The Position Angle Matrix and its Application to Analysis of the Boundary Problems of the Dissymmetrical Many Stage Cascade Connected Polyphase Transmission Lines

| メタデータ | 言語：eng |
| :---: | :--- |
|  | 出版者： |
|  | 公開日：2010－04－06 |
|  | キーワード（Ja）： |
|  | キーワード（En）： |
|  | 作成者：Inagaki，Yoshio，Kido，Masao，Nagahara， |
|  | Toshikuni |
|  | メールアドレス： |
|  | 所属： |
| URL | https：／／doi．org／10．24729／00008594 |

# The Position Angle Matrix and its Application to Analysis of the Boundary Problems of the Dissymmetrical Many Stage Cascade Connected Polyphase Transmission Lines 

Yoshio Inagaki*, Masao Kido* and Toshikuni Nagahara*

(Received Nov. 15, 1982)


#### Abstract

This paper describes new theoretical investigation in which the boundary value problem of the dissymmetrical two stage cascade connected multi-conductor system has been analized by making use of position angle matrices. Our new analysis is very helpful in acquiring a clear physical picture of the phenomena, because it gives very easily the potentials along the lines due to multireflections of the traveling waves. And it will enable the engineer to design for adequate protection of the system.


## 1. Introduction

An analytical method was given in previous paper ${ }^{1)}$ so that the potential and the current of any point of the symmetrical many cascade connected multiconductor system may be exactly obtained as functions of time, taking into consideration arbitrary initial potentials and currents on the lines and assuming boundary conditions on the lines without using the lattice diagram method. But it was assumed there that the system was symmetrical. In this paper the dissymmetrical case has been treated although the arrangement of conductors is the same at all stages of lines.

However, as it is clear from the previous paper ${ }^{1)}$ that these analytical solutions are too complicated to use for numerical calculations and it takes a great deal of trouble to put its method to practical use. Well, the calculation of potential for the cascade connected single phase transmission line can be treated easily by means of the position angle. Therefore, taking into consideration that electrical quantities can be written in matrices for the multi-conductor system, it is evident that such calculation for the cascade connected polyphase transmission system can be done more easily by using the position angle of the matrix form. But, the boundary conditions are so dissymmetrical, that the position angle can not be used for the multi-conductor system, as dissymmetrical matrices is not commutative for its multiplication.

However, it has been found out that the addition theorem of the hyperbolic function of matrix holds good by adopting a new generalized manner under an extending rule on the application of Sylvester's expansion theorem even if matrices do not satisfy commutative law for multiplication, and therefore it has become to be able to introduce the position angle matrix to the multiconductor system.

In this paper, the boundary value problem of the dissymmetrical two stage cascade connected multi-conductor system has been analized by introducing position angle matrices and the results of calculations based upon this method have shown complete agreement with the theory due to the traveling waves along the lines.

[^0]
## 2. Pseudo-Sylvester function

The typical function of a square matrix $[A]$ is defined ${ }^{2)}$ by

$$
f([A])=C_{0}[U]+C_{1}[A]+\cdots+C_{n}[A]^{n}+\cdots
$$

where, $[U]$ is the unit matrix and $C_{0}, C_{1}, \cdots, C_{n}, \cdots$ are scalar constants. Now, the following definition can be introduced.
(Definition 1) The function $f$ is called pseudo-Sylvester function if $[A]$ and $[B]$ are square matrices of order $n$ and

$$
\begin{equation*}
\operatorname{Syl}\{f([A]+[B])\}=\sum_{r}\left[K_{r}\right]\left(S_{r l}\right) \sum_{s}\left[G_{s}\right]^{\left(\mathrm{S}_{s l}\right)} f\left(\alpha_{r l}+\beta_{s l}\right) \tag{1}
\end{equation*}
$$

where, $[A]$ and $[B]$ cannot be added when they are not commutable for multiplication, and $\left[K_{r}\right]^{\left(S_{r l}\right)}$ is calculated by the following equation (2) as $\alpha_{r i}\left(n=\sum_{i=1}^{k} S_{r i}\right)$ is $S_{r i}$-ple characteristic root of matrix [A]

$$
\begin{equation*}
\left[K_{r}\right]^{\left(S_{r l}\right)}=\frac{\substack{m=1,2, \ldots, k \\ \prod}}{\substack{m=l \\ \prod \neq l}}\left(\alpha_{r m}[U]-[A]\right)^{S_{r m}}\left(\alpha_{r m}-\alpha_{r l}\right)^{S_{r m}} \tag{2}
\end{equation*}
$$

and $\left[G_{s}\right]^{\left(S_{s l}\right)}$ may be obtained similarly.
It can be proved easily that matrix hyperbolic function is a kind of pseudoSylvester function, for example the next relation can hold
$\operatorname{Syl}\{\sinh ([A] \pm[B])\}=\operatorname{Syl}\{\sinh [A] \cosh [B] \pm \cosh [A] \sinh [B]\}$
Then next definitions are introduced to use matrix hyperbolic function on the calculation of the electrical potential and to emphasize to be pseudo-Sylvester function.
(Definition 2) $\quad \mathrm{SH}([A] \pm[B])=\sinh [A] \cosh [B] \pm \cosh [A] \sinh [B]$
(Definition 3) $\quad \mathrm{CH}([A] \pm[B])=\cosh [A] \cosh [B] \pm \sinh [A] \sinh [B]$
Then the following theorems for example can hold.
(Theorem 1) $\quad \mathrm{SH}(-[A]+[B])=-\mathrm{SH}([A]-[B])$
(Theorem 2) $\quad \mathrm{SH}([A]+[B]) \cosh [C]+\mathrm{CH}([A]+[B]) \sinh [C]$

$$
\begin{equation*}
=\mathrm{SH}([A]+[B]+[C]) \tag{6}
\end{equation*}
$$

(Theorem 3) $\quad \mathrm{SH}([A]+[B]) \mathrm{CH}([C]+[D])+\mathrm{CH}([A]+[B]) \mathrm{SH}([C]+[D])$

$$
\begin{equation*}
=\mathrm{SH}([A]+[B]+[C]+[D]) \tag{7}
\end{equation*}
$$

(Theorem 4) $\quad \mathrm{SH}([A]+[B]) \mathrm{CH}([B]+[A])$

$$
\begin{equation*}
=\mathrm{CH}([A]+[B]) \mathrm{SH}([B]+[A]) \tag{8}
\end{equation*}
$$

Next, the position angle matrix [ $\delta$ ] is defined as follows:

$$
\text { (Definition 4) } \quad \begin{align*}
\tanh [\delta] & =\sinh [\delta](\cosh [\delta])^{-1} \\
& =(\cosh [\delta])^{-1} \sinh [\delta] \\
& =[z]\left[k^{*}\right]^{-1}[k] \tag{9}
\end{align*}
$$

where, $[z]$ is the square matrix whose elements are the impedances connected on the ends of transmission lines, and

$$
\begin{align*}
& {\left[k^{*}\right]=[p L+R]}  \tag{10}\\
& {[k]^{2}=[p L+R][p C+G]} \tag{11}
\end{align*}
$$

where, $[L],[R],[C]$, and $[G]$ represent square matrices whose elements are inductances, resistances, capacitances and leakances per unit length of transmission lines respectively.

## 3. Analytical method for the dissymmetrical 2-conductors transmission system and applications of position angle matrix

The system illustrated in Fig. 1. is the $i$-stage cascade connected dissymmetrical $n$-conductors transmission system and there are impedances, admittances and sources at the points $A, B, \ldots$, and $J$. The arrangements of conductors at all stages are the same one another, and the line length of the $s$-th stage is $l_{s}\left(l=\sum_{s=1}^{i} l_{s}\right)$.


Fig. 1 Simplified equivalent circuit of cascade connected polyphase transmission system.

In this paper the initial conditions are not taken into account because the application of the position angle matrix is our main purpose. Let $\left[e_{s}(x)\right]$ and $\left[i_{s}(x)\right]$ be the potential and the current matrices in the operational forms at any point $x$ of the $s$-th stage line, the well-known formulas are obtained ${ }^{3)}$ as follows :

$$
\begin{align*}
& {\left[e_{s}(x)\right]=\sinh [k] x \cdot\left[\alpha_{s}\right]+\cosh [k] x \cdot\left[\beta_{s}\right]}  \tag{12}\\
& {\left[i_{s}(x)\right]=-\left[k^{*}\right]^{-1}[k]\left(\cosh [k] x \cdot\left[\alpha_{s}\right]+\sinh [k] x \cdot\left[\beta_{s}\right]\right.} \tag{13}
\end{align*}
$$

where $\left[\alpha_{s}\right]$ and $\left[\beta_{s}\right]$ are the constants of the integration to be determined by boundary conditions.

Now, the analytical solutions in the case of $i=2$ are obtained in the following way, where electrical sources $\left[E_{A}\right],\left[E_{B}\right],\left[E_{C}\right]$ and impedances $\left[Z_{A}\right],\left[Z_{B}\right],\left[Z_{C}\right]$ connected at junctions $A, B, C$ are as shown in Fig. 2. Then the boundary conditions in this case are as follows :

Line $1 \quad$ Line $\mathbb{I}$


Fig. 2 A simple example of cascade connected polyphase transmission system.

$$
\begin{align*}
& {\left[i_{1}(0)\right]=-\left[Z_{A}\right]^{-1}\left[e_{1}(0)\right]+\left[Z_{A}\right]^{-1}\left[E_{A}\right]}  \tag{14}\\
& {\left[i_{1}\left(l_{1}\right)\right]=\left[i_{2}\left(l_{1}\right)\right]+\left[Z_{B}\right]^{-1}\left[e_{1}\left(l_{1}\right)\right]-\left[Z_{B}\right]^{-1}\left[E_{B}\right]}  \tag{15}\\
& {\left[i_{2}(l)\right]=\left[Z_{C}\right]^{-1}\left[e_{2}(l)\right]-\left[Z_{C}\right]^{-1}\left[E_{C}\right]}  \tag{16}\\
& {\left[e_{1}\left(l_{1}\right)\right]=\left[e_{2}\left(l_{1}\right)\right]} \tag{17}
\end{align*}
$$

Denoting the position angles at the points $A, B, C$ by $\left[\delta_{A}\right],\left[\delta_{B}\right],\left[\delta_{C}\right]$, the next equations are derived from eq. (9)

$$
\begin{align*}
& {\left[Z_{A}\right]\left[k^{*}\right]^{-1}[k]=\tanh \left[\delta_{A}\right]}  \tag{18}\\
& {\left[Z_{B}\right]\left[k^{*}\right]^{-1}[k]=\tanh \left[\delta_{B}\right]}  \tag{19}\\
& {\left[Z_{C}\right]\left[k^{*}\right]^{-1}[k]=\tanh \left[\delta_{C}\right]} \tag{20}
\end{align*}
$$

The constants of the integration $\left[\alpha_{1}\right],\left[\alpha_{2}\right],\left[\beta_{1}\right]$ and $\left[\beta_{2}\right]$ are obtained from eqs. (12) ~ (20),

$$
\begin{align*}
& {\left[\alpha_{1}\right]=-\cosh \left[\delta_{A}\right]\left\{\mathrm { SH } ( [ k ] l _ { 1 } + [ \delta _ { A } ] \} ^ { - 1 } \left\{\mathrm{SH}\left([k] l_{2}+\left[\delta_{C}\right]\right)[\mathrm{CH}([k] l\right.\right.} \\
& \left.\left.\quad+\left[\delta_{C}\right]\right)\right\}^{-1}\left[\alpha_{2}\right]+\cosh [k] l_{1} \cdot\left[E_{A}\right]-\cosh [k] l_{1}\left\{\mathrm { CH } \left(\left[\delta_{C}\right]\right.\right. \\
& \left.\quad+[k] l)\}^{-1} \cosh \left[\delta_{C}\right] \cdot\left[E_{C}\right]\right]  \tag{21}\\
& {\left[\alpha_{2}\right]=-\mathrm{CH}\left([\mathrm{k}] l+\left[\delta_{C}\right]\right)\left\{\mathrm{SH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\right\}^{-1}[\Omega]^{-1}\left(\sinh \left[\delta_{B}\right]\right)^{-1}} \\
& \quad \times\left[\sinh \left[\delta_{B}\right]\left[\operatorname{SH}\left(\left[\delta_{A}\right]+[k] l_{1}\right)\right]^{-1} \cosh \left[\delta_{A}\right] \cdot\left[E_{A}\right]+\cosh \left[\delta_{B}\right] \cdot\left[E_{B}\right]\right. \\
& \quad-\left[\operatorname{CH}\left(\left[\delta_{B}\right]-[k] l_{1}\right)+\sinh \left[\delta_{B}\right] \operatorname{CH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\left\{\mathrm { SH } \left([k] l_{1}\right.\right.\right. \\
& \left.\left.\left.\left.\quad+\left[\delta_{A}\right]\right)\right\}^{-1} \cosh [k] l_{1}\right]\left\{\operatorname{CH}\left(\left[\delta_{C}\right]+[k] l\right)\right\}^{-1} \cosh \left[\delta_{C}\right] \cdot\left[E_{C}\right]\right] \tag{22}
\end{align*}
$$

$$
\begin{align*}
& {\left[\beta_{1}\right]=\tanh \left[\delta_{A}\right] \cdot\left[\alpha_{1}\right]+\left[E_{A}\right]}  \tag{23}\\
& {\left[\beta_{2}\right]=\left\{\operatorname{CH}\left(\left[\delta_{C}\right]+[k] l\right)\right\}^{-1}\left\{\cosh \left[\delta_{C}\right] \cdot\left[E_{C}\right]-\operatorname{SH}\left(\left[\delta_{C}\right]+[k] l\right) \cdot\left[\alpha_{2}\right]\right\}} \tag{24}
\end{align*}
$$

where

$$
\begin{align*}
& {[\Omega]=\mathrm{CH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\left\{\mathrm{SH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\right\}^{-1}+\cosh \left[\delta_{B}\right]\left(\sinh \left[\delta_{B}\right]\right)^{-1}} \\
& \quad+\mathrm{CH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\left\{\mathrm{SH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1} \tag{25}
\end{align*}
$$

Now, the pseudo-hyperbolic tangent is defined as follows:
(Definition 5) $\quad \mathrm{TH}([A]+[B])=\mathrm{SH}([A]+[B])\{\mathrm{CH}([A]+[B])\}^{-1}$
Let $\left[\phi_{B}\right]$ be satisfied by eq. (27), then

$$
\begin{equation*}
\mathrm{TH}\left[\phi_{B}\right]=\left[\left\{\mathrm{TH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\right\}^{-1}+\left(\tanh \left[\delta_{B}\right]\right)^{-1}\right]^{-1} \tag{27}
\end{equation*}
$$

It is possible to explain that $\left[\phi_{B}\right.$ ] represents the position angle matrix at the end point of the line $I$ in the case of measuring from the end point $C$ of the line II. Therefore eq. (25) becomes

$$
\begin{equation*}
[\Omega]=\left\{\mathrm{SH}\left(\left[\delta_{A}\right]+[k] l_{1}\right)\right\}^{-1} \mathrm{SH}\left(\left[\delta_{A}\right]+[k] l_{1}+\left[\phi_{B}\right]\right)\left(\mathrm{SH}\left[\phi_{B}\right]\right)^{-1} \tag{28}
\end{equation*}
$$

Let [ $\bar{\phi}_{B}^{\prime}$ ] be satisfied by eq. (29)

$$
\begin{equation*}
\mathrm{TH}\left[\bar{\phi}_{B}^{\prime}\right]=\left[\left(\tanh \left[\delta_{B}\right]\right)^{-1}+\left\{\mathrm{TH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1}\right]^{-1} \tag{29}
\end{equation*}
$$

then it is possible to explain that $\left[\bar{\phi}_{B}^{\prime}\right]$ represents the position angle matrix at the sending point of the line II in the case of measuring from the sending point $A$ of the line I. Therefore eq. (25) is written as follows:

$$
\begin{equation*}
[\Omega]=\left\{\mathrm{SH}\left(\left[\delta_{C}\right]+[k] l_{2}\right)\right\}^{-1} \mathrm{SH}\left(\left[\delta_{C}\right]+[k] l_{2}+\left[\bar{\phi}_{B}^{\prime}\right]\right)\left(\mathrm{SH}\left[\bar{\phi}_{B}^{\prime}\right]\right)^{-1} \tag{30}
\end{equation*}
$$

Hence by eqs. (12), (21) ~(24), (28) and (30), $\left[e_{1}(x)\right]$ and $\left[e_{2}(x)\right]$ are

$$
\begin{align*}
& {\left[e_{1}(x)\right]=\operatorname{SH}\left\{[k]\left(l_{1}-x\right)+\left[\phi_{B}\right]\right\}\left\{\operatorname{SH}\left(\left[\delta_{A}\right]+[k] l_{1}+\left[\phi_{B}\right]\right)\right\}^{-1}} \\
& \quad \times \cosh \left[\delta_{A}\right]\left[E_{A}\right]+\mathrm{SH}\left([k] x+\left[\delta_{A}\right]\right)\left\{\mathrm{SH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1} \mathrm{SH}\left[\bar{\phi}_{B}^{\prime}\right] \\
& \quad \times\left\{\operatorname{SH}\left(\left[\delta_{C}\right]+[k] l_{2}+\left[\bar{\phi}_{B}^{\prime}\right]\right)\right\}^{-1} \operatorname{SH}\left(\left[\delta_{C}\right]+[k] l_{2}\right) \\
& \quad \times\left(\sinh \left[\delta_{B}\right]\right)^{-1} \cosh \left[\delta_{B}\right] \cdot\left[E_{B}\right]+\mathrm{SH}\left([k] x+\left[\delta_{A}\right]\right) \\
& \quad \times\left\{\operatorname{SH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1} \operatorname{SH}\left[\bar{\phi}_{B}^{\prime}\right]\left\{\operatorname { S H } \left(\left[\delta_{C}\right]\right.\right. \\
& \left.\left.\quad+[k] l_{2}+\left[\bar{\phi}_{B}^{\prime}\right]\right)\right\}^{-1} \cosh \left[\delta_{C}\right] \cdot\left[E_{C}\right]  \tag{31}\\
& {\left[e_{2}(x)\right]=\operatorname{SH}\left\{[k](l-x)+\left[\delta_{C}\right]\right\}\left\{\operatorname{SH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\right\}^{-1} \operatorname{SH}\left[\phi_{B}\right]} \\
& \quad \times\left\{\operatorname{SH}\left(\left[\delta_{A}\right]+[k] l_{1}+\left[\phi_{B}\right]\right)\right\}^{-1} \cosh \left[\delta_{A}\right] \cdot\left[E_{A}\right] \\
& \quad+\mathrm{SH}\left\{[k](l-x)+\left[\delta_{C}\right]\right\}\left\{\operatorname{SH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\right\}^{-1} \operatorname{SH}\left[\phi_{B}\right]
\end{align*}
$$

$$
\begin{align*}
& \mathrm{X}\left\{\mathrm{SH}\left(\left[\delta_{A}\right]+[k] l_{1}+\left[\phi_{B}\right]\right)\right\}^{-1} \mathrm{SH}\left(\left[\delta_{A}\right]+[k] l_{1}\right) \\
& \times\left(\sinh \left[\delta_{B}\right]\right)^{-1} \cosh \left[\delta_{B}\right] \cdot\left[E_{B}\right]+\mathrm{SH}\left\{[k]\left(x-l_{1}\right)+\left[\bar{\phi}_{B}^{\prime}\right]\right\} \\
& \times\left\{\operatorname{SH}\left(\left[\delta_{C}\right]+[k] l_{2}+\left[\bar{\phi}_{B}^{\prime}\right]\right)\right\}^{-1} \cosh \left[\delta_{C}\right] \cdot\left[E_{C}\right] \tag{32}
\end{align*}
$$

In the foregoing analysis the application of the general equations derived above is restricted to general circuits, since these simple multiconductor circuits adequately illustrate the methods of analysis with a minimum amount of algebraic exercise. But, in order to explain the phenomena quite easily, some of the transition-point networks must be zero and others must be infinite.

Consider, for example, the case, where the load impedance $z_{A}$ at the point $A$ is directly terminated, and the potential of lines at the point $A$ is given by $\left[E_{A}\right]$. Then

$$
\left[\delta_{A}\right]=[0]
$$

therefore the first term of the right hand in eq. (31) becomes

$$
\begin{equation*}
\mathrm{SH}\left\{[k]\left(l_{1}-x\right)+\left[\phi_{B}\right]\right\}\left\{\mathrm{SH}\left([k] l_{1}+\left[\phi_{B}\right]\right)\right\}^{-1}\left[E_{A}\right] \tag{33}
\end{equation*}
$$

The position angle matrices at the points $A$ and $B$, when the point $C$ is chosen to be the standard point, are written by $[k] l_{1}+\left[\phi_{B}\right]$ and $[k]\left(l_{1}-x\right)+\left[\phi_{B}\right]$, respectively, that is to say, by making use of the position angle matrix, eq. (33) is the acceptable solution for the potential of the cascade connected transmission line.

On the lines of arbitrary series impedance (load or internal impedance) in the power source $\left[E_{A}\right]$ at the sending point $A$, these solutions are of interest in showing the eq.(31)'s right hand with position angle matrix $\left[\delta_{A}\right]$.

Assuming the load impedance $\left[z_{C}\right]$ at $C$ to be zeros and the potential of lines at the point $A$ to be $\left[E_{C}\right]$, the third term of the right hand in eq. (31) is, by putting $\left[\delta_{C}\right]=$ [0],

$$
\begin{align*}
& \mathrm{SH}\left([k] x+\left[\delta_{A}\right]\right)\left\{\mathrm{SH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1} \mathrm{SH}\left[\bar{\phi}_{B}^{\prime}\right] \\
& \quad \times\left\{\mathrm{SH}\left([k] l_{2}+\left[\bar{\phi}_{B}^{\prime}\right]\right)\right\}^{-1}\left[E_{C}\right] \tag{34}
\end{align*}
$$

SH $\left[\bar{\phi}_{B}^{\prime}\right]\left\{\operatorname{SH}\left([k] l_{2}+\left[\bar{\phi}_{B}^{\prime}\right]\right)\right\}^{-1}\left[E_{C}\right]$ in eq. (34) is obviously the potential of line at the point $B$, and then eq. (34) represents the potential of line at the point $x$. Thus, eq. (34) gives the potentials of the transmission lines in the position angle matrix form. It is clear that the influence of the position angle matrix [ $\delta_{C}$ ] in eq. (31) is so vital as to change the characteristics of the potential at the receiving point $C$ through the medium of a series impedance (load or internal impedance) in the power source [ $E_{C}$ ].

In general, in the study of traveling waves due to multireflections, cosh [ $\delta] \cdot[E]$ must be used instead of the power source matrix $[E]$, when there is a load (or an internal) impedance in series of the power source, where [ $\delta$ ] is the position angle matrix at the point that the power source is located.

The second term of the right hand in eq. (31) involves numerical calculations that are time-consuming and prohibit the use of these expressions for rapid engineering calculations, because there are lines on both sides of power source $\left[E_{B}\right]$. Next equation in this term

$$
\begin{equation*}
\mathrm{SH}\left(\left[\delta_{C}\right]+[k] l_{2}\right)\left(\sinh \left[\delta_{B}\right]\right)^{-1} \cosh \left[\delta_{B}\right] \cdot\left[E_{B}\right] \tag{35}
\end{equation*}
$$

represents a rigid distribution of the line potential at the point $C$ that is calculated by using the position angle matrix of the base point $B$. Therefore the second term of the right hand in eq. (31) is the potential of the point $x$ in the form of the position angle matrix as well as the third term of the right hand in eq. (31).

Furthermore, let $\left[z_{B}\right]$ be $[0]$, then $\left[\delta_{B}\right]$ becomes [ 0 ], but $\left[\delta_{B}\right]$ can not be put in [0] directly at the second term of the right hand in eq. (31). Then this term must be reformed as follows :

$$
\begin{align*}
& \mathrm{SH}\left([k] x+\left[\delta_{A}\right]\right)\left\{\mathrm{SH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1}\left[[U]+\left(\cosh \left[\delta_{B}\right]\right)^{-1} \sinh \left[\delta_{B}\right]\right. \\
& \quad \times \mathrm{CH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\left\{\operatorname{SH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1}+\left(\cosh \left[\delta_{B}\right]\right)^{-1} \sinh \left[\delta_{B}\right] \\
& \left.\quad \times \mathrm{CH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\left\{\operatorname{SH}\left([k] l_{2}+\left[\delta_{C}\right]\right)\right\}^{-1}\right]^{-1}\left[E_{B}\right] \tag{36}
\end{align*}
$$

Therefore, let $\left[\delta_{B}\right.$ ] be [ 0 ] in eq. (36), then next equation is obtained

$$
\mathrm{SH}\left([k] x+\left[\delta_{A}\right]\right)\left\{\mathrm{SH}\left([k] l_{1}+\left[\delta_{A}\right]\right)\right\}^{-1}\left[E_{B}\right]
$$

and this expression gives the line potential at the point $x$ in the form of the position angle matrix of the base point $A$, where $\left[E_{B}\right]$ is the potential of the line at the point $B$.

Now, in addition to those wave trains may be found from the following transformation in eq. (31),

$$
\begin{array}{rlll}
{\left[E_{A}\right]} & \leftarrow & \rightarrow & {\left[E_{C}\right]} \\
{\left[\delta_{A}\right]} & \leftarrow & \rightarrow & {\left[\delta_{C}\right]} \\
l_{1} & \leftarrow & \rightarrow & l_{2} \\
{\left[\phi_{B}\right]} & \leftarrow & \rightarrow & {\left[\bar{\phi}_{B}^{\prime}\right]} \\
x & \leftarrow & \rightarrow & (l-x) \\
\left(l_{1}-x\right) & \leftarrow & \rightarrow & \left(x-l_{1}\right)
\end{array}
$$

It is sometimes possible to obtain the solution for a given problem by an ingenious interpretation of the solution for an entirely different one. In our analysis, it should be remarked that eq. (32), which has been obtained purely analytically, is properly matched with the results get from eq. (31) and Fig. 2.

As shown before, it is evident that the idea of the position angle matrix is useful for calculations of the potential at the multi-conductor system; owing to the following reasons :
(1) It will serve our porpose better to consider the position angle matrix, rather than the multireflections of the traveling waves.
(2) The required equations of the line potential can be written simply and directly from the system diagram without so much as calculating by making use of position angle matrices.

## 4. Conclusion

It has been found out that the addition theorem of the hyperbolic function of matrix holds good by adopting a new generalized manner under an extending rule on the application of Sylvester's expansion theorem even if matrices do not satisfy commutative law for multiplication. Then, by introducing position angle matrices to the multi-conductor system we can very easily obtain the potential on the dissymmetrical multi-conductor system with three boundary conditions. This does not lead only to carry out the computations with ease, but also this leads to get equations of potentials of the cascade connected multi-conductor system from the diagram directly using the position angles; thus the solution can be got without solving the boundary problem of partial differential equations.

## References

1) Y. Inagaki and M. Kido, Trans. of IEE of Japan, 99-A, 17 (1979)
2) S. Hayashi, A. C. Circuit Theory and Transient Phenomena, P. 502, OHM-SHA, Tokyo (1961)
3) S. Hayashi, Surges on Transmission Systems, P. 106, DENKI-SHOIN, INC., Kyoto (1955)

[^0]:    * Department of Electrical Engineering, College of Engineering.

