## An Approximation Method for Planar Assignment Problem

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# An Approximation Method for Planar Assignment Problem 

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#### Abstract

The present paper describes an approximation method for the assignment problem under the conditions that all vertices are on the same plane and the distance between two vertices is taken as the cost of edge connecting their vertices. This method obtains first an initial solution by setting a certain restriction to the exploratory area on the plane and then finds out a final approximate solution by improving successively the initial solution. It is also shown that this approximation method will be able to apply to the large-scale problem with the vertices of the order of 10,000 .


## 1. Introduction

The assignment problem, one of the fundamental problems in graph theory, is applied to many fields including operations research and therefore theoretical and practical researches have been carried out up to this time. In general, the assignment problem is defined as a problem which finds an assignment such as total sum of the costs of edges comes to minimum, when whole vertices are partitioned into two sets, $A$ and $B$, and a vertex belonging to the set $A$ is assigned by one-to-one correspondence to a vertex belonging to the set $B$ and an edge with cost is inserted between the assigned vertices. The exact solution of this problem is obtained by using Hungarian method ${ }^{\mathbf{1}}{ }^{\mathbf{1}, 2)}$ published in 1957. However, though many researchers repeated the improvement, the computation time of this method stands in need of $O\left(N^{3}\right)$, where $N$ is the number of vertices. Consequently, Hungarian method is available in the case of small number of vertices, but it is of no practical use in large-scale problem such as the number of vertices is more than 10,000 . The approximation method, which saves the computation time, has also been studied in the nature of things. Some approximation methods published after that, however, were comparatively inferior in accuracy and therefore it was of no practical use.

The historical processes of the assignment problem stated above suggest that it is difficult to contrive immediately an excellent approximation method for general assignment problem. From this point of view, the present paper deals with an assignment problem adding to general assignment problem the conditions that all vertices are on the same plane and Euclidean distance between the vertices is taken as the cost of edge connecting their vertices. We give a definition of planar assignment problem to such the assignment problem and propose a method for obtaining the approximate solution. Even if the problem is restricted to on the plane, it has a close relation with

[^0]the problem concerning planar diagrams like the placement problem of electronic circuits on the print boards or LSI chips. Recently, the scale of various systems including electronic circuits becomes increasingly large and then the scale of problems to be solved is also in a large way. For example, the number of gates in the latest LSI will amount to the order of 10,000 . However, since the approximation method which is applied in practice to such the large-scale problem has not been published at the present, the development of actual method with a high efficiency for solving the large-scale assignment problem on the plane is demanded.

The approximation method described in this paper obtains structually first an initial solution by setting a certain restriction to the exploratory area on the plane and then finds out a final approximate solution by improving successively the initial solution. In the latter case, the pairwise interchange method ${ }^{3}$, one of heuristic method which is widely applied in the combinatorial optimization problems, is used as the method for improving the initial solution. As the results of Monte Carlo simulation based on the data up to 1,000 vertices, we confirmed that the present approximation method is excellent in accuracy as compared with the other methods.

## 2. Assignment Problems

### 2.1 General assignment problem

Figure 1 shows a complete bipartite graph with two sets, $A$ and $B$, which contain $N$ vertices, respectively. When the cost $c_{a_{i} b_{j}}$ is given to the edge between any vertex belonging to the set $A, a_{i} \in A$, and any vertex belonging to the set $B, b_{j} \in B$, and the vertex $a_{i}$ is assigned to the vertex $b_{j}$ by one-to-one correspondence, we consider the objective function $I$ defined by the following equation :

$$
\begin{equation*}
I=\sum_{i=1}^{N} c_{a_{i} b_{j}} \tag{1}
\end{equation*}
$$



Fig. 1 General assignment problem.
The general assignment problem is defined as the problem finding an assignment such as the value of $I$ of Eq. (1) comes to minimum. From this definition, the assignment problem described in this paper means the minimum-weight matching problem in a complete bipartite graph with two sets, $A$ and $B$, containing the same number of vertices. This is also called the assignment problem of the first degree.

### 2.2 Planar assignment problem

We now consider a regular square with the area of $1 \times 1$ on the planar coordinates as shown in Fig. 2 and let us suppose that two sets of vertices, $A$ and $B$, have $N$ vertices, respectively, in that square. Furthermore, Euclidean distance between any vertex belonging to the set $A, a_{i} \in A$, and any vertex belonging to the set $B, b_{j} \in B$, is taken as the cost of the edge connecting their vertices. The planar assignment problem is defined as the problem finding an assignment on the plane such as the value of the objective function $I$ which is geven by Eq. (1) comes to minimum. At the start of solving this problem, it is assumed that only coordinates of all the vertices are given but the costs $c_{a_{i} b_{j}}$ are not given.


Fig. 2 Planar assignment problem.

## 3. Approximation Method

### 3.1 Method for obtaining the initial solution

From the observation of the exact solutions obtained by solving in practice some planar assignment problems, it is understood that the vertices existing in the neighborhood with each other are assigned and in contract with this, the vertices locating at a long distance have almost no chance of the assignment. Fig. 3 shows the distributions of the length of edges in the exact solution (optimum assignment) which are obtained by solving in practice some examples. The axes of ordinates of these figures show the rate of existence of edges with the length expressing along the axes of abscissas. Fig. 3 gives two characteristics of the distribution of edges in planar assignment problem. One is that short edges are used in plenty for the assignment and the other is that this distribution inclines toward the shorter edges according to increasing of the number of vertices, $N$. These characteristics suggest that though the assignment is executed within a suitable area restricted on the plane, which is determined by considering the number of vertices, its result makes no great difference compared with that of the exact solution
which is obtained by considering the assignment to all the vertices on the plane. This is the point of our observation in the approximation method described in this paper.

The method for obtaining the initial solution is able to classify broadly into three operations as (1) preprocessing, (2) sorting and (3) initial assignment.


Fig. 3 Distribution of the length of edges in the exact solution of planar assignment problem.

### 3.1.1 Preprocessing

The preprocessing includes the determination of the suitable exploratory area as stated above, and the computation of the costs between a vertex belonging to the set $A$ and a vertex belonging to another set $B$. However, since it is very complex to determine at every one vertex the suitable exploratory ares on the plane, we make it a point to consider approximately such an area.

First, we partition a given square with the area of $1 \times 1$ on planar coordinates into $K \times K$ small squares, where $K \geqq 1$. Hereinafter referred to as "cell" such the small square. The number of partitions in this operation, $K$, is given by

$$
\begin{equation*}
K=[(\sqrt{N}-1) / \sqrt{0.5}+1], \tag{2}
\end{equation*}
$$

where $[x]$ denotes the greatest integer less than or equal to $x$. Furthermore, the value of Eq. (2) becomes 1 when $N=1$. From Eq. (2), we have

$$
\begin{equation*}
\frac{0.5 N}{(\sqrt{N}-1+\sqrt{0.5})^{2}} \leqq \frac{N}{K^{2}}<\frac{0.5 N}{(\sqrt{N}-1)^{2}} \tag{3}
\end{equation*}
$$

Approximating Eq.(3) by a large number of $N$, we also have

$$
\begin{equation*}
N / K^{2} \cong 0.5 \tag{4}
\end{equation*}
$$

Equation (4) shows that a cell contains half of a vertex on the average, when the value of $K$ expressing by Eq. (2) is used and the number of vertices is very large. The reason which adopted Eq. (2) as a equation determining the value of $K$ is explained in Chapter 5.

Next, we consider nine cells made by a cell containing a vertex $a_{i} \in A$ and eight cells surrounding that cell, as shown by oblique lines in Fig. 4, and compute the distances between $a_{i} \in A$ and some vertices $b_{j} \in B$ existing in the nine cells. The obtained


Fig. 4 Cells surrounding the vertex $a_{i}$.
value of the distance is given as the cost $c_{i} b_{j}$ between $a_{i} \in A$ and $b_{j} \in B$. The suitable exploratory area, stated before, means the area which is covered by the nine cells, shown in Fig. 4.

### 3.1.2 Sorting ${ }^{4}$ )

In order to execute the assignment with small value of the costs $c_{a_{i} b_{j}}$ in the succeeding operation, we arrange in the order of increasing value of the costs $c_{a_{i} b_{j}}$ obtained in the preprocessing. Since the range of the value of $c_{a_{i} b_{j}}$ is given by

$$
\begin{equation*}
0 \leqq c_{a_{i} b_{i} \leqq 2 \sqrt{2} l=2 \sqrt{2} / K} \tag{5}
\end{equation*}
$$

where $l$ denotes the length of a side of one cell, the cost $c_{a_{i} b_{j}}$ was quantized by using $l / 100$ as the unit and then the sorting was excuted.

### 3.1.3 Initial assignment

The assignment is determined in order from the edge with small value of the cost $c_{a_{i} b_{i}}$ which was obtained by the sorting. At this time, in the case where either of the two end vertices of the edge was already assigned, the assignment to that edge is not executed and then the assignment to the next edge is considered. If all the vertices were assigned perfectly by this operation, that result is accepted as an initial solution. However, since the assignment is considered within the restricted exploratory area, shown in Fig. 4, it may well be that some vertices are not assigned and remains to the last. Two vertices, $a_{i}$ and $b_{j}$, shown in Fig. 5, are the examples of such vertices.


Fig. 5 Remaind vertices at the end of initial assignment.

Let $N^{\prime}$ be the number of vertices which were not assigned and remained to the last. We notice only the remained vertices and consider again as a new planar assignment problem with $N^{\prime}$ vertices. The three operations of the preprocessing are executed again, and they are repeated until all the vertices are assigned perfectly and an initial solution is obtained.

### 3.2 Method for improving successively the initial solution

When two pairs of vertices assigned by two edges as shown in Fig. 6 (a) are interchanged as shown in Fig. 6-(b), let us now suppose that the sum of costs of two edges in the latter is less than that in the former. The pairwise interchange method ${ }^{3}$ ) is a
procedure which improves the sum of costs of two edges by interchanging the pair of vertices.


Fig. 6 Pairwise interchange.
In practice, if we consider all the edges as the object of interchange, the computation times of $O\left(N^{2}\right)$ is required. However, since it is considered that the interchange of two edges locating at a distance is of no available, as stated in the previous section, a certain restriction is made to the exploratory area in the same manner as the method for obtaining the initial solution, and the pairwise interchange method is used within that restricted area.

The method for improving successively the initial solution described here is able to classify broadly into two operations as (1) preprocessing and (2) pairwise interchange.

### 3.2.1 Preprocessing

A given square with the area of $1 \times 1$ on the planar coordinates is partitioned into $K \times K$ cells by the same menner as the preprocessing in the method for obtaining the initial solution. The number of partitions, $K$, in this preprocessing is given by

$$
\begin{equation*}
K=[(\sqrt{N}-1) / 2+1] \tag{6}
\end{equation*}
$$

It is evident that Eq. (6) is different in a constant number from Eq. (2). From Eq. (6), we have

$$
\begin{equation*}
\frac{4 N}{(\sqrt{N}+1)^{2}} \leqq \frac{N}{K^{2}}<\frac{4 N}{(\sqrt{N}-1)^{2}} \tag{7}
\end{equation*}
$$

Approximating Eq. (7) by a large number of $N$, we also have

$$
\begin{equation*}
N / K^{2} \cong 4 \tag{8}
\end{equation*}
$$

From Eq. (8), we understand that a cell contains 4 vertices on the average, when the value of $K$ expressing by Eq. (6) is used and the number of vertices is very large. The reason which adopted Eq. (6) as a equation determining the value of $K$ is described in Chapter 5.

### 3.2.2 Pairwise interchange

The assignments, which are considered as the object of pairwise interchange to an assignment ( $a_{i}, b_{j}$ ), are those having the vertices belonging to the set $B$ within nine cells made by a cell containing $a_{i} \in A$ and eight cells surrounding that cell. For example, in Fig. 7, they are four assignments, $\left(a_{i+1}, b_{j+1}\right),\left(a_{i+2}, b_{j+2}\right),\left(a_{i+3}, b_{j+3}\right)$ and $\left(a_{i+4}\right.$,
$b_{j+4}$ ). If the number of assignments to be improved by executing the pairwise interchange is more than one, only one among them, which has the highest grade of improvement, is accepted. In Fig. 7, such an assignment executing the pairwise interchange to the assignment $\left(a_{i}, b_{j}\right)$ is an assignment $\left(a_{i+1}, b_{j+1}\right)$.


Fig. 7 Exploratory area for pairwise interchange.
When the pairwise interchange was considered on all the assignment, if the assignment which executed the pairwise interchange exists one at least, the pairwise interchange on all the assignments is considered again. If the assignment to be executed the pairwise interchange is not in existence, this operation is over and all the operations for improving successively the initial solution are finished.

### 3.3 Algorithm and its complexity

The algorithm for the approximation method described in this paper is as follows:
(Step 1) Partition into $K \times K$ cells a given square with the area of $1 \times 1$ on planar coordinates and giving some necessary informations to each verte $x$ and each cell. Provided that the number of partitions of the plane, $K$, use the value obtaining by Eq. (2).
(Step 2) Computing the distances between a vertex $a_{i} \in A$ and a vertex $b_{j} \in B$ within the area which is covered by the nine cells, shown in Fig. 4, and taking the obtained distances as the costs $c_{a_{i} b_{j}}$.
(Step 3) Arranging the given costs $c_{a_{i} b_{j}}$ in the order of increasing value.
(Step 4) Executing the assignments in the order of increasing value of the costs $c_{a_{i} b_{j}}$. If $N^{\prime}$ vertices in the set $A$ were not assigned and remained to the last, considering again as a new planar assignment problem with $N^{\prime}$ vertices and returning to step 1.
(Step 5) Executing the same operation as that in step 1. Provided that the number of partitions of the plane, $K$, use the value obtaining by Eq. (6).
(Step 6) Considering the pairwise interchange within the area which is covered by the nine cells stated in 3.2.2. When the pairwise interchange is considered on all of the vertices belonging to the set $A$, if the assignment which is improved by this operation exists one at least, considering again the pairwise interchange on all of the vertices
belonging to the set $A$. If there is no room for improvement, all the operations are finished.

The complexity in each step of the algorithm stated above is able to obtain comparatively easily by cosidering the number of cells which partitioned into small cells and the number of costs to be computed on the average. If we suppose that the pairwise interchange in step 6 is executed only once to all the assignments, the complexity in each step of the present algorithm is give by $O(N)$. From this matter, it is understood that each of the methods for obtaining the initial solution and for improving successively the initial solution has the complexity of $O(N)$, when they are executed only once. However, since some steps in either methods are necessary to execute repeatedly in practice, it is desirable to consider the complexity in such a case. in the following, the complexity, which were considered from the experimental results in the case where the number of vertices, $N$, is from 10 to 1,000 , is described.

First, we make a consideration on the method for obtaining the initial solution. Observing the relation between the number of vertices, $N$, and the number of vertices, $N^{\prime}$, remained at the end of the first initial assignment, which were obtained by the experiments, we can understand that $N^{\prime}$ is proportional to $N$ and is approximately given by

$$
\begin{equation*}
N^{\prime}=0.23 N \tag{9}
\end{equation*}
$$

Consequently, the aggregate number of vertices, $W$, in this method is expressed by

$$
\begin{equation*}
W=N+0.23 N+0.23^{2} N+\cdots \cdots \cdots \tag{10}
\end{equation*}
$$

From Eq. (10), we have

$$
\begin{equation*}
W=N /(1-0.23)=1.3 N \tag{11}
\end{equation*}
$$

Equation (11) shows that the problem, which each of two sets, $A$ and $B$, have $N$ vertices and some steps in this method are executed repeatedly, is equivalent to the problem, which each of the sets, $A$ and $B$, have $1.3 N$ vertices and all the steps in this method are executed only once. Therefore, though some steps of the method is repeated, it is considered that the complexity in this method is also $O(N)$.

Next, we make a consideration on the method for improving successively the initial solution. Observing the experimental result which gives the relation between the number of improvement, $R$, when the pairwise interchange is considered to all the vertices, and the number of vertices, $N$, we can understand that $R$ increases proportionally to $N^{0,3}$. Since the required number of computation in the first pairwise interchange is equal to that in the second pairwise interchange, the required number of computations in the method for improving successively the initial solution is proportional to

$$
\begin{equation*}
N \times N^{0.3}=N^{1.3} \tag{12}
\end{equation*}
$$

From Eq. (12), it is considered that the complexity of this method is $O\left(N^{1.3}\right)$.

### 3.4 Example

We now try to obtain the approximate solution by applying the algorithm described in 3.3 to the planar assignment problem which $N=25$ as shown in Fig. 8. In this figure, the vertices belonging to the set $A$ are shown by a symbol " $O$ " and the vertices belonging to the set $B$ are shown by another symbol " $\bullet$ ".


Fig. 8 Given problem for obtaining the approximate solution.
(Step 1) Since the value of $K$ becomes 6 from Eq. (2), we partition into 36 cells the given square, shown in Fig. 8.
(Step 2) The distances between $a_{l} \in A$ and $b_{\mathrm{j}} \in B$ are computed within the area which is covered by the nine cells, shown in Fig. 4, and they are given as the costs $c_{a_{i} b_{j}}$. Here, the number of costs to be computed is 127 in total. If we take the vertex $a_{7}$, shown in Fig. 9 as an example, the distances between $a_{7}$ and eight vertices, $b_{1}, b_{2}$, $b_{3}, b_{6}, b_{7}, b_{8}, b_{11}$ and $b_{12}$, are computed, and their costs become as follows:

$$
\begin{array}{llll}
c_{a 7 b_{1}}=0.0745, & c_{a 7 b_{2}}=0.1213, & c_{a 7 b_{3}}=0.2877, \quad c_{a 7 b_{6}}=0.1509, \\
c_{a 7 b 7}=0.1509, & c_{a 7 b 8}=0.2192, & c_{a 7 b_{11}}=0.3236 \text { and } c_{a 7 b_{12}}=0.2677 .
\end{array}
$$

(Step 3) The costs $c_{a_{i} b_{j}}$ obtained in step 2 are arranged in the order of increasing value.
(Step 4) The assignments are executed in order from small value of the costs $c_{a_{i}} b_{j}$. In the end of this operation, two vertices, $a_{13}$ and $b_{11}$, as shown in Fig.10, remain at the last. Consequently, it returns to step 1 as a problem which $N^{\prime}=1$.
(Step 1) Since $N^{\prime}=1$, the value of $K$ becomes 1 from Eq. (2). This shows that the partition of the plane is not executed.
(Step 2) Since the vertices are only $a_{13}$ and $b_{11}$, only the cost $c_{a_{13} b_{11}}$ is given


Fig. 9 Computation of costs $c_{a_{i}} b_{j}$.


Fig. 10 Initial solution to the problem of Fig. 8.
and its value is 0.5357 .
(Step 3) Since the number of costs is only one, the rearrange is not executed.
(Step 4) The vertex $a_{13} \in A$ is assigned to the vertex $b_{11} \in B$. Hereupon, since all the vertices have been assigned, the procedures for obtaining the initial solution have been finished and it goes to step 5. Fig. 10 shows the initial solution obtained at the end of this step.
(Step 5) Since the value of $K$ becomes 3 from Eq. (6), the given square is partitioned into 9 cells.
(Step 6) The pairwise interchange to all the assignments is considered within the exploratory area which is covered by the nine cells stated in 3.2.2. In practice, the assignments to be executed the pairwise interchange are those of three groups as follows:

$$
\left(a_{1}, b_{1}\right) \text { and }\left(a_{7}, b_{6}\right),\left(a_{6}, b_{7}\right) \text { and }\left(a_{13}, b_{11}\right) \text {, and }\left(a_{12}, b_{13}\right) \text { and }\left(a_{13}, b_{7}\right)
$$

These pairwise interchanges are shown in Fig. 11 (a), (b) and (c) according to the order stated above. Since the value of costs was improved, the similar operation is executed in succession. Here, the assignments to be executed the pairwise interchange are those of two groups as follws:

$$
\left(a_{6}, b_{11}\right) \text { and }\left(a_{12}, b_{7}\right), \text { and }\left(a_{11}, b_{12}\right) \text { and }\left(a_{12}, b_{11}\right)
$$



Fig. 11 Pairwise interchanges for improving the initial solution of Fig. 10.

Since the value of costs was improved again, the similar operation is also executed. However, since any improvement is not expected beyond this value, the algorithm of the approximation method has been completed.

The value of the objective function $I$ obtained by this approximation method is 2.4008, and the final result of the assignment is shown in Fig. 12. We obtained the exact solution of this example by using Hungarian method. From this result, we confirmed that the value of the objective function and the assignment in the exact solution coincided with those in the final solution obtained by using this approximation method.

## 4. Experimental Results

We executed the computer experiments on planar assignment problems with the vertices from 10 to 1,000 by using the approximation method proposed in this paper


Fig. 12 Approximate solution to the problem of Fig. 8.
and the existing methods. In this chapter, the approximate solution and the computation time obtained by the experimental results are discussed. The coordinates of vertices on the plane were entirely given by using the uniform random numbers and the average value of 100 data was adopted as the value to the specified number of vertices. Futhermore, the computer experiments are executed by using NEC-ACOS 77/700 at the Computer-Center of the University of Osaka Prefecture.

### 4.1 Comparison of approximate solutions

Figure 13 shows the relation between the objective function $I$ obtained by the experiments and the number of vertices, $N$. The numerals enclosed by the circles in this figure express the classification of five methods for obtaining the exact or approximate solutions as shown in the following:
(1) Method for obtaining the initial solution prposed in this paper,
(2) Approximation method proposed in this paper,
(3) Hungarian method ${ }^{1), 2)}$,
(4) $X Y$-sorting ${ }^{5}$,
and (5) Spiral rack algorithm ${ }^{6)}$.
The result obtained by Hungarian method, (3), gives the exact solution. The ratios of objective functions obtained by the method for obtaining the initial solution, (1), and by the approximation method, (2) , proposed in this paper to that obtained by Hungarian method, (3), come to within about 1.3 and within about 1.03 , respectively. Furthermore, these values are almost constant in regard to increasing of the number of vertices.

These characteristics are worthy of attention compared with the characteristic that the accuracies of the solutions obtained by the approximation methods based on XY-sorting, (4), and the spiral rack algotithm, (5), get worse according to increase of


Fig. 13 Comparison of objective functions.
the number of vertices, $N$. From these matters, it is understood that the approximation method proposed in this paper is very accurate and will be able to apply sufficiently to the practical problems.

### 4.2 Comparison of computation times

Figure 14 shows the relation between the computation time $T_{c}$ and the number of vertices, $N$. The numerals enclosed by the circles correspond to those in Fig. 13. The slope of each line in Fig. 14 expresses the order of computation time. The complexity of each method obtained from the slope of the part of straight line in Fig. 14 becomes as follows:
(1) $O\left(N^{1.03}\right)$, (2) $O\left(N^{1.39}\right),(3) O\left(N^{3.12}\right)$, (4) $O\left(N^{1.11}\right)$ and (5) $O\left(N^{1.01}\right)$.

The complexities of the method for obtaining the initial solution, (1), and the approximation method proposed in this paper, (2), are slightly different from those described in 3.3, but they almost coincide. The method for obtaining the initial solution, (1) , is excellent in accuracy, and the absolute value of computation time is


Fig. 14 Comparison of computation times.
several times at most compared with those of XY-sorting, (4), and the spiral rack algorithm, (5). Consequently, when only this method is applied to the very large scale problems with the vertices of the order of $10^{6}$, it is expected to solve the problems in about 30 minutes. Furthermore, the complexity of the approximation method, (2), is worse than that of the method obtaining the initial solution, (1). However, when this method is applied to the problems with the vertices of the order of 10,000 , it is expected that the value close to the exact solution is obtained in about 30 minutes.

## 5. Discussions

In this chapter, the number of partitions of the plane used in the case of partitioning into some cells is discussed and the reason for using Eqs. (2) and (6) is described.

The fundamental line of thinking in the approximation method proposed in this paper is to make a exploration only within the restricted area on the plane. The matter to be attended here is to give the property that though the number of vertices, $N$, chan-
ged, the number of vertices existing in a cell, $N / K^{2}$, is the same on the average. We now discuss the optimum number of vertices existing in a cell, $N / K^{2}$, in the methods for obtaining the initial solution and for improving successively the initial solution.

Figure 15 shows the relation between the computation time, $T_{c}$, required for obtaining the initial solution and the number of vertices existing in a cell, $N / K^{2}$. Since it is definitely shown by the experimental result that the value of the objective function $I$ is almost constant in regardless to the change of the value of $N / K^{2}$, it is suitable to take the value corresponding to the minimum computation time, as the optimum value of $N / K^{2}$. From Fig. 15, it can be seen that such a value is within from 0.4 to 0.6 . Consequently, we adopted 0.5 on the average as the optimum value of $N / K^{2}$ in the method for obtaining the initial solution. This is the value given in Eq. (4), and then Eq. (2) was determined as the equation for obtaining the value of $K$.


Fig. 15 Relation between computation time and the number of vertices existing in a cell.

We can understand from Fig. 15 that the value of $N / K^{2}$ corresponding to the minimum computation time have a tendency to increase with the number of vertices, $N$. Therefore, it is conjectured that the optimum value of $N / K^{2}$ becomes more than 0.5 in very large-scale problems such as the number of vertices is more than $10,000$. In the present paper, we could not explain theoretically a detailed circumstance concerning such the large-scale problems. This is the subject for a future study.

Next, we describe the discussion on the value of $N / K^{2}$ in the method for improving successively the initial solution. Fig. 16 shows the relation between the objective function $I$ of the approximate solution obtained by the approximation method
proposed in this paper and the number of vertices existing in a cell, $N / K^{2}$, as a parameter the number of vertices. The straight lines in this figure show the values of $I$ of the initial solution for $N / K^{2}=0.5$. Therefore, these are not concerned in the value of $N / K^{2}$, shown along the axis of abscissa. We can understand that the value of $I$ in the method for improving successively the initial solution is improved until the value of $N / K^{2}$ comes to 4 , but when that value becomes more than 4 , the value of $I$ is almost not improved.


Fig. 16 Improvement of objective function by increasing the number of vertices existing in a cell.

On the other hand, it is definitely shown that the computation time required for improving successively the initial solution, $T_{c}$, increases according as the value of $N / K^{2}$ increases. From these facts, we adopted 4 as the value of $N / K^{2}$ in the method for improving successively the initial solution. This is the value given in Eq. (8), and then Eq. (6) was determined as the equation for obtaining the value of $K$.

## 6. Conclusions

In the present paper, we freshly defined the planar assignment problem and proposed an approximation method. The characteristics of the present approximation method are summarized as follows:
(1) The present approximation method consists of two methods for obtaining structurally the initial solution and for improving successively the initial solution by using pairwise interchange, and both methods restrict the exploratory area on the plane by partitioning the given square into $K \times K$ cells.
(2) In the method for obtaining the initial solution, the complexity is $O(N)$ and the ratio of the objective function of obtained initial solution to that of the exact solution is about 1.3 and constant.
(3) In the method for improving successively the initial solution, the complexity is $O\left(N^{1.4}\right)$ and the ratio of the objective function of obtained approximate solution to that of the exact solution becomes about 1.03.

The conventional methods in the planar assignment problems have the restrictions of the computation time and the accuracy from practical point of view. Therefore, judging from the existing condition, these methods are impossible to apply to the problems with the vertices more than hundreds. Since this problem, however, is important in connection with the placement problems of very LSI and others, the development of the approximation method for solving the problems with the vertices of the order of 1,000 to 10,000 is demanded. From a consideration of the experimental results, we consider that the present approximation method is possible to apply as well to the problems with the vertices of the order of 10,000 and is very available.

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