A Hybrid Method to Improve Forecasting Accuracy Utilizing Genetic Algorithm

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Yuki Higuchi, Kazuhiro Takeyasu

Abstract. *Focusing that the equation of exponential smoothing method (ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the following method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the stock market price data of glass and the quarrying companies. Genetic Algorithm is utilized to search optimal weights for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. The new method shows that it is useful for the time series that has various trend characteristics. The effectiveness of this method should be examined in various cases.*

Key Words: *minimum variance, exponential smoothing method, forecasting, trend, genetic algorithm*

1. INTRODUCTION

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM) [1]-[4]. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend[5] , utilizing Kalman Filter[6], Bayes Forecasting[7], adaptive ESM[8], exponentially weighted Moving Averages with irregular updating periods[9], making averages of forecasts using plural method[10] are presented. For example, Maeda[6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he couldn't grasp observation noise. It can be said that it doesn't pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii[11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before[13]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as shipping data, trend removing method is devised. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the stock market price data of glass and the quarrying companies. Genetic Algorithm is utilized to search optimal weights for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of the previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3.The Monthly Ratio is referred in section 4. Measuring method of forecasting accuracy is exhibited in section 5. GA model to search optimal weights for the weighting parameters of liner and non-linear function is introduced in 6. Forecasting is executed in section 7, and estimation accuracy is examined.

2. DESCRIPTION OF ESM USING ARMA MODEL[13]

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha(x_t - \hat{x}_t) \quad (1)$$

$$= \alpha x_t + (1-\alpha)\hat{x}_t \quad (2)$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$
x_t : realized value at t
α : smoothing constant $(0 < \alpha < 1)$

(2) is re-stated as:

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l} \quad (3)$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \quad (4)$$

Generally, (p,q) order ARMA model is stated as:

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \quad (5)$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1,2,\cdots,N,\cdots$
$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (5) is supposed to satisfy convertibility condition.
Utilizing the relation that:

$$E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0$$

we get the following equation from (4).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \quad (6)$$

Operating this scheme on $t+1$, we finally get:

$$\begin{aligned}\hat{x}_{t+1} &= \hat{x}_t + (1-\beta)e_t \\ &= \hat{x}_t + (1-\beta)(x_t - \hat{x}_t)\end{aligned} \tag{7}$$

If we set $1-\beta=\alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model.

Comparing (4) with (5) and using (1) and (7), we get:

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta \end{cases} \tag{8}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model becomes non-linear equations which are described below.
Let (5) be:

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \tag{9}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{10}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (9), (10), we get the following non-linear equations which are well known[3].

$$\left.\begin{aligned} &\tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{cases} \\ &\tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \end{aligned}\right\} \tag{11}$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (4) (5) (8) (11), we get:

$$\begin{aligned} &q = 1 \\ &a_1 = -1 \\ &b_1 = -\beta = \alpha - 1 \\ &\tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ &\tilde{r}_1 = b_1\sigma_e^2 \end{aligned} \tag{12}$$

If we set:

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \tag{13}$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2} \tag{14}$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \tag{15}$$

In order to have real roots, ρ_1 must satisfy

$$\left|\rho_1\right| \le \frac{1}{2} \tag{16}$$

As

$$\alpha = b_1 + 1$$

b_1 is within the range of

$$-1 < b_1 < 0$$

Finally we get

$$b_1 = \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1}$$

$$\alpha = \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \quad (17)$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way.

3. TREND REMOVAL METHOD

As ESM is a one of a linear model, forecasting accuracy for the time series with non-linear trend is not necessarily good. How to remove trend for the time series with non-linear trend is a big issue in improving forecasting accuracy. In this paper, we devise to remove this non-linear trend by utilizing non-linear function.

As a trend removal method, we describe linear and non-linear function, and the combination of these.

[1] Linear function

We set:

$$y = a_1 x + b_1 \quad (18)$$

as a linear function, where x is a variable, for example, time and y is a variable, for example, shipping amount, a_1 and b_1 are parameters which are estimated by using least square method.

[2] Non-linear function

We set:

$$y = a_2 x^2 + b_2 x + c_2 \quad (19)$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \quad (20)$$

as a 2nd and a 3rd order non-linear function. (a_2, b_2, c_2) and (a_3, b_3, c_3, d_3) are also parameters for a 2nd and a 3rd order non-linear functions which are estimated by using least square method.

[3] The combination of a linear and a non-linear function
We set:

$$y = \alpha_1 (a_1 x + b_1) + \alpha_2 (a_2 x^2 + b_2 x + c_2) + \alpha_3 (a_3 x^3 + b_3 x^2 + c_3 x + d_3) \tag{21}$$

$$0 \le \alpha_1 \le 1 \quad , 0 \le \alpha_2 \le 1 \quad , 0 \le \alpha_3 \le 1$$
$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \tag{22}$$

as the combination linear and 2nd order non-linear and 3rd order non-linear function. Trend is removed by dividing the original data by (21). The optimal weighting parameter $\alpha_1, \alpha_2, \alpha_3$ are determined by utilizing GA. GA method is precisely described in section 6.

4. MONTHLY RATIO

For example, if there is the monthly data of L years as stated bellow:

$$\{x_{ij}\} \quad (i = 1, \cdots, L) \quad (j = 1, \cdots, 12)$$

where $x_{ij} \in R$ in which j means month and i means year and x_{ij} is a data of i-th year, j-th month, then, monthly ratio $\tilde{x}_j \ (j = 1, \cdots, 12)$ is calculated as follows.

$$\tilde{x}_j = \frac{\frac{1}{L} \sum_{i=1}^{L} x_{ij}}{\frac{1}{L} \cdot \frac{1}{12} \sum_{i=1}^{L} \sum_{j=1}^{12} x_{ij}} \tag{23}$$

Monthly trend is removed by dividing the data by (23). Numerical examples for both of the monthly trend removal case and the non-removal case are discussed in section 7.

5. FORECASTING ACCURACY

Forecasting accuracy is measured by calculating the variance of the forecasting error.
Variance of forecasting error is calculated by:

$$\sigma_\varepsilon^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\varepsilon_i - \bar{\varepsilon})^2 \tag{24}$$

where, forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{25}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{26}$$

6. SEARCHING OPTIMAL WEIGHTS UTILIZING GA

6.1 Definition of the problem

We search $\alpha_1, \alpha_2, \alpha_3$ of (21) which minimizes (24) by utilizing GA. By (22), we only have to determine α_1 and α_2. $\sigma_\varepsilon^{\ 2}$((24)) is a function of α_1 and α_2, therefore we express them as $\sigma_\varepsilon^2(\alpha_1, \alpha_2)$. Now, we pursue the following:

$$\begin{aligned} &\text{Minimize: } \sigma_\varepsilon^2(\alpha_1, \alpha_2) \\ &\text{subject to: } 0 \le \alpha_1 \le 1 \quad , 0 \le \alpha_2 \le 1 \quad , \alpha_1 + \alpha_2 \le 1 \end{aligned} \tag{27}$$

We do not necessarily have to utilize GA for this problem which has small number of variables. Considering the possibility that variables increase when we use logistics curve etc. in the near future, we want to ascertain the effectiveness of GA.

6.2 The structure of the gene

Gene is expressed by the binary system using {0,1} bit. Domain of variable is [0,1] from (22). We suppose that variables take down to the second decimal place. As the length of domain of variable is 1-0=1, seven bits are required to express variables. The binary bit strings <bit6, ~ ,bit0> is decoded to the [0,1] domain real number by the following procedure. [14]

Procedure 1: Convert the binary number to the binary-coded decimal.

$$\left(\langle bit_6, bit_5, bit_4, bit_3, bit_2, bit_1, bit_0 \rangle\right)_2 = \left(\sum_{i=0}^{6} bit_i 2^i\right)_{10} = X' \qquad (28)$$

Procedure 2: Convert the binary-coded decimal to the real number.

$$\text{The real number} = (\text{Left hand starting point of the domain}) + X'((\text{Right hand ending point of the domain})/(2^7 - 1)) \qquad (29)$$

The decimal number, the binary number and the corresponding real number in the case of 7 bits are expressed in Table 6-1.

Table 6-1: Corresponding table of the decimal number, the binary number and the real number

The decimal number	The binary number Position of the bit							The Corresponding real number
	6	5	4	3	2	1	0	
0	0	0	0	0	0	0	0	0.00
1	0	0	0	0	0	0	1	0.01
2	0	0	0	0	0	1	0	0.02
3	0	0	0	0	0	1	1	0.02
4	0	0	0	0	1	0	0	0.03
5	0	0	0	0	1	0	1	0.04
6	0	0	0	0	1	1	0	0.05
7	0	0	0	0	1	1	1	0.06
8	0	0	0	1	0	0	0	0.06
…								…
126	1	1	1	1	1	1	0	0.99
127	1	1	1	1	1	1	1	1.00

1 variable is expressed by 7 bits, therefore 2 variables needs 14 bits. The gene structure is exhibited in Table 6-2.

Table 6-2: The gene structure

α_1							α_2						
Position of the bit													
13	12	11	10	9	8	7	6	5	4	3	2	1	0
0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1

6.3 The flow of Algorithm

The flow of algorithm is exhibited in Figure 6-1.

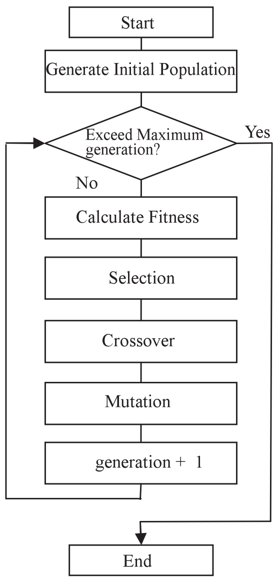

Figure 6-1: The flow of algorithm

A. Initial Population

Generate M initial population. Here, $M=100$. Generate each individual so as to satisfy (22).

B. Calculation of Fitness

First of all, calculate forecasting value. There are 36 monthly data for each case. We use 24 data (1st to 24th) and remove trend by the method stated in section 3. Then we calculate monthly ratio by the method stated in section 4. After removing monthly trend, the method stated in section 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (24). Calculation of fitness is exhibited in Figure 6-2.

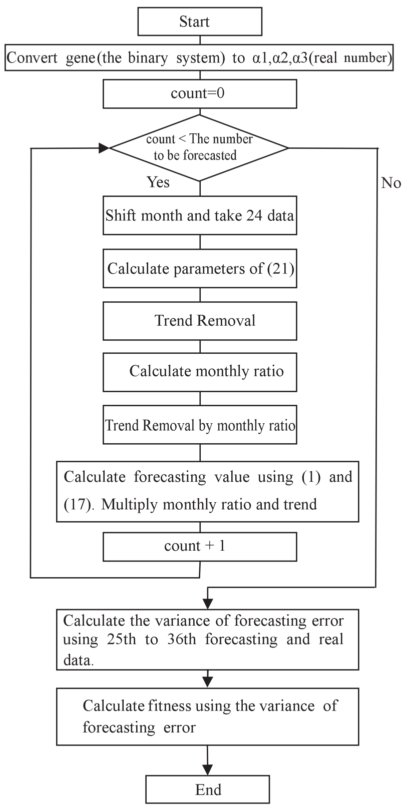

Figure 6-2: The flow of calculation of fitness

Scaling[15] is executed such that fitness becomes large when the variance of forecasting error becomes small. Fitness is defined as follows.

$$f(\alpha_1,\alpha_2)=U-\sigma_\varepsilon{}^2(\alpha_1,\alpha_2) \tag{30}$$

Where U is the maximum of $\sigma_\varepsilon{}^2(\alpha_1,\alpha_2)$ during the past W generation. Here, W is set to be 5.

C. Selection

Selection is executed by the combination of the general elitist selection and the tournament selection. Elitism is executed until the number of new elites reaches the predetermined number. After that, tournament selection is executed and selected.

D. Crossover

Crossover is executed by the uniform crossover. Crossover rate is set as follows.

$$P_c=0.7 \tag{31}$$

E. Mutation

Mutation rate is set as follows.

$$P_m=0.05 \tag{32}$$

Mutation is executed to each bit at the probability P_m, therefore all mutated bits in the population M becomes $P_m \times M \times 14$.

7. NUMERICAL EXAMPLE

7.1 Application to stock market price data

The stock market price data of glass and the quarrying companies for 4 cases from January 2008 to December 2010 are analyzed. Furthermore, GA results are compared with the calculation results of all considerable cases in order to confirm the effectiveness of GA approach. First of all, graphical charts of these time series data are exhibited in Figure 7-1, 7-2, 7-3, 7-4.

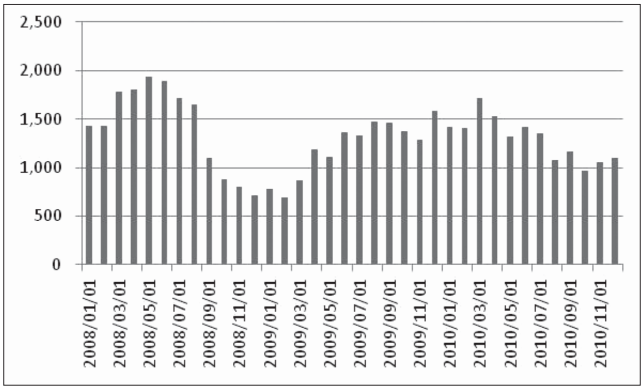

Figure 7-1: Data of OHARA INC

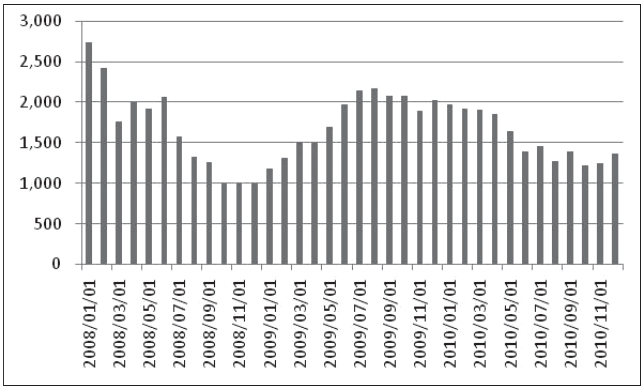

Figure 7-2: Data of NGK INSULATORS, LTD.

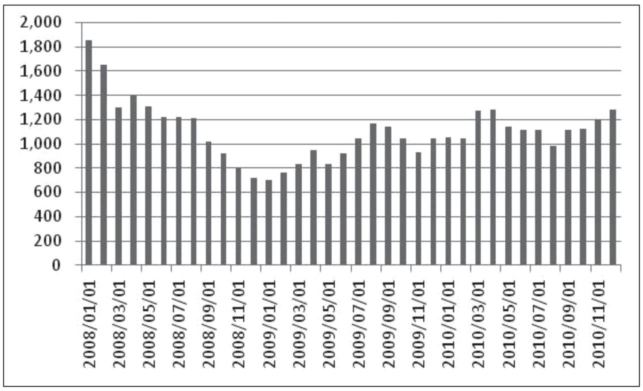

Figure 7-3: Data of NGK SPARK PLUG CO., LTD.

7.2 Execution Results

GA execution condition is exhibited in Table 7-1.

Table 7-1: GA Execution Condition

GA Execution Condition	
Population	100
Maximum Generation	50
Crossover rate	0.7
Mutation ratio	0.05
Scaling window size	5
The number of elites to retain	2
Tournament size	2

We made 10 times repetition and the maximum, average, minimum of the variance of forecasting error and the average of convergence generation are exhibited in Table 7-2 and 7-3.

Table 7-2: GA execution results (Monthly ratio is not used)

Food No	The variance of forecasting error			Average of convergence generation
	Maximum	Average	Minimum	
OHARA	22,213.29371	19,129.17239	18,801.24219	13.8
NGK INSULATORS	12,800.70989	9,976.67064	9,919,03719	10.6
NGK SPARK PLUG	8,988.04498	8,898.95340	8,895.00481	7.2

Table 7-3: GA execution results (Monthly ratio is used)

Food No	The variance of forecasting error			Average of convergence generation
	Maximum	Average	Minimum	
OHARA	27,614.95566	27,557.59505	27553.6306	8.5
NGK INSULATORS	26,820.46758	25,046.31301	24,962.91542	7.2
NGK SPARK PLUG	12,746.25679	12,330.50866	12,181.61376	5.2

The case monthly ratio is not used is smaller than the case monthly ratio is used concerning the variance of forecasting error in every company. It may be because stock market price does not have definite seasonal trend in general.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

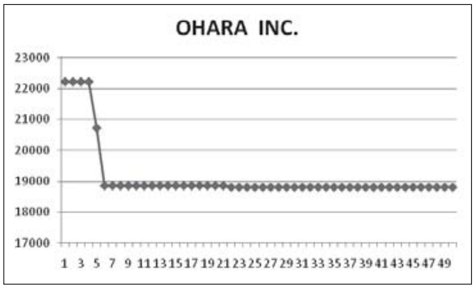

Figure 7-5: Convergence Process in the case of OHARA INC. (Monthly ratio is not used)

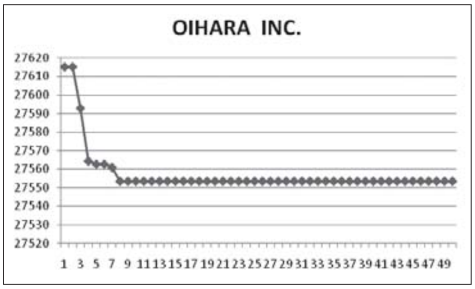

Figure 7-6: Convergence Process in the case of OHARA INC. (Monthly ratio is used)

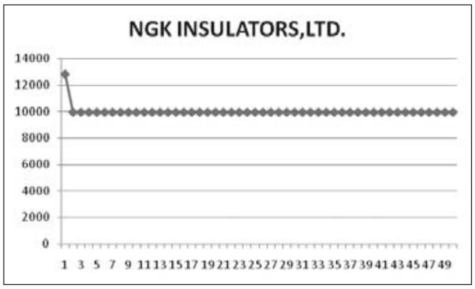

Figure 7-7: Convergence Process in the case of NGK INSURATORS,LTD. (Monthly ratio is not used)

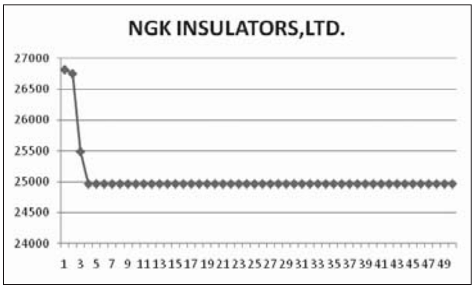

Figure 7-8: Convergence Process in the case of NGK INSULATORS,LTD. (Monthly ratio is used)

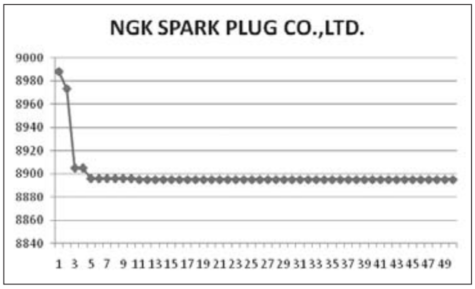

Figure 7-9: Convergence Process in the case of NGK SPARK PLUG CO., LTD（Monthly ratio is not used）

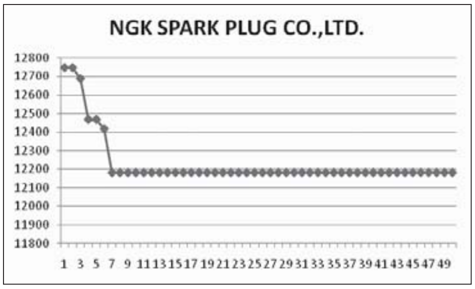

Figure 7-10: Convergence Process in the case of NGK SPARK PLUG CO.,LTD.（Monthly ratio is used）

Next, optimal weights and their genes are exhibited in Table 7-4, 7-5.

Table 7-4: Optimal weights and their genes（Monthly ratio is not used）

Data	α_1	α_2	position of the bit													
			13	12	11	10	9	8	7	6	5	4	3	2	1	0
OHARA	0.76	0.24	1	0	1	1	1	0	1	0	0	0	0	1	0	0
NGK INSULATORS	0.54	0.46	1	0	0	0	1	0	1	0	0	0	0	0	0	0
NGK SPARK PLUG	0.40	0.60	0	1	1	0	0	1	1	0	0	0	0	0	0	0

Table 7-5: Optimal weights and their genes（Monthly ratio is used）

Data	α_1	α_2	position of the bit													
			13	12	11	10	9	8	7	6	5	4	3	2	1	0
OHARA	0.20	0.80	0	0	1	0	1	0	0	0	0	0	0	1	0	1
NGK INSULATORS	0.20	0.80	0	0	1	1	0	0	1	0	0	0	0	0	0	0
NGK SPARK PLUG	0.00	1.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0

In the case monthly ratio is not used, the combination of linear and $2^{nd}+3^{rd}$ order non-linear function model is best in OHARA. On the other hand, the combination of linear and 3^{rd} order non-linear function model is best in NGK INSULATORS and NGK SPARK PLUG. In the case monthly ratio is used, the combination of linear and $2^{nd}+3^{rd}$ order non-linear function model is best in OHARA. On the other hand, the combination of linear and 3^{rd} order non-linear function model is best in NGK INSULATORS. NGK SPARK PLUG case

selected the only 3rd order non-linear function model as the best one.
Parameter estimation results for the trend of equation (21) using least square method are exhibited in Table 7-6 for the case of 1st to 24th data.

Table 7-6: Parameter estimation results for the trend of equation (21)

Data	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3	d_3
OHARA	-15.47	1512.82	5.03	-141.24	2057.82	0.37	-8.82	0.13	1733.68
NGK INSULATORS	-1.10	1748.97	8.77	-220.25	2698.61	-0.64	32.89	-466.39	3263.00
NGK SPARK PLUG	-23.83	1380.77	4.50	-136.38	1868.48	-0.17	10.76	-200.23	2014.89

Trend curves are exhibited in Figure 7-13 - 7-16.

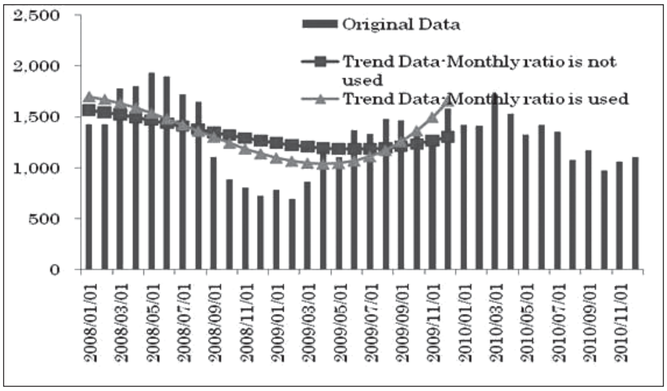

Figure 7-13: Trend of OHARA INC.

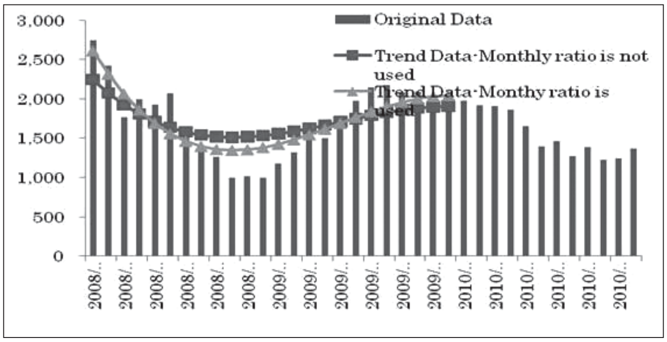

Figure 7-14: Trend of NGK INSULATORS, LTD.

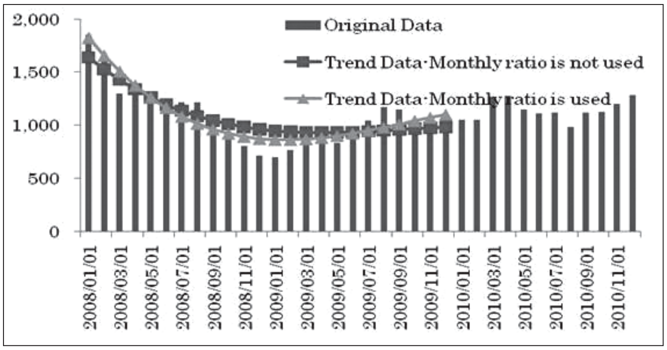

Figure 7-15: Trend of NGK SPARK PLUG CO.,LTD.

Calculation results of Monthly ratio for 1st to 24th data are exhibited in Table 7-7.

Table 7-7: Parameter Estimation result of Monthly ratio

Date.	1	2	3	4	5	6	7	8	9	10	11	12
OHARA	0.78	0.76	0.96	1.15	1.17	1.29	1.21	1.24	1.01	0.87	0.77	0.90
NGK INSULATORS	0.95	0.98	0.93	1.01	1.09	1.24	1.14	1.06	1.01	0.90	0.85	0.87
NGK SPARK PLUG	0.91	0.94	0.91	1.05	0.99	1.02	1.11	1.20	1.10	1.00	0.88	0.89

Estimation result of the smoothing constant of minimum variance for the 1st to 24th data are exhibited in Table 7-8, 7-9.

Table 7-8: Smoothing constant of Minimum Variance of equation (17)
(Monthly ratio is not used)

Date	$\rho 1$	α
OHARA	-0.049840	0.950076
NGK INSULATORS	-0.032747	0.967218
NGK SPARK PLUG	-0.352967	0.586756

Table 7-9: Smoothing constant of Minimum Variance of equation (17)
(Monthly ratio is used)

Date,	ρ1	α
OHARA	-0.394392	0.511489
NGK INSULATORS	-0.330435	0.622468
NGK SPARK PLUG	-0.130233	0.867480

Forecasting results are exhibited in Table 7-17 - 7-20.

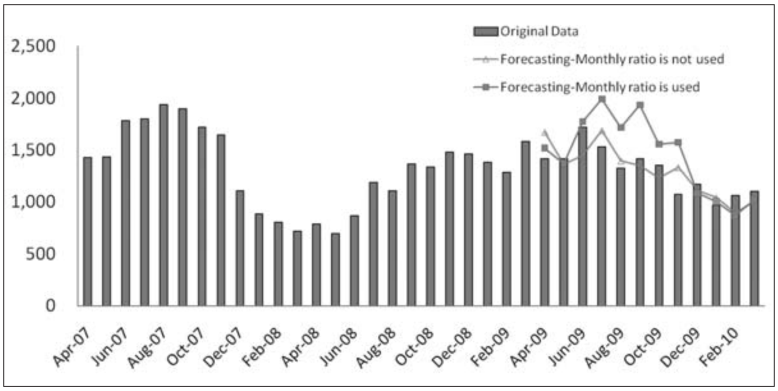

Figure 7-17: Forecasting Result of OHARA INC.

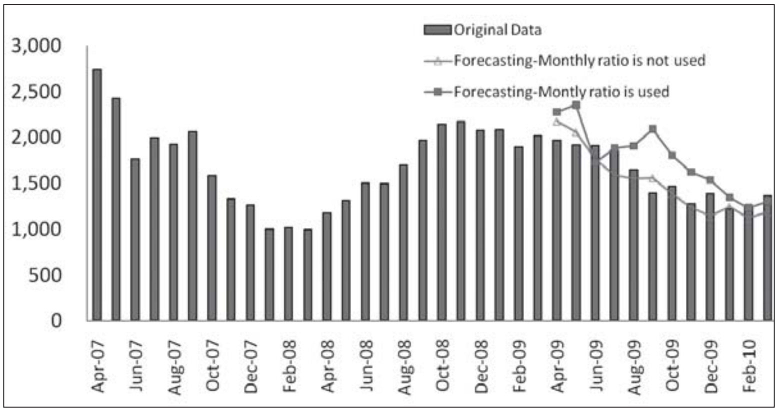

Figure 7-18: Forecasting Result of NGK INSULATORS, LTD.

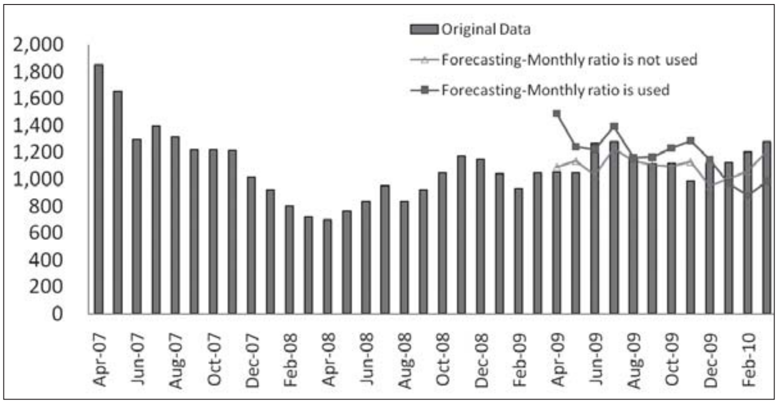

Figure 7-19: Forecasting Result of SPARK PLUG CO,.LTD.

7.3 Remarks

In all cases, that monthly ratio was not used had a better forecasting accuracy. OHARA had a good result in the combination of linear, 2^{nd} and 3^{rd} order non-linear function model. NGK INSUKATORS had a good result in the combination of linear and 3^{rd} order non-linear function model. NGK SPARK PLUG case selected the only 3^{rd} function model as the best one.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

8. CONCLUSION

Focusing on the idea that the equation of exponential smoothing method (ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the stock market price data of glass and the quarrying companies. The combination of linear and non-linear function was also introduced in trend removal. Genetic Algorithm was utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data. The new method shows that it is useful for the time series that has various trend characteristics. The effectiveness of this method should be examined in various cases.

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