Estimation and Sensitivity Analysis for Tool Life Equation Nonlinear in Parameters

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A nonlinear tool life model is proposed, in which the cutting conditions and the amount of tool wear are treated as independent variables. The nonlinear model is constructed to fit the process of tool wear in three stages, i.e. rapid initial wear, gradual wear and catastrophic wear.

The nonlinear parameters are estimated by Gauss linearization method and prediction accuracy of tool life is investigated within confidence region of the estimated parameters.

1. Introduction

Taylor equation is widely used for determination of economically optimum cutting conditions. The equations which treat the amount of tool wear are necessary when surface roughness is one of the constraints in the machining process, and cutting speed varies in cutting material such as a stepped part. In answer to the request, a multiplication model (a modified form of Taylor equation) and a polynomial model are suggested. The models agree well with the wear process which involves gradual wear after rapid initial wear, however, disagreement between observations and the models is frequently perceived at the third stage of the process. This paper proposes a tool life equation which describes the three stages of the wear process as a mathematical model. The sensitivity analysis of the proposed model is tried by investigating the effect of variation in the parameters on its accuracy.

2. Tool Life Equation

Flank wear V_B is related to cutting time t, if cutting conditions are constant, as shown in Fig. 1 (a). The wear process follows three stages; rapid initial wear, gradual wear and catastrophic wear as the cutting time increases.[1] Assume that the wear process curve approximates to a curve having an asymptote $t=T_o$ as shown in Fig. 1 (b), then the relationship between V_B and t is given by

$$t=T_o \exp[-\exp(b)V_B{}^n] \qquad n<0 \tag{1}$$

where b and n are constants and T_o stands for the critical cutting time. Provided that T_o in eq. (1) is a function of the cutting conditions,

$$T_o=aV^{n_1}f^{n_2} \tag{2}$$

where a, n_1 and n_2 are constants, V is the cutting speed and f is the feed rate. The depth of cut can be disregarded because its effect on tool life is almost negligible.[2] Eqs. (1) and (2) give

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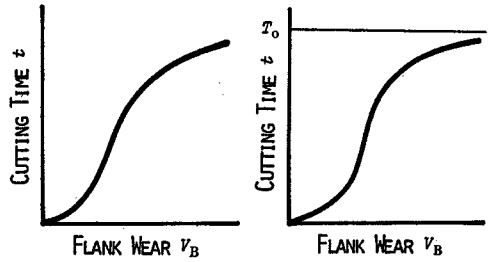

Fig.1 (a) Fig. 1(b)

Fig. 1(a) Ordinary wear process curve
Fig. 1(b) Approximate wear process curve

$$t=aV^{n_1}f^{n_2}\exp\left[-\exp(b)V_B{}^n\right] \tag{3}$$

Substitution of the appropriate value of V_B in eq. (3) as the tool life criterion yields the ordinary Taylor equation. Taking logarithms of both sides of eq. (3) to estimate the parameters, we have

$$\ln t=\ln a+n_1\ln V+n_2\ln f-e^b V_B{}^n \tag{4}$$

Since eq. (4) is nonlinear, the least square method can not estimate simultaneously the parameters, a, n_1, n_2, b and n. The application of Gauss linearization method can be examined.

Equation (4) can be written as

$$\eta_i=\beta_1+\beta_2 x_{i1}+\beta_3 x_{i2}-\beta_4 x_{i3}^{\beta_5} \tag{5}$$

where η, x_1, x_2 and x_3 are the logarithmic transformation of the cutting time, the cutting speed, the feed rate and the amount of flank wear, respectively. The sensitivity coefficients in eq. (5) are

$$\begin{aligned} &X_{i1}=\frac{\partial\eta_i}{\partial\beta_1}=1\,,\quad X_{i2}=\frac{\partial\eta_i}{\partial\beta_2}=x_{i1}\,,\quad X_{i3}=\frac{\partial\eta_i}{\partial\beta_3}=x_{i2}\,,\\ &X_{i4}=\frac{\partial\eta_i}{\partial\beta_4}=-x_{i3}^{\beta_5}\quad\text{and}\quad X_{i5}=\frac{\partial\eta_i}{\partial\beta_5}=-\beta_4(\log x_{i3})x_{i3}^{\beta_5}\,. \end{aligned} \tag{6}$$

The sensitivity matrix corresponding to the coefficients is

$$\boldsymbol{X}=\begin{pmatrix} \frac{\partial\eta_1}{\partial\beta_1}\cdots\frac{\partial\eta_1}{\partial\beta_5}\\ \vdots\qquad\vdots\\ \frac{\partial\eta_n}{\partial\beta_1}\cdots\frac{\partial\eta_n}{\partial\beta_5}\end{pmatrix}=\begin{pmatrix} 1 & x_{11} & x_{12} & -x_{13}^{\beta_5} & -\beta_4(\log x_{13})x_{13}^{\beta_5}\\ \vdots & \vdots & & & \\ 1 & x_{n1} & x_{n2} & -x_{n3}^{\beta_5} & -\beta_4(\log x_{n3})x_{n3}^{\beta_5}\end{pmatrix} \tag{7}$$

The nonlinear parameter vector $\boldsymbol{b}$ can be estimated by

$$\boldsymbol{b}^{(k+1)}=\boldsymbol{b}^{(k)}+\boldsymbol{P}^{(k)}[\boldsymbol{X}^{T(k)}(\boldsymbol{Y}-\hat{\boldsymbol{Y}}^{(k)})] \tag{8a}$$

$$\boldsymbol{P}^{-1(k)}=\boldsymbol{X}^{T(k)}\boldsymbol{X}^{(k)} \tag{8b}$$

where the notation k and $k+1$ indicate the iteration number of calculation required

Table 1 Cutting conditions and experimental results

Trial No.	Cutting Speed V m/min	Feed Rate f mm/rev.	Coding x_1	Coding x_2	Flank Wear V_B mm	Cutting Time t min
1	252	0.40	1	1	0.143	5.01
2	252	0.15	1	−1	0.151	13.39
3	178	0.40	−1	1	0.098	7.78
4	178	0.15	−1	−1	0.100	6.35
5	300	0.25	2	0	0.080	1.90
6	211	0.60	0	2	0.108	1.68
7	150	0.25	−2	0	0.165	44.15
8	211	0.10	0	−2	0.187	8.80
9	252	0.40	1	1	0.342	9.28
10	252	0.15	1	−1	0.191	21.54
11	178	0.40	−1	1	0.122	13.73
12	178	0.15	−1	−1	0.183	31.47
13	300	0.25	2	0	0.155	4.93
14	211	0.60	0	2	0.195	3.83
15	150	0.25	−2	0	0.295	151.42
16	211	0.10	0	−2	0.270	31.46
17	252	0.40	1	1	0.511	15.81
18	252	0.15	1	−1	0.259	40.21
19	178	0.40	−1	1	0.178	17.79
20	178	0.15	−1	−1	0.300	78.87
21	300	0.25	2	0	0.260	6.84
22	211	0.60	0	2	0.258	8.27
23	150	0.25	−2	0	0.450	293.75
24	211	0.10	0	−2	0.448	103.90
25	252	0.40	1	1	0.670	21.90
26	252	0.15	1	−1	0.282	48.79
27	178	0.40	−1	1	0.210	35.45
28	178	0.15	−1	−1	0.414	197.85
29	300	0.25	2	0	0.331	12.09
30	211	0.60	0	2	0.341	12.99
31	150	0.26	−2	0	0.550	323.04
32	211	0.10	0	−2	0.624	117.32
33	252	0.40	1	1	0.802	25.65
34	252	0.15	1	−1	0.448	61.00
35	178	0.40	−1	1	0.261	52.63
36	178	0.15	−1	−1	0.643	333.70
37	300	0.25	2	0	0.536	15.86
38	211	0.60	0	2	0.434	16.28
39	150	0.25	−2	0	0.749	344.20
40	211	0.10	0	−2	0.969	283.35
41	211	0.25	0	0	0.250	21.58
42	211	0.25	0	0	0.250	34.08
43	211	0.25	0	0	0.250	50.70
44	211	0.25	0	0	0.250	41.15

for estimation of nonlinear parameters, and $\hat{Y}^{(k)}$ implies the estimate of Y, which is obtained at k-th iteration.

Design of the Experiment

A central composite design of twelve tests in five levels with two factors (cutting speed and feed rate), consisting of nine different cutting conditions and three repetitions at the center point is adopted for the nonlinearity of the postulated model. Each of the eight tests is performed at five levels of amount of tool wear, and total number of experiments is forty-four.

The levels of x_1 and x_2 are

$$x_1=\frac{2(\ln V-\ln 252)}{\ln 252-\ln 178}+1\,,\quad x_2=\frac{2(\ln f-\ln 0.4)}{\ln 0.4-\ln 0.15}+1\,. \tag{9}$$

x_3 is equivalent to V_B on a logarithmic scale, since control of flank wear level is difficult and time consuming.

4. Experiments and Results

Tool life tests were performed in an engine lathe equipped with a 7.5 KW motor. The work material was as-received S55C carbon steel; the specimens were 100 mm in diameter and 400 mm in length. The cutting tool was a throw-away type made of P10 and its geometrical shape was (−0.5, −0.5, 5, 5, 15, 15, 0.8). The cutting conditions, flank wear and corresponding cutting time are shown in Table 1.

Using the experimental results and the estimated parameters b_1, b_2, b_3, b_4 and b_5, we obtain

$$y=e^{6.79}-0.62x_1-0.42x_2-2.04V_B^{-0.40} \tag{10}$$

The above result and the iterations are summarized in Table 2. From eqs. (9) and (10), the required tool life equation becomes

Table 2 Summary of calculations by iterations

Iteration Number	Parameter Values					Sum of Squares
	b_1	b_2	b_3	b_4	b_5	
0	20.0	−3.0	−1.0	0.6	−0.7	10384.0
1	6.224	−0.621	−0.422	1.471	−0.286	44.00
2	6.102	−0.620	−0.423	1.361	−0.557	11.143
3	6.572	−0.620	−0.422	1.814	−0.408	9.948
4	6.795	−0.620	−0.422	2.042	−0.395	9.226
5	6.789	−0.620	−0.422	2.037	−0.397	9.223

$$t=e^{24.67}V^{-3.57}f^{-0.86}\exp(-2.04V_B^{-0.40})\,. \tag{11}$$

Application of F-statistics for purpose of comparison is described in Table 3. It is apparent from the table that the proposed model is satisfactory.

Table 3 Analysis of variance table

Source of Variation SV	Sum of Squares SS	Degree of Freedom DF	Mean Square MS	Calculated F_0 Value	
Regression	545.1	5			
Residual	9.223	39			$F(36, 3, 0.95)$ $=8.62$ $>F_0$
Lack of Fit	8.825	36	0.245	1.842	
Pure Error	0.398	3	0.133		
Total	554.323	44			

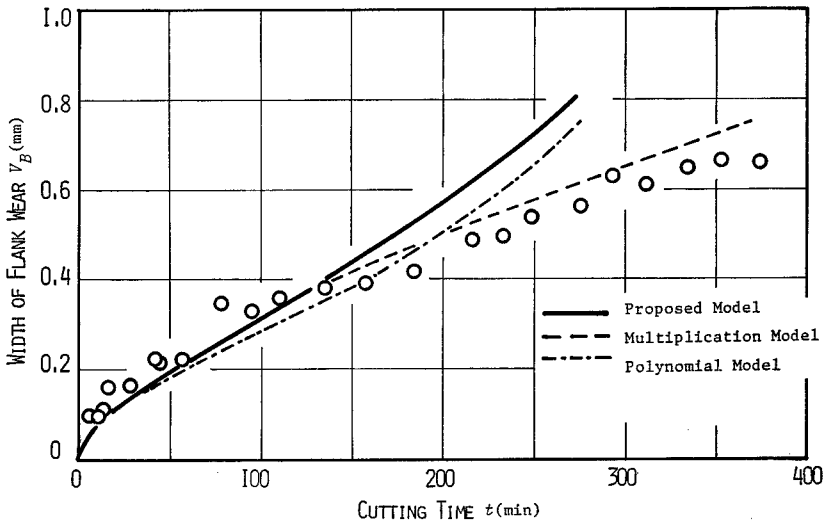

Fig. 2(a) Wear process curve (V=178 m/min, f=0.15 mm/rev., d=1 mm)

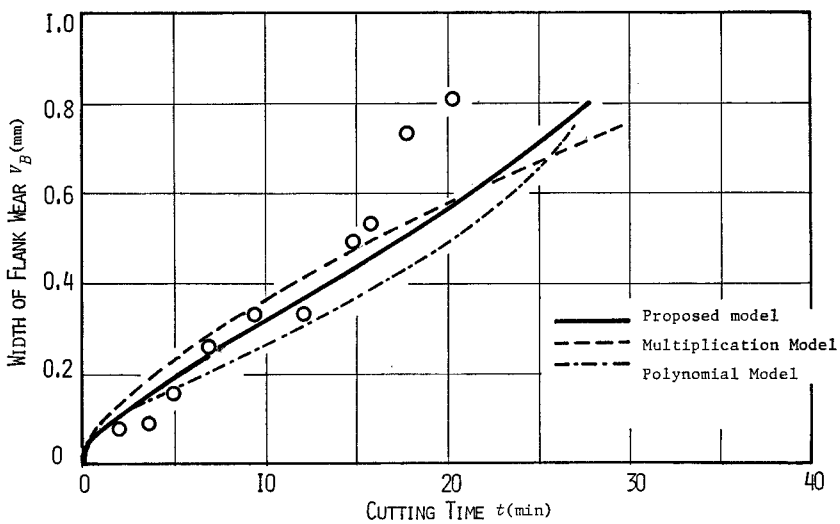

Fig. 2(b) Wear process curve (V=300 m/min, f=0.25 mm/rev., d=1 mm)

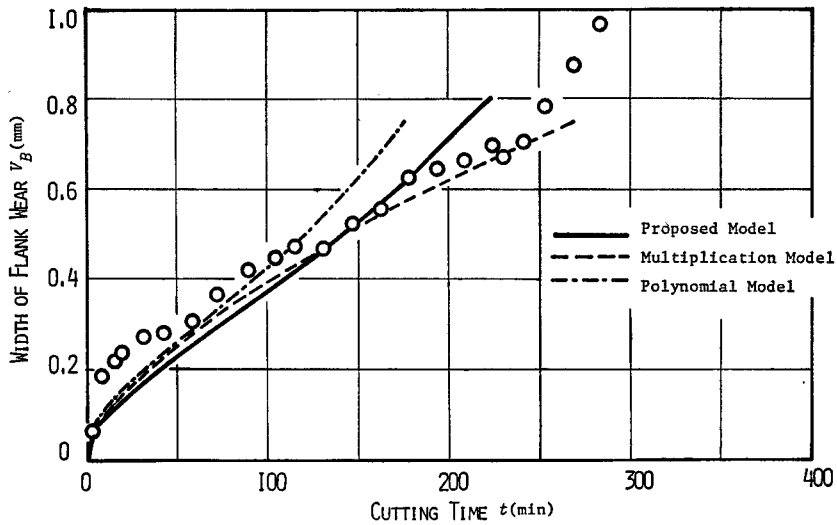

Fig. 2(c) Wear process curve (V=211 m/min, f=0.10 mm/rev., d=1 mm)

The curves of the tool wear process by the proposed model and the others together with observed values are shown in Figs. 2 (a), 2 (b) and 2 (c). In these figures the multiplication model is a modified form of Taylor equation:

$$t=a_0V^{a_1}f^{a_2}V_B^{a_3}$$

where a_0, a_1, a_2 and a_3 are constants. The polynomial model can be obtained by a stepwise regression procedure.[3] The parameters of the two models are estimated with the data used in the proposed model. These figures tell us that whereas the multiplication and the polynomial models are valid for the tool life in the second stage, the proposed model is valid for the tool life in the third stage. The mean square errors for each model shown in Table 4 suggest that the proposed model is not so inferior to the others in accuracy.

Table 4 Comparison of the accuracy of the models

Tool Life Equation	Tool Life Criterion	Residual	Degree of Freedom	Mean Square
Proposed Model	V_B=0.7	9.223	39	0.237
Multiplication	0.4	7.328	24	0.305
Model	0.7	10.009	40	0.250
Polynomial	0.4	2.049	21	0.098
Model	0.7	3.645	37	0.099

For practical application of tool life equation to analytical method such as the determination of economically optimum cutting conditions, the polynomial model cannot be used because of its complexity. The multiplication model is intended for optimization of cutting processes.[4] From the facts that the proposed model is similar in form to the multiplication model and that use of non-linear estimation enables us to obtain a tool life model readily without estimating

the critical cutting time individually and employing a linear estimation twice, one may regard the model as practcial.

5. Sensitivity Analysis of the Tool Life Model

Since parameters are probabilistic, not deterministic, at a workshop parameter estimation should be tried again in making use of the proposed model which is composed of two parts; a function of the cutting condition and a function of the amount of tool wear. If the optimization of cutting conditions for a specified tool life criterion is to be accomplished, the parameter values in the former function can be updated by fixing the parameters in the latter. And if the tool life time having the relation to V_B under constant cutting conditions is to be predicted, updating the parameter values in the latter would be possible by fixing the parameters in the former. If the fixed ones are greatly affected by these variation, this method will be useless.

The objective of this section is the determination of the joint confidence regions of the parameters in each function and the sensitivities of tool life to the variations in the parameters within the joint confidence regions.

5.1 Confidence Region of the Parameters in the Model

Unlike linear estimation, the expression for the confidence region of nonlinear parameter is approximate, because the sensitivity coefficients are function of parameters and the first two terms of Taylor series in linearization method.

Let $\boldsymbol{\beta}$ be a vector of p parameters and $\boldsymbol{b}$ be its least squares estimator, the $100(1-\alpha)\%$ joint confidence region for $\boldsymbol{\beta}$ is given by

$$(\boldsymbol{b}-\boldsymbol{\beta})^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}(\boldsymbol{b}-\boldsymbol{\beta})\leqq pS^2F_{1-\alpha}(p, n-p) \tag{12}$$

where $\boldsymbol{X}$ is the sensitivity matrix of which sensitivity coefficients are function of b, and S^2 is approximately

$$S^2\approx\frac{(\boldsymbol{Y}-\hat{\boldsymbol{Y}})^{\mathrm{T}}(\boldsymbol{Y}-\hat{\boldsymbol{Y}})}{n-p} \tag{13}$$

The approximate confidence region given by eq. (12) is ellipse or circle for $p=2$.

The sensitivity matrix for estimating the parameters in $aV^{n_1}f^{n_2}$ is equivalent to the first $n\times 3$ matrix of eq. (7). From the matrix we obtain

$$\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}=\begin{bmatrix} 44 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix}$$

The eigenvalues of $\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}$ are

$$\lambda_1=44\,,\quad \lambda_2=60\,,\quad \lambda_3=60$$

and corresponding eigenvectors are

$$e_1=\begin{bmatrix}1\\0\\0\end{bmatrix}\quad e_2=\begin{bmatrix}0\\1\\0\end{bmatrix}\quad e_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Hence, the end points of the axes are given by

$$[(\boldsymbol{b}-\boldsymbol{\beta})_{maj}(\boldsymbol{b}-\boldsymbol{\beta})_{min}]=\pm(pS^2F_{1-\alpha}(5,39))^{1/2}$$
$$\times\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}44^{-1/2}&0&0\\0&60^{-1/2}&0\\0&0&60^{-1/2}\end{bmatrix}$$

The reason for putting $p=5$ is that the estimation of the three parameters in $aV^{n_1}f^{n_2}$ includes estimation of the two parameters in $\exp(-e^bV_B{}^n)$. Thus the region forms into the ellipsoid, of which each cross section is shown in Fig. 3. Since the sensitivity matrix of $\exp(-e^bV_B{}^n)$ consists of the fourth and fifth columns in eq. (7), $\boldsymbol{X}^T\boldsymbol{X}$ is

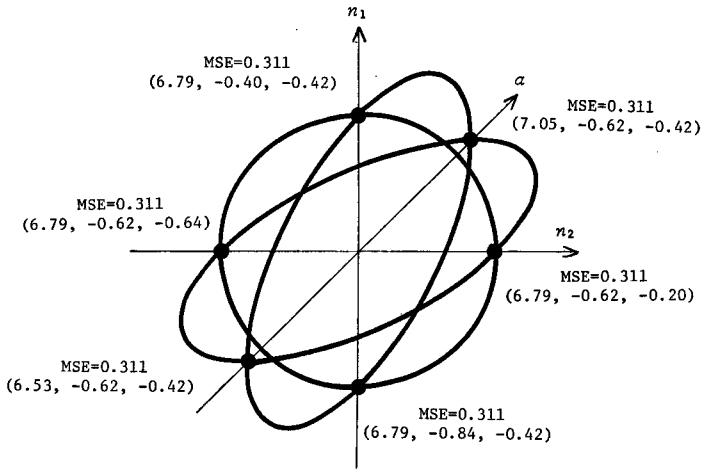

Fig. 3 Joint confidence region of the parameters for $aV^{n_1}f^{n_2}$ and values of mean square errors

$$\boldsymbol{X}^T\boldsymbol{X}=\begin{bmatrix}135.2&-427.3\\-427.3&1538.1\end{bmatrix}$$

For the eigenvalues,

$$\lambda_1=15.31\quad\lambda_2=1658$$

and foı the eigenvectors,

$$e_1=\begin{vmatrix}0.963\\0.270\end{vmatrix}\quad e_2=\begin{vmatrix}-0.270\\0.963\end{vmatrix}$$

Consequently, the end points of the axes are given by

$$[(\boldsymbol{b}-\boldsymbol{\beta})_{maj}(\boldsymbol{b}-\boldsymbol{\beta})_{min}]=\pm(pS^2F_{1-\alpha}(5,39))^{1/2}$$

$$\times \begin{vmatrix} 0.963 & -0.270 \\ 0.270 & 0.963 \end{vmatrix} \begin{vmatrix} 15.31^{-1/2} & 0 \\ 0 & 1658^{-1/2} \end{vmatrix}$$

The region is the ellipse as shown in Fig. 4.

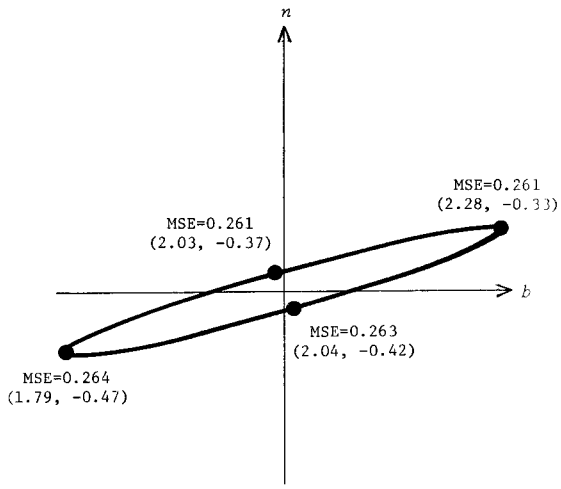

Fig. 4 Joint confidence region of the parameters for $\exp(-e^b V_B{}^n)$ and values of mean square errors

5.2 The sensitivity of the Tool Life Equation

In order to examine how the accuracy of the tool life changes within the joint confidence regions of the parameters, the residual sums of squares of the estimated tool lives from the observed may be obtained from the 95% confidence regions of the calculated parameters. The mean square errors are shown in Figs. 3 and 4. From these figures a significant difference cannot be perceived. Therefore, if desired, the parameters in one function can be updated with fixed parameters in the other.

6. Conclusion

(1) The nonlinear model for prediction of the tool life is identified, which can be applied to the wear process consisting of the three stages of tool wear with the cutting conditions. The parameters in the model are estimated by Gauss linearization method.

(2) The accuracy and utility of the model are compared with multiplication and polynomial models to show its usefulness.

(3) From the results of the sensitivity analysis of the model, a simple way of practical use of the model is considered.

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