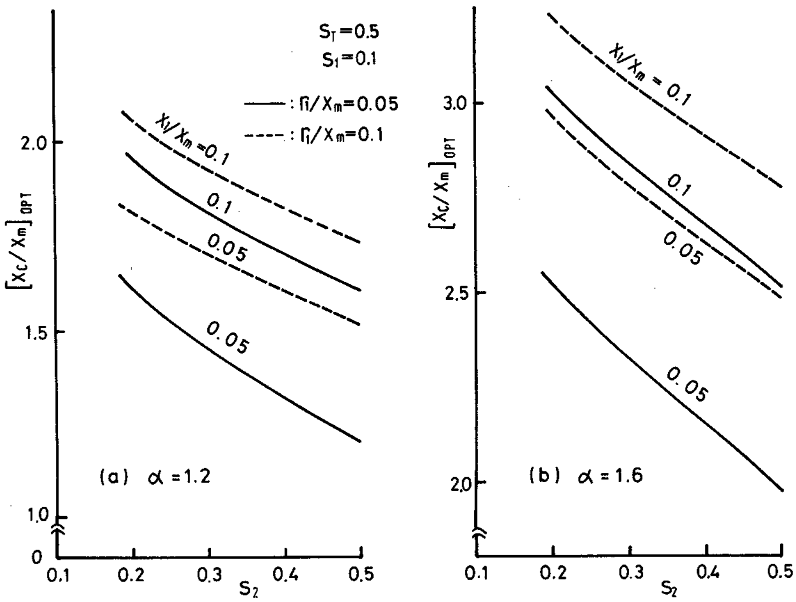

Fig. 7 Relations of optimum reactance and speed range.

3.2.3 Experimental results

The motor used in this work is rated 100 watt, 100 volt, 60Hz, 4-pole single phase induction motor of which machine parameters are as follows : r_1 = *4.75* (Ω), r_2 = *15.8* (Ω), x_1 = x_2 = *3.14* (Ω), r_a=*15.1* (Ω), x_a = *8.03* (Ω), r_m = *156* (Ω), x_m = *83.2* (Ω), and x_{cb} = *305* (Ω).

The motor is operated within the speed range between s_1 = *0.15* and s_2 = *0.3*, and with a constant load torque τ = 7.5 Kg-cm. And the stator voltage is excited by typical two sources respectively that are the auto-transformer, and the inverse parallel connected thyristors whose output voltage is distored sinusoidal waveform.

Measured and calculated energy efficiency against reactance of condenser at supply frequency are illustrated in Fig. 8. Although the harmonics contained in the stator voltage produce additional loss which offset somewhat the gain in energy efficiency, the experimental results demonstrate that the computed optimum reactances favorably predict the actual ones and are almost independent with the distortion of the supply voltage, and that they might be able to approximate with the reactances which yield balanced operation. These results verify the practical applications of the analytical solutions proposed in this paper.

4. Conclusions

Optimum machine parameters, which yield the maximum energy efficiency for variable speed operation of induction motors are simply determined under the appropriate assumptions. With respect to a three phase induction motor the optimum rotor resistance, and to a single phase condenser motor the optimum reactance of the condenser are obtained by the functions of a lower speed range for any given load characteristics respectively. And also, the optimum reactance may be roughly considereed to

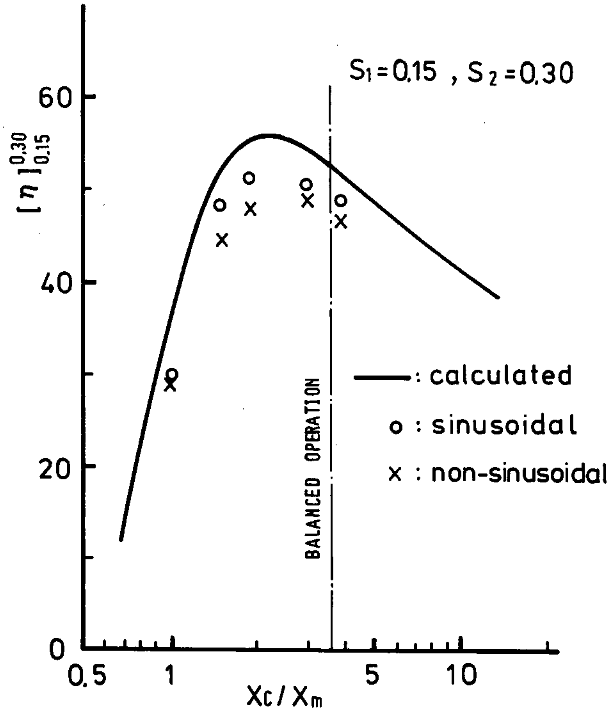

Fig. 8 Measured results of energy efficiency vs. reactance.

be the reactance for balanced operation of a single phase condenser motor.

Although, in this paper it is assumed that the operating time for each speed in the duty cycle to be equal, generally, the time should be given by the function of load which varies in terms of probabilities.

References

1) D. W. Novotny and T. A. Lipo, U. S - JAPAN Seminar S6-1, 199 (1981)
2) N. Mohan, Trans. IEEE, PAS-99, 1466 (1980)
3) Y. Yokozuka, Trans. IEEJ, 90-10, 2031 (1970)