Particle Movement In a Vertically Oscillating Liquid

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The hydrodynamic retardation of the liquid drops and the bubbles in an oscillating liquid has been calculated on the basis of the non-linear Langevin equation with the quasi-steady state assumption. For gas bubbles, the theoretical explanations of the retardation coincide with the experimental findings. For liquid drops, the experimental retardation due to fluid oscillation is more than that predicted by the theory. The holdup data in multistage vibrating disk columns are discussed in connection with the theoretical calculations of the hydrodynamic retardation.

1. Introduction

Fluid pulsation and vibrating agitation are widely applied in the chemical process industries in order to promote phase contacting efficiency; pulsed columns or vibratory disk columns have been successfully employed as extractors or chemical reactors in which more than two phases of gases, liquids, and/or solids are present simultaneously[12), 13), 15)]. A high efficiency of phase contacting in these types of equipment is due mainly to two factors: increased retention time of dispersed particles in the fluid and increased interfacial area by complete suspension and breakup of particles in the flow field with high velocity gradient. These two factors, velocity retardation and particle breakup, give rise to a remarkable increase in holdup of the dispersed phase and the resulting increase in interfacial area between the phases involved.

So far as the particle size is concerned, the effects of pulsation or vibration on the dispersed particle size have been investigated by many researchers both theoretically and experimentally, and one can estimate the particle size as a function of the intensity of pulsation or vibration by using a correlating equation. On the other hand, in spite of the fact that considerable light has been thrown on the mechanism of particle motion in a pulsating liquid[22-29)] there are many problems still unsolved. In particular it is quite uncertain whether retardation or breakup contributes more to the enhancement of the contacting efficiency in gas-liquid or liquid-liquid systems. In the present study, we have discussed the effect of liquid oscillation on the mean settling velocity of small bubbles or droplets based on some theoretical studies[3), 4), 5), 6), 7), 8)]. A non-linear Langevin equation describing particles movement in an oscillating liquid is numerically solved and the effect of oscillation on the mean settling velocity of suspended particles is discussed under a quasi-steady state assumption with respect to the drag coefficient. The contribution of the velocity retardation to an increase in dispersed phase holdups is also calculated and the results are compared with the experimental ones obtained in a multistage vibrating disk column.

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2. Theory

The general differential equation for a spherical particle travelling in a straight line through a viscous liquid at low Reynolds number was given by Basset[1), 2)].

$$m\left(\frac{du}{dt}\right) = (m - m')g - \chi m' \frac{du}{dt} + \frac{18\, m'\, \mu u}{d^2\, \rho} + \frac{9\, m'}{d}\sqrt{\frac{\mu}{\rho \pi}} \cdot \int_0^t F'(t-\tau) / \sqrt{\tau}\, d\tau \tag{1}$$

where m = mass of the particle, m' = mass of the liquid displaced by the particle, u = velocity of the particle, χ = coefficient of virtual mass, d = diameter of the particle, ρ = density of the liquid, μ = viscosity of the liquid and $F' = du/dt$. The first term in the right hand side of Eq. (1) is the gravitational force acting on the particle. The second term is the resistance of a fluid to the accelerated motion of the particle. It is equivalent to an added mass of the particle by the factor χ of the mass of the liquid displaced. The third term is the viscous resistance given by Stokes' law. The last term, called the Basset term, accounts for the effect of the deviation in liquid flow pattern from steady state. If the particle is situated in a liquid moving at the velocity of v, the equation of motion of the particle can be obtained by replacing u in the right hand side of Eq. (1) by $(u\text{-}v)$ as follows:

$$(m + m'\chi)\frac{du}{dt} = (m - m')g + (1 + \chi)\, m' \frac{dv}{dt} + 3\pi d\mu\,(u - v) + \frac{3}{2} d^2 \sqrt{\pi \rho \mu} \int_0^t \frac{dv/dt - du/dt}{\sqrt{t - t'}}\, dt' \tag{2}$$

The virtual mass coefficient χ for sinusoidally oscillating liquid equals to 1/2 at low Reynolds number ($Re < 1$). The Basset theoretical treatment is limited to the particle motion in a laminar flow field. In a turbulent flow field, on the other hand, very little is known about the interaction between particle motion and turbulent agitation in the surrounding fluid.

If the Stokes viscous term $3\pi d\mu\ (u\text{-}v)$ in Eq. (2) can be replaced by the equivalent steady state expression for higher Reynolds number, Eq. (2) is rewritten as follows :

$$(m + m'\chi)\frac{du}{dt} = (m - m')\, g + (1 + \chi)\, m' \frac{dv}{dt} + \frac{\pi}{8} C_D d^2 \rho\, |u - v|^n . \operatorname{sgn}(u - v) + \frac{3}{2} d^2 \sqrt{\pi \rho \mu} \int_0^t \frac{dv/dt - du/dt}{\sqrt{t - t'}}\, dt' \tag{3}$$

where $n = 1$ for $Re < 1$ and $n = 2$ for $Re > 1$.

The velocity of a sinusoidally oscillating fluid is given by

$$v = 2\pi A f \cos(2\pi f t). \tag{4}$$

For the case that a spherical particle rises through an oscillating liquid where a turbulent flow field is fully developed, the virtual mass coefficent is assumed by

$$\chi = 1/2 \tag{5}$$

In the present study, the fundamental equation Eq. (3) is solved numerically by using a

quasi-steady state approach, i.e., when the Basset term is zero and the drag coefficient can be calculated from the instantaneous Reynolds number ($d \mid u - v \mid \rho/\mu$).

The validity of the quasi-steady state assumption (QSSA) is supported by Shih et al.[9] and Keulegan et al.[10] who dealt with drag to oscillating flat plates, and found that the coefficient of drag decreases with the period of oscillation, from a larger initial value to a steady state one, they also found that when the period parameter, defined by

$$Per = u_{max}\, T/W$$

is larger than about 10, the drag coefficient is almost independent of the period of oscillation. Here u_{max}, T and W are the maximum velocity of the plate, period of oscillation and width of the plate, respectively. Therefore, it can be assumed that the developed flow is established in an operational range of $Per > 10$. If the plate width is taken as the particle diameter d, the condition for developed flow is as follows:

$$\frac{u_{max}\, T}{W} = \frac{2\pi A f\,(1/f)}{d} > 10 \qquad \text{or} \quad \frac{A}{d} > 1.59 \tag{6}$$

In most cases of the present study, the value of A/d exceeds 2.0 and it seems reasonable to assume the quasi-steady state approach.

Neglecting the transient component of the drag (the last term in Eq. (3)), we evaluate the viscous term in Eq. (3) by the following relation:

$$\text{(viscous term)} = \frac{\pi}{8} C_D\, d^2 \rho\, (u - v)^2 \ \operatorname{sgn}(u - v) \tag{7}$$

where

$$C_D = \begin{cases} 24/Re & (10^{-4} < Re < 2.0) \\ 18.5/Re^{0.6} & (2.0 < \mathrm{Re} < 500) \\ 0.44 & (500 < Re < 2 \times 10^5) \end{cases} \tag{8}$$

for the solid particle in the fluid, and

$$C_D = \begin{cases} 5 \times 10^{-4}\, Re + 0.4 & (400 < Re \leqq 700) \\ 8.2 \times 10^{-4}\, Re + 0.17 & (700 < Re \leqq 1000) \\ 2.5 \log_{10} Re - 6.5 & (1000 < Re \leqq 4000) \\ 2.55 & (4000 < Re) \end{cases} \tag{9}$$

for the gas bubble. Equation (8) is applied as an approximation to the liquid-liquid system for $Re < 1000$ and the gas-liquid system for $Re < 400$[11].

3. Experimental

Experimental apparatus for measuring the mean settling velocity in an oscillating fluid is shown in Fig. 1. Pyrex glass columns (5.0 cm, 2.8 cm and 2.5 cm I.D.) were used. The fluid oscillation was introduced by using a compressed-air-triggered pulse generator. The vibration was operated at frequencies from 0 to 3.5 Hz with amplitude up to 4.7 cm. The

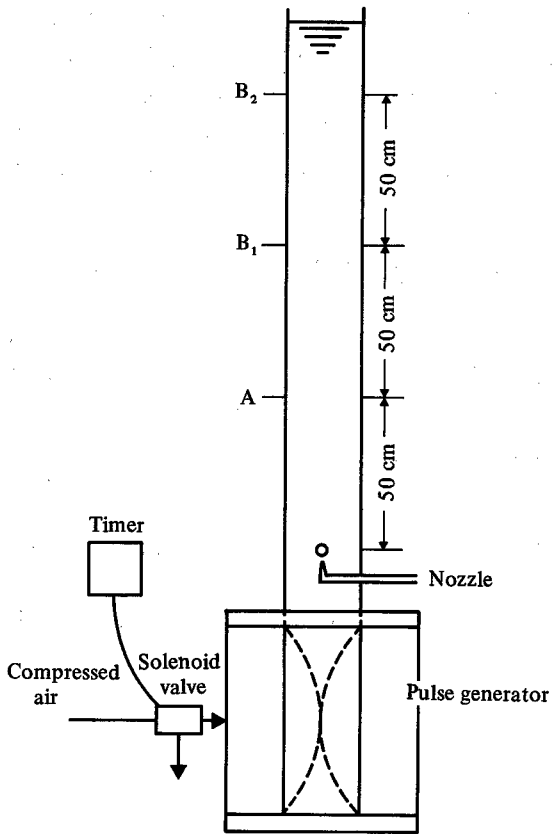

Fig. 1. Experimental apparatus for particle rising velocity measurement.

amplitude of vibration could be varied by altering the inside diameter of the glass column. The air bubbles with sizes from 0.2 to 0.6 cm and toluene drops 0.1 to 0.5 cm were rapidly and conveniently introduced by a syringe injector. The sizes of the particles were evaluated by analyzing the free settling velocities determined under no vibration operation between two parts (A-B1 or B2 in Fig. 1.). Immediately after introduction of the particles through the nozzle, the vibrator was operated and then the rise of the particles between two fixed marks, 50 cm or 100 cm apart, was visually timed by a stop watch, as shown in Fig. 1. The measurements were conducted at room temperature (18 to 25°C). Although the wave form of vibration may not be exactly sinusoidal, especially in low vibration frequencies, we assume the sinusoidal wave form in the present study. Judging from the structure of the pulse generator used, the wave form of fluid oscillation introduced seems to be symmetrical triangular (in higher frequency operation) or trapezoidal wave form (in lower frequency operation). If the wave form is symmetrical, however, the effect of wave form on the mean rising velocities may not be significant[30].

4. Results and Discussion

The trajectory and the mean rising velocity of a particle suspended in an oscillating liquid were numerically calculated by using a digital computer. The Runge-Kutta-Gill method was employed to solve Eq. (3) subject to the drag coefficient of Eqs. (8) or (9).

The computed effect of fluid (water) oscillation on the particles (toluene drops or air bubbles) retardation is shown in Figs. 2 and 3 as a function of oscillating velocity with the amplitude as a parameter. As can be seen from these figures, the mean settling velocity of a particle decreases appreciably with increasing the oscillating velocity of the fluid. The retardation is increased with decreasing the amplitude when the product values of Af are constant. Several experimental findings that claim the dispersed phase holdup and mass transfer characteristics in pulsed columns or vibrating disk columns can be described only in terms of the product value of A and f are thus inconsistent with the present calculated results if the hydrodynamic retardation due to fluid oscillation is a predominant factor in controlling the dispersed phase flow characteristics in these contactors. When the contactors with vibratory agitation are operated under wide ranges of amplitude and frequency, their holdup and mass transfer characteristics should be examined by taking account of the effects of A and f separately.

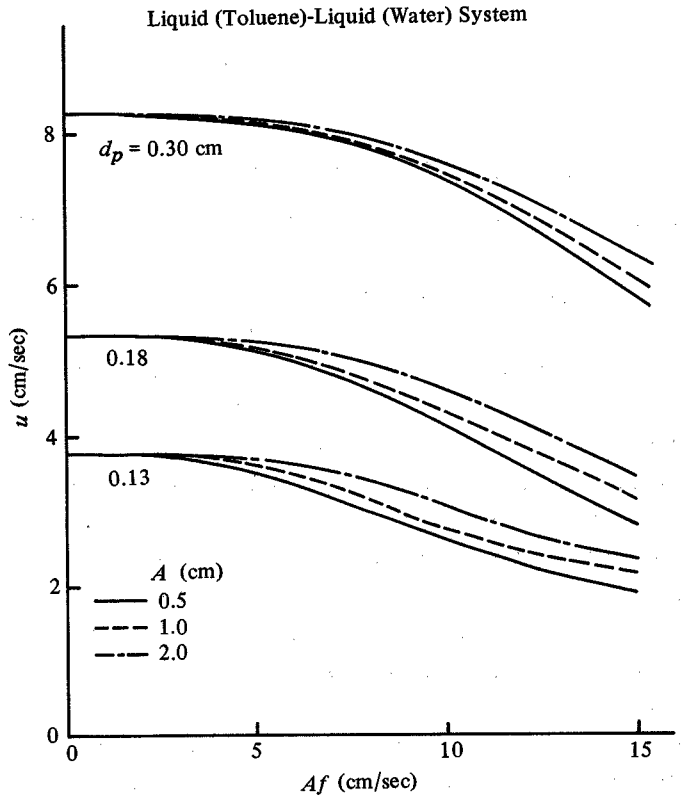

Fig. 2. Effect of the oscillating velocity of water on the mean rising velocity of the toluene drops.

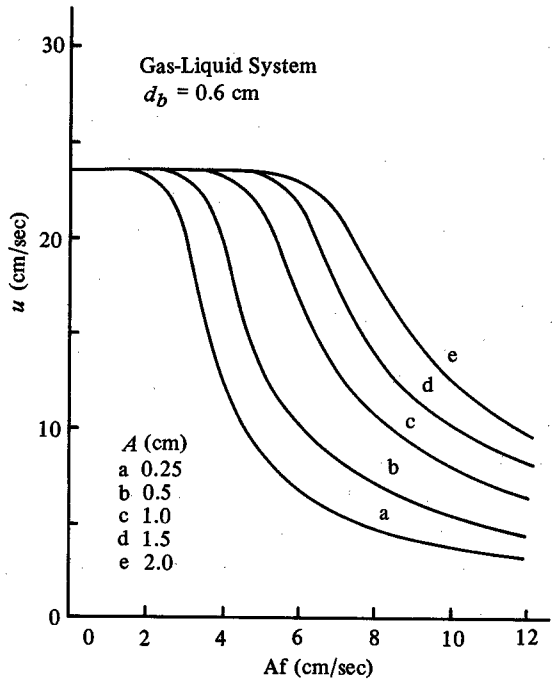

Fig. 3. Effect of the oscillating velocity of water on the mean rising velocity of the air bubbles.

4.1. Comparison between the Experimental and Calculated Results

The experimental rising velocities of liquid drops (toluene) and air bubbles in oscillating water are shown in Figs. 4 and 5, respectively. The solid lines are calculated by Eq. (3). For the liquid-liquid system, the experimental rising velocity is more retarded than that predicted based on the QSSA. On the other hand, for gas-liquid system, the calculated velocity on the assumption of the QSSA is found to agree approximately with the experimental results. The findings of the liquid-liquid system in the present study are similar to those obtained for solid particles retardation by Tunstall and Houghton[22] and Baird et al.[7]. Tunstall and Houghton found that for the larger solid spheres (same order in sizes as that in this study), the experimental velocities were lower than those predicted theoretically and those differences are primarilly attributed to oscillation-induced increases in drag coefficient. Baird et al. have concluded that the increased drag force is caused by the periodic vortex shedding during oscillation. In addition to these phenomena, the particle shape for gas-liquid and liquid-liquid systems may be easily deformed by continuous fluid oscillation and this may increase more the drag force. In Fig. 4, the dashed line corrects the drag coefficient to meet the experimental velocities. The increased drag coefficient due to oscillation is plotted in Fig. 6 as a function of Af. The increased drag coefficient, for the larger particles is more than that for the smaller ones. This experimental finding may be mainly due to shape deformation during the oscillation; the larger drops are de-

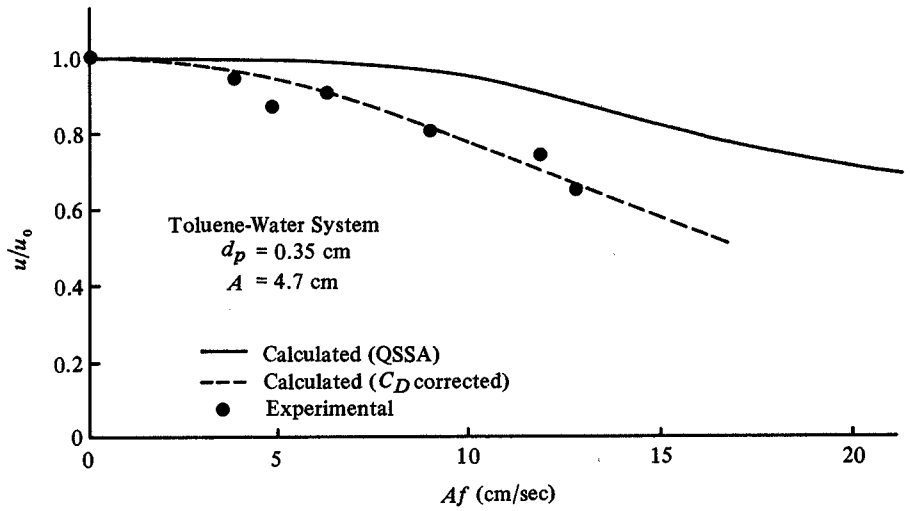

Fig. 4 a. Experimental rising velocity of toluene drops in oscillating water.
a) large droplets (d_p = 0.35 cm).

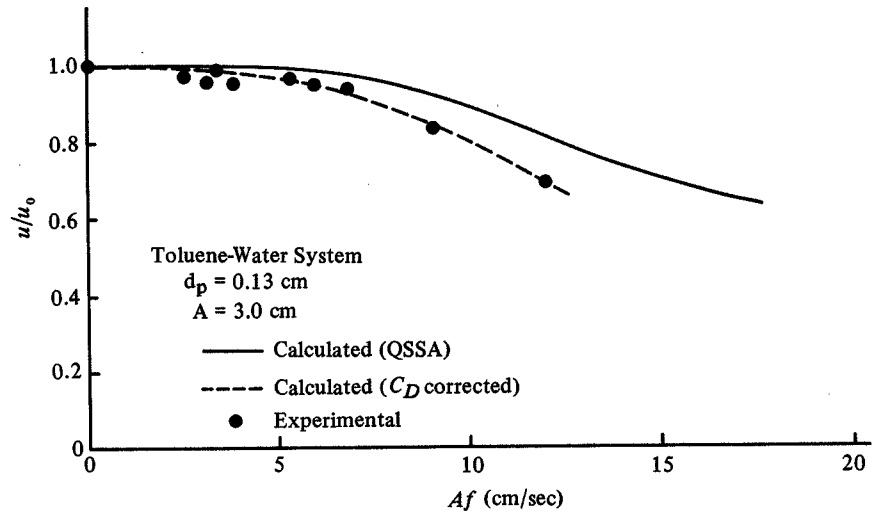

Fig. 4 b. Experimental rising velocity of toluene drops in oscillating water.
b) small droplets (d_p = 0.13 cm).

formed more easily than the smaller ones by fluid oscillation. It can be seen from Fig. 6, that the maximum increase in drag coefficient is about 50% under the present experimental conditions (Af = 13 cm/sec). For the gas-liquid system, on the other hand, there is little increase in the drag coefficient. This finding coincides with the experimental result on the free rising velocity which is almost independent of the bubble size if the bubble size is about 0.3 to 1 cm[21] in spite of the bubble deformation due to fluid oscillation. When the vibrating velocity is much higher than that in the present study, however, the effect of gas compressibility caused by the oscillating fluid motion may become significant[24].

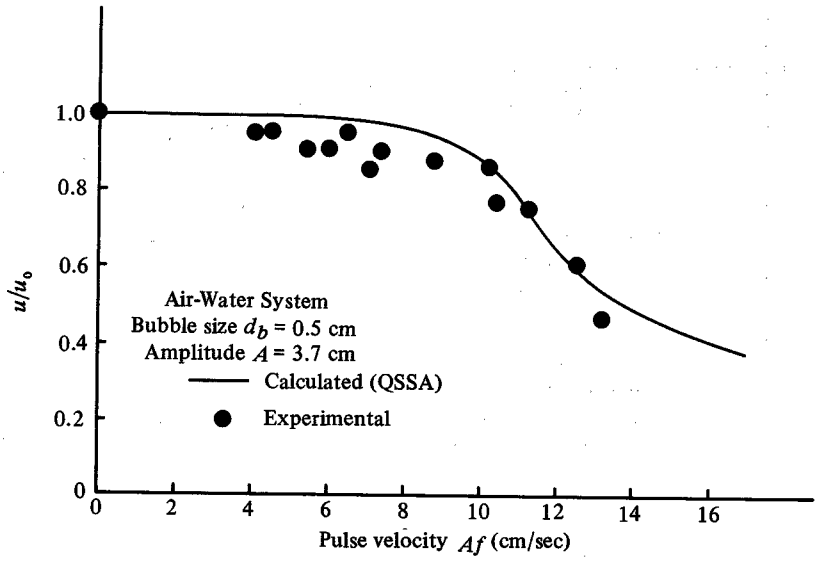

Fig. 5. Experimental rising velocity of air bubbles in oscillating water.

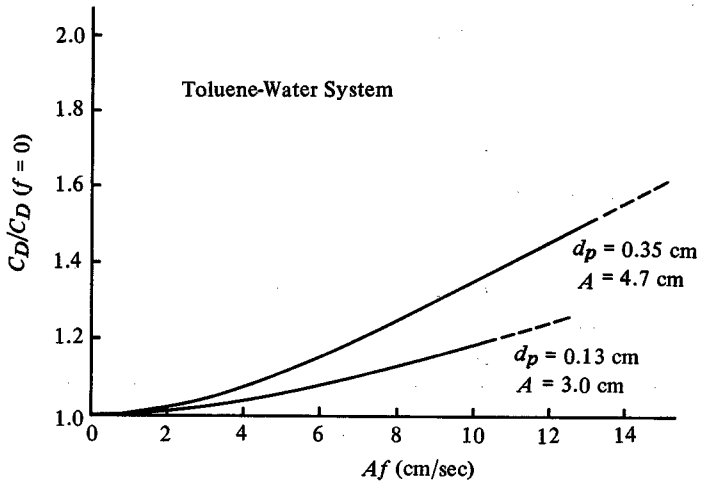

Fig. 6. Increased drag coefficient due to fluid oscillation.

The holdup data experimentally obtained in a multistage vibrating disk column[17), 19)] have been compared with the results calculated by the following relation:

$$\phi = u_d/u \tag{10}$$

where u_d and ϕ are the superficial velocity and the holdup of the dispersed phase, respectively.

For the multistage vibrating disk column, it has been shown that the particle diameter can be evaluated by the following equations[16), 17), 19)]:

Gas-liquid system : $d_b = 0.60$ cm ($Af < 8$ cm/sec) (11)

for the small vibrating disk column (d_i = 5 cm), and

$$d_b = 0.11\,(d_d/d_i)^{-1.2}\ We^{-0.32}\quad (20 < We < 400) \tag{12}$$

for the large vibrating disk column (d_i = 17.2 cm).

Liquid-liquid system:

$$d_p/d_d = 0.18\ We^{-0.6}\quad (We < 0.032\,Bo^{0.834}) \tag{13}$$

$$Bo = d_p^2\,\Delta\rho\, g/\sigma = 2.46\quad (We > 0.032\,Bo^{0.834}) \tag{14}$$

where $We = d_d\,(2\pi Af)^2\,\rho/\sigma$

d_d = diameter of the vibrating disk,

ρ = density of the continuous phase,

$\Delta\rho$ = density difference between particle and fluid,

σ = interfacial tension.

In order to estimate the fluid velocity along the axial direction in the multistage vibrating disk column, the flow pattern was investigated by analyzing the streak photographs taken by using small styrene balls suspended in the column as a tracer. Figure 7 shows one of the typical flow patterns. It is found that the circulating rings as shown in Fig. 7 are formed alternately above and under the vibrating disk. This means that the fluid flow in the vibrating disk column is substantially two dimensional, axially and horizontally. In spite of this, the emphasis is placed only on the vertical direction because the rising velocity of the

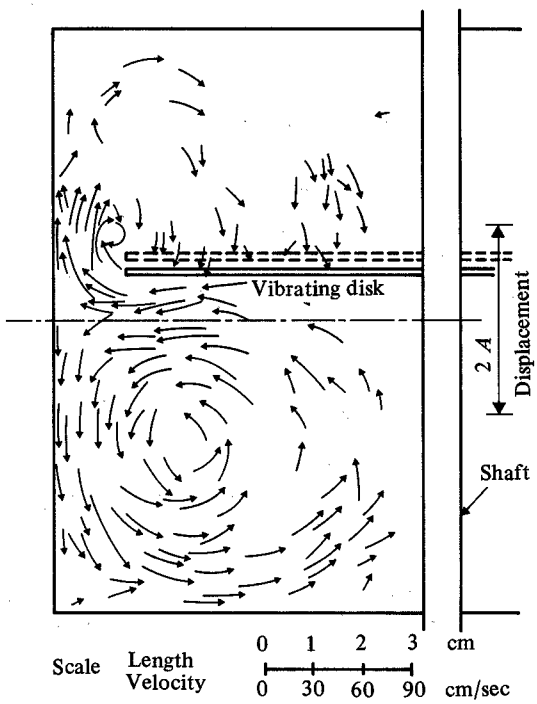

Fig. 7. Typical flow pattern of fluid in a multistage vibrating disk column; d_i = 17.2 cm, d_d = 13.5 cm, A = 2.0 cm, f = 1Hz.

particles is examined in the present study. The average values of amplitude and frequency of flowing liquids are found to be dependent on the stage height, d_h, the disk amplitude, A and the disk frequency, f. Under the present operating conditons where the diameter ratio between the disk and the column inside is 0.6 to 0.7, the average amplitude a_f and frequency f_f of the oscillating fluid may be approximately given by the following relation:

$$a_f = \alpha d_h \text{ (cm)}$$

$$f_f = 2.5\,(A/a_f)f \text{ (1/sec)}$$

where the value of coefficient α is 0.35 for the small column of d_i = 5 cm and 0.18 for the large column of d_i = 17.2 cm respectively. The liquid flow pattern in the vibrating disk column will be shown in more detail elsewhere by the authors[20].

The holdup data obtained experimentally for the liquid-liquid system are compared with those calculated by Eq. (3) in Fig. 8. As can be seen from this figure, the contribution of the effect of the hydrodynamic retardation based on the QSSA is nearly 6% of the total dispersed phase holdups. Even if the drag coefficients are corrected according to the experimetal rising velocity as mentioned earlier, the contribution to the retardation of the rising velocity by fluid oscillation is about 9%. This finding shows that the dispersed phase holdup can not be explained satisfactorily only by taking account of the effect of hydrodynamic particle retardation. Therefore, the effect of the other factors influencing the rising velocity, that is, multi-dispersion of the particles, the column geometries and particles interaction, is predominant to determine the dispersed phase holdups for the liquid-liquid system.

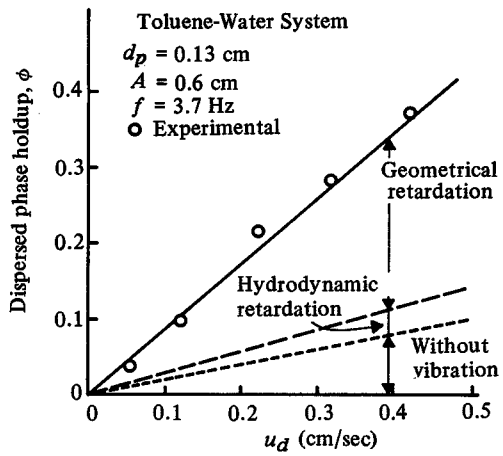

Fig. 8. Comparison of experimental holdup with the calculated ones for the liquid-liquid system in the multistage vibrating disk column.

Figure 9 shows the effect of Af on the gas holdup in the multistage vibrating disk columns in which the bubble size is given by Eq. (11) and (12) as a function of operating variables. In the vibrating disk columns where the mean bubble size varies from 2 to 6 mm

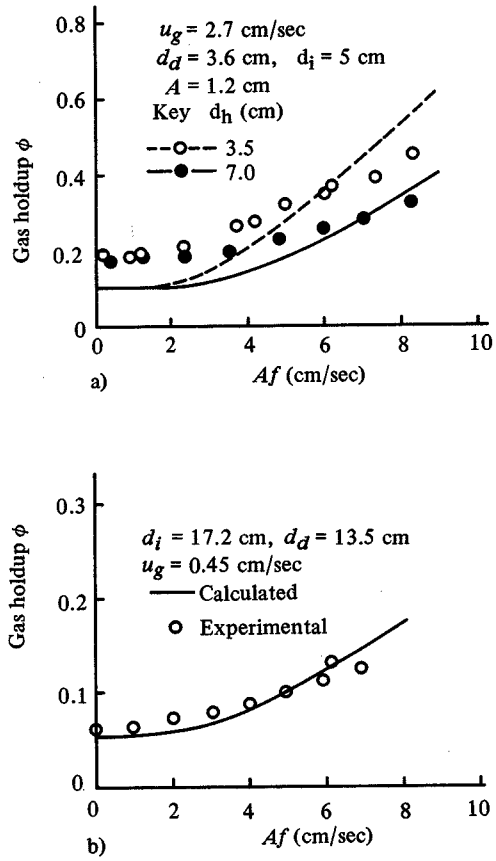

Fig. 9. Effect of vibrating velocity Af on the gas holdup in the multistage vibrating disk column; a) small column, b) large column.

depending on the vibrating velocity, the free rising velocity of bubbles is hardly influenced by the sizes and nearly equal to 23 cm/sec[21]. Therefore the effect of bubble breakup on the retardation of the rising velocity and the resulting increase in the holdup is not appreciable. It can be seen that the gas holdup in the large column well agrees with the calculated one. For the small column, however, the experimental holdup is not as well explained by the theory based on the QSSA. This is probably because the bubble rising in the small column is retarded not only by the oscillating flow but also by the partition plate or vibrating disk in a complicated manner. The critical velocity given as 2.4 cm/sec in the previous paper at which the holdup starts to increase by vibration is approximately equal to the theoretically estimated velocity at which the particle retardation and resulting holdup increase are observed here.

5. Conclusion

The particle settling velocity in an oscillating liquid has been calculated by numerically integration of the non-linear Langevin equation with the assumption of the quasi-steady state approach to elucidate the particle movement in a multistage vibrating disk column. The findings obtained are as follows:

The extent of retardation of particle movement in the oscillating water is a function of the amplitude A and the frequency f. Generally the higher the frequency, the more the retardation occurs while the vibrating velocity Af is kept to be constant. For a liquid-liquid system, the rising velocity of the particle is decreased more than that calculated based on the quasi-steady state assumption. This phenomenon is mainly attributed to the drop shape deformation during oscillation and the resultant increase in drag force. The effect of hydrodynamic retardation on the dispersed phase holdup increase for the liquid-liquid system is less than 10% of the total holdups in the present study and the effect of column internals, drops interaction and particle size dispersion on the dispersed phase behavior in the column is found to be more significant. For gas-liquid system, the rising velocity is approximately explained by the calculated hydrodynamic reterdation based on the QSSA. By employing these findings, the gas holdup in a multistage vibrating disk column is satisfactorily evaluated.

Nomenclature

A = amplitude of oscillation or vibration, cm
a_f = amplitude of fluid oscillation in the MVDC, cm
Bo = modified Bond number, -
C_D = drag coefficient, -
d_d = diameter of vibrating disk, cm
d_h = height of the stage of the MVDC, cm
d_i = inside diameter of the column, cm
d_b = mean bubble diameter, cm
d_p = mean droplet diameter, cm
d = mean particle diameter, cm
f = frequency of oscillation or vibration, Hz
f_f = frequency of fluid oscillation in the MVDC, Hz
g = gravitational constant, cm/sec^2
m = mass of particle travelled through liquid medium, g
m' = mass of continuous liquid displaced by particle, g
Per = Period number, –
Re = Reynolds number, –
t = time, sec
T = period of oscillation, sec
u = velocity of the particle, cm/sec

u_0 = velocity of the particle without fluid oscillation, cm/sec
u_d = superficial velocity of the dispersed liquid phase, cm/sec
u_g = superficial velocity of the gas phase, cm/sec
u_{max} = maximum velocity of the vibrating plate, cm/sec
v = velocity of the oscillating fluid, cm/sec
We = Weber number, $d_d (2\pi Af)^2 \rho/\sigma$
μ = viscosity of the liquid, g cm/sec
ρ = density of the liquid, g/cm^3
$\Delta\rho$ = density difference, g/cm^3
σ = interfacial tension. g/sec^2
ϕ = dispersed phase holdup, -
χ = virtual mass coefficient, -

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