One Method Analysing Two-Dimensional Steady Flow in Very Low Reynolds Number

One Method Analysing Two-Dimensional Steady Flow in Very Low Reynolds Number

Osamu KINOSHITA*, Hidebumi ITÔ* and Yasuyuki MASUDA*

(Received June 15, 1983)

This paper describes that characteristics of the steady flow of a viscous fluid can be analysed if a minute velocity distribution is obtained experimentally. As an example, two-dimensional cavity flow in very low Reynolds number is taken up, where the cavity has somewhat complicated shape. The experiment is performed using water glass as an operating fluid under the Reynolds number 3.9×10^{-4}. The velocity distribution is examined by using the tracer technique and the velocity components (u, v) for the coordinates (x, y) are determined in detail. From these data, the stream function ψ is calculated to be a volume of flow between a streamline and the cavity wall, and the vorticity ζ is evaluated by $\partial v/\partial x-\partial u/\partial y$. Then from the vorticity obtained, the pressure p is got by means of $-\int(\partial\zeta/\partial y)dx$ and $\int(\partial\zeta/\partial x)dy$. Thus distributions of the stream function, the vorticity and the pressure are drawn.

1. Introduction

In these days, the various problems of fluid dynamics have been solved making use of computer. However, there are still much works to be required in stability, and even when the solution is obtained in the stable condition, the accuracy should be checked further with helps of experimental data or basic knowledges of fluid mechanics. Accordingly the experimental study must proceed simultaniously with the study of computer technique.

In this paper the authors take up a flow in a cavity whose shape is somewhat complicated. The problems of cavity flow have been treated so far by several investigaters[1)-6)] theoretically or experimentally. Using a computer, Kawaguti[2)] has been studied the two-dimensional rectangular cavity flow to get distributions of stream function, vorticity and pressure for low Reynolds numbers ranging from 0 to 64. Experimental studies[5),6)] of the cavity flow have been done to get streamlines, though the other characteristics (stream function, vorticity and pressure) are not analysed. However, the characteristics can be calculated, if the velocity distribution

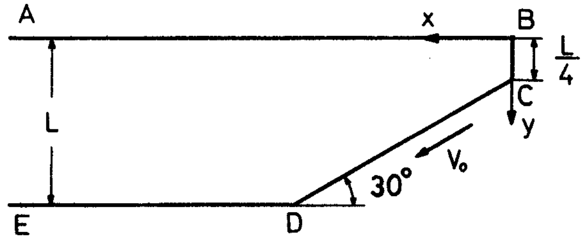

Fig. 1 Experimental model.

* College of Integrated Arts and Sciences.

is obtained by experimental technique. Under this consideration, the authors analyse the steady flow in the cavity shown in Fig. 1. Here the three rigid walls AB, BC and DE are fixed, while the wall CD moves with constant velocity V_0. The parallel walls AB and DE are long as compared with the distance L between them. The authors are studying similar cavity flow from a geotectonical view point. From the study, fundamental problems concerning the fluid mechanics have arisen as will be mentioned in this paper.

2. Experimental Equipment and Procedure

For following explanations, let us employ the length L and the velocity V_0 to be the reference quantities, and the Reynolds number R is given by

$$R=\rho V_0 L/\mu \,, \tag{1}$$

where ρ and μ are density and viscosity of an operating fluid respectively. The experiment has been planed in which the Reynolds number is very small, that is $R \ll 1$. The flow with very low Reynolds number has following characteristics: (1) a pattern of streamlines is independent of the Reynolds number, (2) the pattern does not change, even if a direction of flow is made to reverse and (3) the flow becomes steady as soon as it starts.

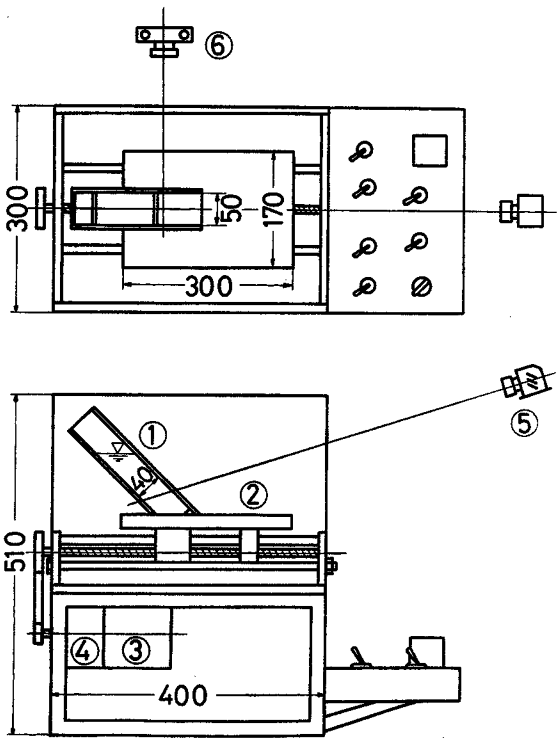

Fig. 2 Schematic diagram of experimental apparatus. 1: the model containing viscous fluid, 2: driving bed, 3: motor, 4: reduction gear, 5: light projector, 6: camera.

Figure 2 shows a schematic diagram of the experimental apparatus. The case inclined in the upper part of the apparatus contains the operating fluid, which is the model of the cavity flow shown in Fig. 1. The case is made of clear acrylic resin, the distance L (Fig. 1) being 40 mm. The bed is driven very slowly by an induction motor with a reduction gear. The bed speed is variable and 1.16 mm/sec in maximum. Clear water glass (sodium silicate solution) is used as the operating fluid and is poured into the case settled on the bed until the surface level becomes high enough not to affect the first vortex flow. Density and viscosity of the water glass have been measured to be 1.7 g/cm^3 and 2.0×10^2 Pa.sec, respectively. Taking the bed speed of 1.16 mm/sec, the Reynolds number of the model is 3.9×10^{-4} (ref. eq. 1), which would be sufficiently small for a simulation of very slow motion. The fluid might be regared to be the Newtonian liquid under such very low Reynolds number.

In order to visualize the flow pattern, minute air bubbles have been mixed in the water glass previously. The above model is illuminated by the light projector as shown in Fig. 2. The light flux through a narrow slit of the projector has about 2 mm thickness when it passes through the model. Hence the minute bubbles distributed only in the light flux can be observed in a dark room and trajectories of the bubbles can be photographed in a long exposure (Fig. 3). Since each bubble draws a long trajectory, it migrates within the narrow light flux. Even when the light flux has been shifted parallelly from the middle position, the flow pattern has not been changed. These mean that the two-dimensional flow is held widely in the model.

In order to observe the velocity distribution of flow, the trajectories of bubbles are photographed by means of switching on and off the light. The switching intervals were taken 4 sorts of 5, 10, 20 and 30 seconds, because lengths of the trajectories (velocities) depend largely on the position. Two examples shown in Fig. 3 are ones for the switching intervals 5 and 30 seconds respectively.

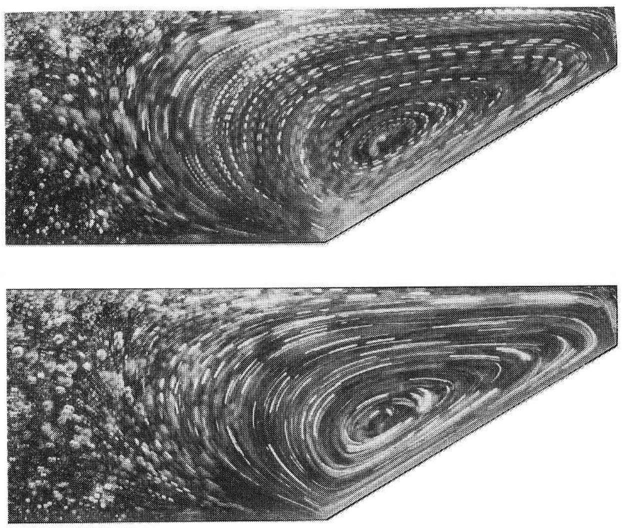

Fig. 3 Flow patterns photographed by means of switching on and off a light.
Upper: switching interval 5 sec, lower: switching interval 30 sec.

The first vortex is seen sharply in the right hand region in Fig. 3, but the second vortex is not observed, which is induced by the first vortex in the left hand region of the photograph. Each pattern of Fig. 3 has been obtained by advancing the bed to the direction from C to D in Fig. 1. Though the pattern obtained when the bed advances reversely (from D to C) is not shown here, it is almost the same with Fig. 3 for the first vortex. In this paper, only the first vortex are analysed.

3. Velocity Distribution

The photographs were taken not only for 4 sorts of the switching interval but also for normal and reverse advances of bed. Analysing all these photographs, the velocity v_t tangential to streamline and the rectangular components of velocity (u, v) in the directions of the axes of x and y shown in Fig. 1 were obtained. Diagrams y vs. u for $x=x_i$ $(i=1, \dots 50)$ and x vs. v for $y=y_j$ $(j=1, \dots 24)$ were drawn. Some of them are shown in Fig. 4, where (x, y) and (u, v) are normalized as follows:

$$X=x/L, \quad Y=y/L; \quad U=u/V_0, \quad V=v/V_0. \tag{2}$$

Thus velocity components (u_i, v_j) at a mesh point (x_i, y_j) were determined.

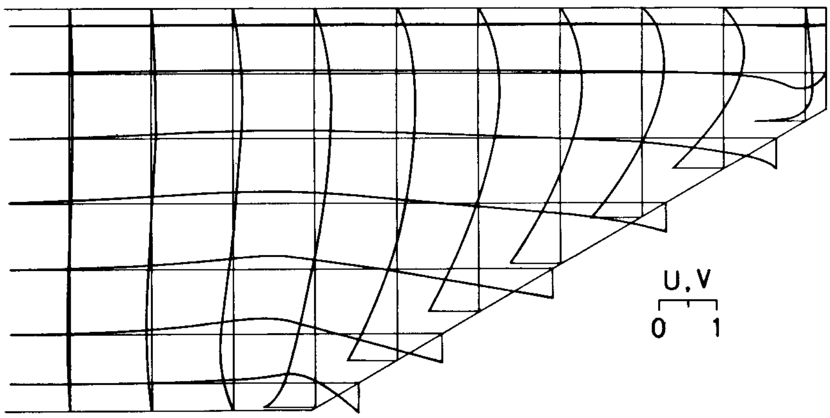

Fig. 4 Velocity distributions of U vs. Y and V vs. X.

4. Streamline and Stream Function

It takes so long times while one bubble (marker) in the water glass makes one circulation in the first vortex to draw a closed trajectory (streamline). But the closed streamlines will be able to be drawn refering to the photographs obtained above. The boundary ABCDE is also one streamline. The stream function ψ' is given by a volume of flow between the streamlines and the boundary, provided $\psi'=0$ at the boundary. Therefore the stream function is calculated by

$$\psi'=\int v_t \cdot dn \,, \tag{3}$$

where dn is an infinitesimal increment of the curve n which is orthogonal to the streamlines.

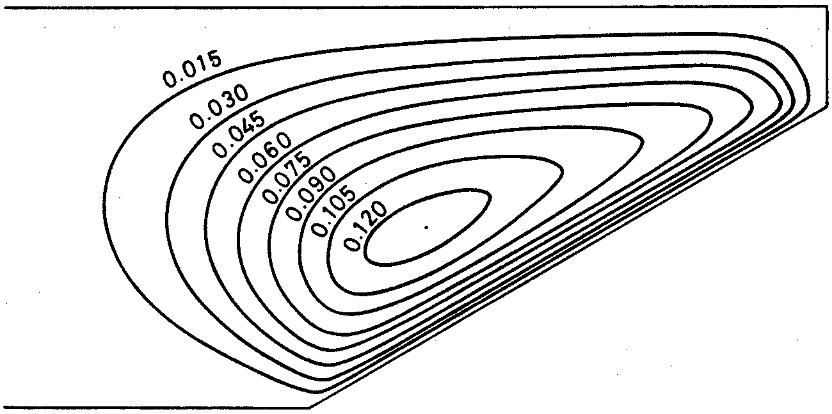

Fig. 5 Streamlines with values of stream function ψ.

At first several streamlines were drawn arbitrarily. Secondly three orthogonal lines cutting perpendiculaly the streamlines were drawn from the boundaries AB, CD and DE (Fig. 1). The orthogonal lines should be jointed at a center of vortex. Thirdly values of the stream function were calculated by eq. (3) along each of the three orthogonal lines. Three values for each streamline were confirmed to coincide. To do this confirmation it is necessary to take plural orthogonal lines. At last the modified streamlines, whose values of the stream function have a equal difference, were drawn as shown in Fig. 5. Here ψ is the normalized stream function to be given by

$$\psi=\psi'/V_0L\,. \tag{4}$$

An interval of the streamlines is inversely proportional to the velocity.

5. Vorticity Distribution

The vorticity ζ' is defined by

$$\zeta'=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\,, \tag{5}$$

and the normalized vorticity ζ is given by

$$\zeta=L/V_0\cdot\zeta'. \tag{6}$$

Velocity gradients $\partial v/\partial x$ and $\partial u/\partial y$ were measured on the graphs v vs. x and u vs. y in Fig. 4, and values of vorticity were got at the mesh points. Thus the equi-vorticity lines were drawn as shown in Fig. 6, in the dimensionless vorticity ζ, where the dotted points were ones interpolated graphically from values of vorticity at the mesh points.

6. Pressure Distribution

If the pressure p is normalized by

$$P=p/\rho V_0^2, \tag{7}$$

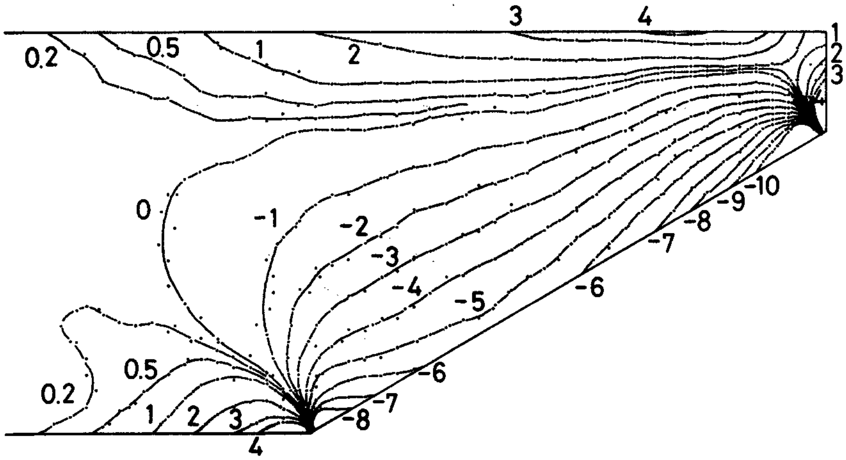

Fig. 6 Equi-vorticity lines of ζ.

the Navier-Stokes equations for two-dimensional steady flow in very slow motion are written in dimensionless form as follows;

$$\frac{\partial P}{\partial X}=\frac{1}{R}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)=-\frac{1}{R}\frac{\partial \zeta}{\partial Y} \tag{8}$$

$$\frac{\partial P}{\partial Y}=\frac{1}{R}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)=\frac{1}{R}\frac{\partial \zeta}{\partial X}, \tag{9}$$

provided the equation of continuity

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\,. \tag{10}$$

Integrating eq. (8) along a line Y=constant between points 1 and 2, we get

$$R(P_2-P_1)=-\int_1^2\frac{\partial \zeta}{\partial Y}\,dX\,. \tag{11}$$

For integration of eq. (9) between points 3 and 4 on a line X=constant, we get

$$R(P_4-P_3)=\int_3^4\frac{\partial \zeta}{\partial X}\,dY \tag{12}$$

(ref. Thom and Apelt[1]). The equations (11) and (12) are used to evaluate the pressure difference between a reference point and any point in the fluid.

Using these equations, values of RP, instead of P itself, were calculated at the mesh points, assuming $P=0$ at the reference point shown in Fig. 7. Thus the isobaric lines, RP=constant were drawn in this figure.

From eqs. (8) and (9), we get easily the relation;

$$\frac{\partial P}{\partial X}\frac{\partial \zeta}{\partial X}+\frac{\partial P}{\partial Y}\frac{\partial \zeta}{\partial Y}=0\,. \tag{13}$$

This relation shows that the isobaric lines intersect perpendicularly to the equi-vorticity lines. It is found out that the isobaric lines in Fig. 7 and the equi-vorticity lines in Fig. 6 intersect perpendicularly each other.

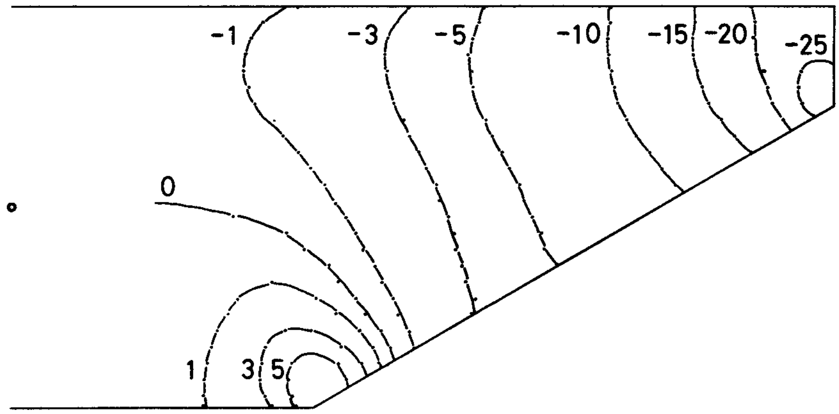

Fig. 7 Isobaric lines represented by *RP*. The hollow circle is the reference point, at which $P=0$ is assumed.

7. Concluding Remarks

Taking up the cavity flow in Fig. 1, the authors have determined the velocity distribution experimentally. Using the velocity, the stream function and the vorticity have been calculated by eqs. (3) and (5) respectively. Then using the vorticity obtained, the pressure has been calculated by eqs. (11) and (12). In order to proceed these successive calculations, the velocity must be determined in detail. For this reason, many photographs as shown in Fig. 3 have been required.

The values of stream function obtained can be checked by calculating eq. (3) along plural orthogonal lines cutting the streamlines normally. The vorticity and the pressure obtained may be checked by confirming whether the equi-vorticity lines and the isobaric lines intersect normally or not. The results thus obtained seem to be fairly well, though all the calculations have been carried out manually. It is needless to say that the method described in this paper can be applied to all steady flows in very low Reynolds number.

References

1) A. Thom and C.J. Apelt, Brit. Aero. Res. Coun. R and M. No. 3090 (1958)
2) M. Kawaguti, J. Phys. Soci. Japan, **16,** 12, 2307 (1961)
3) H.K. Moffatt, J. Fluid Mech. **18,** 1 (1964)
4) M. Takematsu, J. Phys. Soci. Japan, **21,** 9, 1816 (1966)
5) T. Niida, doctor's thesis in Univ. of Osaka Pref. (1978) (in Japanese)
6) S. Taneda, Nagare, **11,** 1, 3 (1979) (in Japanese)