Material Damping and Radiation Sound Pressure of Simply Supported Beam under Vibration

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The purpose of this paper is to describe a method for estimating material damping by the loss factor for a higher mode of vibration and radiation sound pressure of a beam under vibration. Application of this method to a simply supported beam comes to the following conclusions: The material damping of the beam at each mode of vibration can be obtained from the stress distribution function of the beam at each mode of vibration and energy absorption function. The relationships between the amplitude of acceleration and the radiation sound pressure, and between the material damping and the radiation sound pressure are not influenced by any mode of vibration.

1. Introduction

The method for estimating the loss factor from the stress distribution function and the energy absorption function has been reported by Lazan[1]. As the stress distribution function is based on the quasi-static deformation of a member, however, the stress distribution function at higher mode of vibration can not be given by Lazan's method. This paper describes a method for calculating the stress distribution function from the normal functions acquired by the analysis of undamped vibration and for evaluating the material damping of higher modes of vibration by the loss factor.

The vibration of the structures such as a machine and building generated by exciting force radiates sound. Although the mechanism of the sound radiation has been investigated from many aspects, the relationship between the acceleration amplitude of the structure and the sound pressure is not yet satisfactorily clarified. Layleigh[2] and Morse[3] have studied on the sound pressure due to the sound radiation out of a beam with baffle. The sound pressure radiated by the vibration of the beam without baffle has scarcely studied.

In the case of the vibration of the beam without baffle, the sound is radiated from the front side and rear side of the beam with the phase difference of 180°, and interference takes place between sound waves. Thus we cannot neglect the dimension of the cross-section, the acceleration and frequency of vibration and the distance from a sound source to a receiving point when the sound pressure is to be estimated. Therefore, the relationship among the external excitation, the vibration of the beam and the radiated sound pressure should be analyzed by taking into account Young's modulus, the moment of

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inertia, the length of the beam, the amplitude of acceleration, the exciting frequency and the boundary condition at beam ends. The mechanism of the sound radiation from a vibrating beam without baffle is assumed to be equivalent to that from two pulsating spheres that have a phase difference of 180° and the same acceleration as the beam.

2. Estimation of material damping

The material damping of the member can be evaluated by various measures, but this paper applies the measure described below. Let U and D be the maximum elastic energy and dissipating energy per unit cycle, respectively. Then the loss factor[4] is given by

$$\eta = \frac{D}{2\pi U}. \tag{1}$$

If the whole member is distributed by uniform stress, Eq. (1) can be rewritten as

$$\eta = \Delta U/(2\pi U/V_s), \tag{2}$$

where V_s is the total volume of the member, ΔU is the specific damping energy or the dissipating energy per unit cycle and unit volume.

In general, it is said that the specific damping energy is affected by stress amplitude, frequency, temperature and stress history, and is influenced considerably in paticular by the stress amplitude. The specific damping energy is calculated from the loss factor which is given by the experiment of a simple shape test specimen. Lazan[5] has obtained the experimental equation for steel

$$\Delta U(\sigma) = 0.0703\left(\frac{\sigma}{\sigma_f}\right)^{2.3} + 0.422\left(\frac{\sigma}{\sigma_f}\right)^8. \tag{3}$$

This equation is called the energy absorption function, in which the dimensions of $\Delta U(\sigma)$, the stress amplitude σ and fatigue limit σ_f are *kgf/(cm²·cycle)*, *kgf/cm²* and *kgf/cm²*, respectively. If the deformation of material is linear, the specific damping energy ΔU is proportional to $(\sigma/\sigma_f)^2$ and the energy absorption function is approximately linear to the maximum stress amplitude at the low stress region.

When the member is deformed by applying force, the internal stress distribution is nonuniform. The function expressing the state of the stress distribution is called the stress distribution function. The stress distribution function becomes

$$f(\sigma/\sigma_m) = \mathrm{d}(V/V_s)/\mathrm{d}(\sigma/\sigma_m), \tag{4}$$

where σ_m stands for the maximum stress amplitude and V the volume subjected to the stress below σ.

As the maximum elastic energy $\mathrm{d}U$ acting on a volume element $\mathrm{d}V$ is expressed as $\sigma^2\mathrm{d}V/2E$,

$$U=\frac{V_s}{2E\sigma_m}\int_0^{\sigma_m}\sigma^2 f(\sigma/\sigma_m)\mathrm{d}\sigma\,. \tag{5}$$

is obtained from Eq. (4). On the other hand, the same computation as in Eq. (5) yields

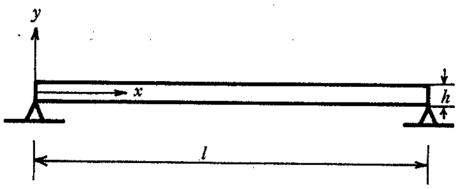
$$D=\frac{V_s}{\sigma_m}\int_0^{\sigma_m}\Delta U(\sigma)f(\sigma/\sigma_m)\mathrm{d}\sigma\,, \tag{6}$$

since $\mathrm{d}D$ is $\Delta U \mathrm{d}V$. From Eqs. (1), (5) and (6), the loss factor can be written in the form[6)]

$$\eta=\frac{E}{\pi}\frac{\int_0^{\sigma_m}\Delta U(\sigma)f(\sigma/\sigma_m)\mathrm{d}\sigma}{\int_0^{\sigma_m}\sigma^2 f(\sigma/\sigma_m)\mathrm{d}\sigma}\,. \tag{7}$$

3. Stress distribution under flexural vibration

The slender beam of isotropic body whose state is uniform in the direction of the axis as shown in Fig. 1 is simply supported. In this case, the normal function of n th mode of vibration is



l: length of beam, h: depth of beam

Fig. 1. Model for a simply supported beam

$$W_n = C\sin(n\pi x/l)\,, \tag{8}$$

where C is a constant and l is the length of beam. The stress amplitude of n th mode of vibration $\sigma_n(x, y)$ can be written as

$$\sigma_n=-Ey\frac{\mathrm{d}^2W_n}{\mathrm{d}x^2}=EyC\left(\frac{n\pi}{l}\right)^2\sin\left(n\pi\frac{x}{l}\right). \tag{9}$$

If the coordinates where σ_n becomes maximum are (x_m, y_m) and h is the thickness of beam, they become

$$x_m = l/(2n), \quad \text{and} \quad y_m = h/2\,, \tag{10}$$

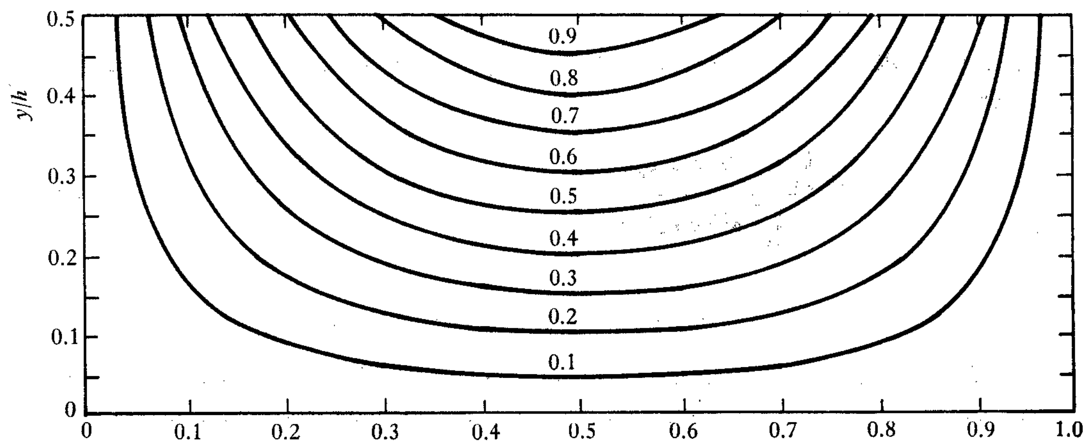
which settle the place where W_n is maximum. For the maximum values of σ_n, W_n, we have

$$\sigma_{mn}=\frac{h}{2}E\left(\frac{n\pi}{l}\right)^2 W_{mn}\,, \tag{11}$$

from Eqs. (8), (9) and (10). Equal stress contour lines can be written in the form

$$\frac{y}{h}=\frac{(\sigma_n/\sigma_{mn})}{2\sin(n\pi x/l)}\,. \tag{12}$$

These results are summarized in Fig. 2, and the state of the stress distribution is shown in Fig. 3.



σ: stress, σ_m: maximum stress, h: depth of beam, n : mode number
l: length of beam, parameter: σ/σ_m

Fig. 2. Equal stress contours of the simply supported beam

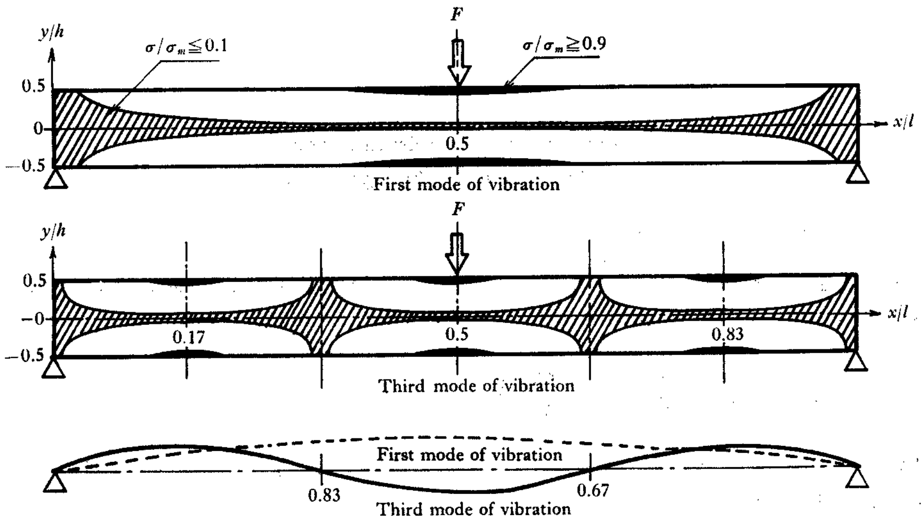


Fig. 3. Stress distribution and mode of vibration

4. Stress distribution function and material damping

If the stress distribution function $f(\sigma/\sigma_m)$ and the energy absorption function $\Delta U(\sigma)$ are given, we can express the material damping by the loss factor η from Eq. (7). From Eq. (12), the intersect of the equal stress contour line with the outer fiber of the beam is written as

$$x/l = 1/(n\pi)\sin^{-1}(\sigma_n/\sigma_{mn}). \tag{13}$$

Since the equal stress contour line of the beam is symmetric with respect to x axis, the part of beam volume in which stress is less than σ_m can be calculated by doubling the area of the range of $0 \leqq y \leqq h/2$. Let the width of the beam be replaced by w, V becomes

$$V = 4nw\left(\frac{h}{2}x_1 + \int_{x_1}^{l/2n} y\,dx\right). \tag{14}$$

Now, let $\zeta = \sigma_n/\sigma_{mn}$, $K = \sigma_f/\sigma_{mn}$, $\varphi = (1-\zeta^2)^{1/2}$, $\phi = \tan\{\sin^{-1}\varphi/2\}$. Since $V_s = whl$, the volume stress function becomes

$$V/V_s = (2/\pi)(\sin^{-1}\zeta - \zeta\log\phi). \tag{15}$$

The stress distribution function expressing the rate of volume variation by means of the variation of σ_n can be expressed by

$$f = (2/\pi)\left\{\varphi - \log\varphi - \frac{\zeta\varphi}{2}\left(\frac{1}{\phi} + \phi\right)\right\}. \tag{16}$$

The results of Eqs. (15) and (16) are shown in Fig. 4. Substitution of Eq. (13) and (16) into Eq. (7) conducts us finally to

$$\eta = \frac{E}{\pi\sigma_{mn}^2}\,\frac{\int_0^1\left\{0.0703\left(\frac{\zeta}{K}\right)^{2.3} + 0.422\left(\frac{\zeta}{K}\right)^8\right\}d\zeta}{\int_0^1 \zeta^2 f\,d\zeta}. \tag{17}$$

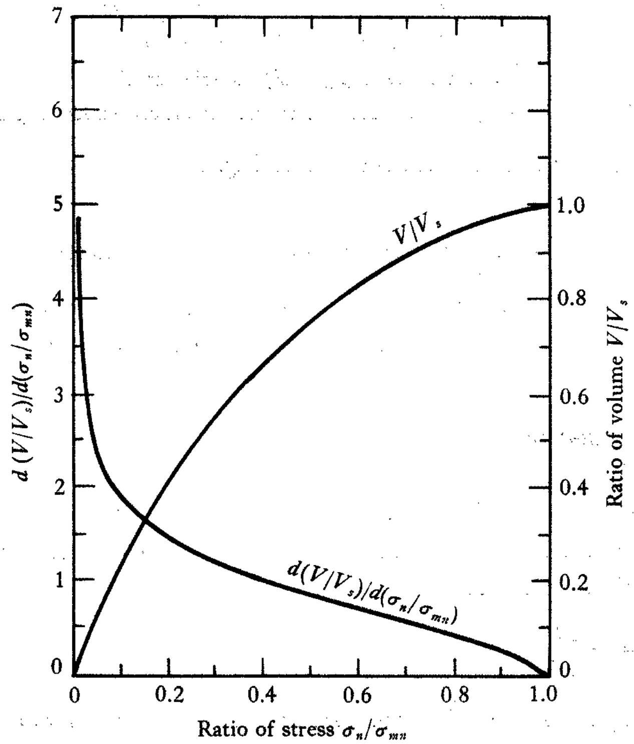
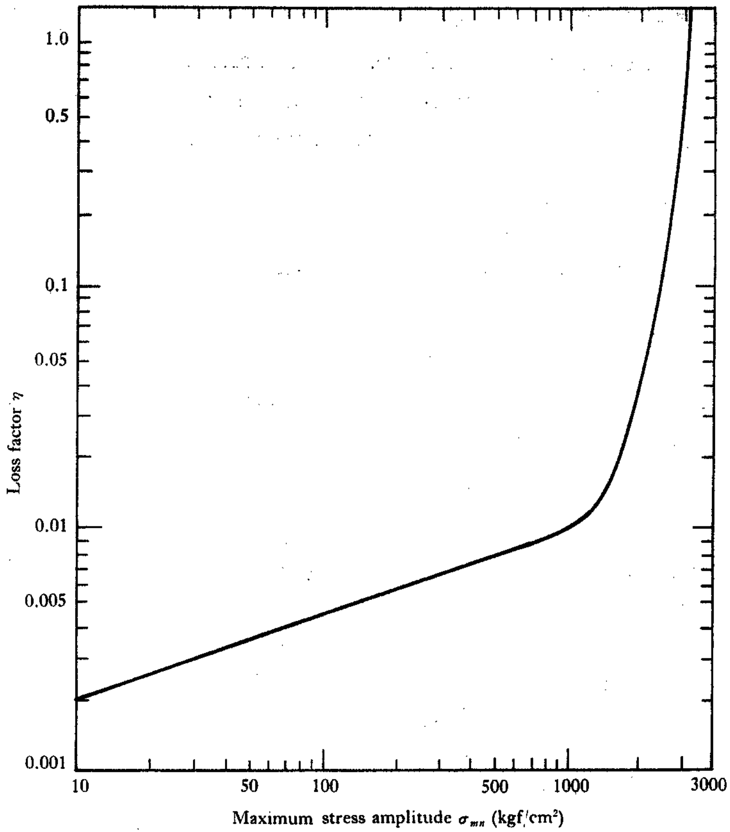


Fig. 4. Volume stress function and stress distribution function



fatigue limit 1900 (kgf/cm²) material (steel)

Fig. 5. Relationship between loss factor and maximum stress amplitude

The calculated result of this equation is shown in Fig. 5.

5. Estimation of radiation sound pressure

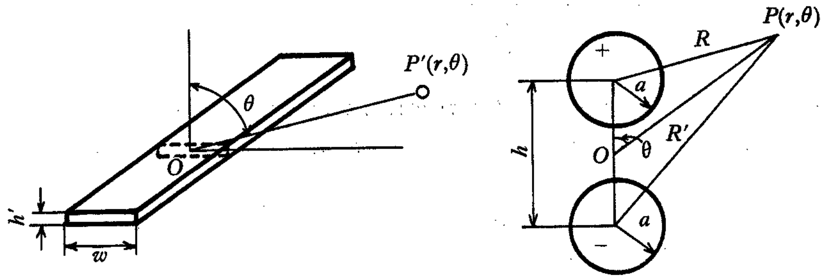
The mechanism of radiation from the beam can be modelled as shown in Fig. 6. When the vibration of the pulsating sphere radiates spherical waves, the velocity potential on a sphere of radius R is given by

$$\phi = \frac{A}{R} e^{j(\omega t - kR)}, \tag{18}$$

where R, A, ω, k and t denote the distance from the center of the sphere O to the receiving point P, an unknown constant, angular frequency, wavenumber and time, respectively. The particle velocity becomes

$$\frac{du}{dt} = -\frac{\partial \phi}{\partial R}. \tag{19}$$

The present analysis assumes that the particle velocity at the radius of the pulsating sphere a is equal to the vibration velocity of the beam $V_{on} e^{j\omega t}$.



Vibrating beam without baffle

A_{on} Acceleration amplitude of surface of beam
h' Thickness of beam
O Origin (center of cross-section of beam)
r Receiving distance
θ Sound receiving angle

A_{on} Acceleration amplitude of spherical sound surface
h Distance between pulsating spheres
r Receiving distance
θ Sound receiving angle

Fig. 6. Relationship between cross-section of beam and pulsating spheres

$$\left[\frac{\mathrm{d}u}{\mathrm{d}t}\right]_{R=a} = A\frac{1+jka}{a^2}e^{j(\omega t-ka)} = V_{on}e^{j\omega t} \tag{20}$$

is obtained from Eqs. (18) and (19). The unknown constant A is determined as

$$A = V_{on}\frac{a^2}{1+jka}e^{jka}, \tag{21}$$

where V_{on} is the amplitude of vibration velocity. From Eqs. (18) and (21), the velocity potential is reduced to

$$\phi = \frac{a^2}{R}\frac{V_{on}}{1+jka}e^{jka}e^{j(\omega t-kR)}. \tag{22}$$

If the sound radiation of the pulsating sphere is elucidated, the model shown in Fig. 6 can be analyzed on the basis of Eq. (22). Since the sound radiated from the vibrating beam without baffle is equivalent to that from two pulsating spheres, the radius of the pulsating sphere a depending on the dimension of cross-section of the beam is determined experimentally. The sound pressure at the receiving point P' corresponds to that on the circle of the pulsating sphere model including the receiving point P. Consequently, the velocity potentials at the receiving point P

$$\phi_+ = \frac{a^2}{R}\frac{V_{on}}{1+jka}e^{jka}e^{j(\omega t-kR)}, \quad \text{and} \tag{23}$$

$$\phi_- = -\frac{a^2}{R'}\frac{V_{on}}{1+jka}e^{jka}e^{j(\omega t-kR')} \tag{24}$$

are obtained from Eq. (22).
R and R' in Eqs. (23) and (24) are expressed as

$$R = r\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}, \quad \text{and} \tag{25}$$

$$R' = r\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}} \tag{26}$$

with reference to Fig. 6, where r and θ denote the receiving distance and the receiving angle, respectively. Substitution of Eqs. (25) and (26) into Eqs. (23) and (24) results in

$$\phi_+ = \frac{a^2}{(1+jka)}\frac{V_{on}e^{jka}}{r\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}e^{j\left(\omega t-r\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}\right)} \tag{27}$$

and

$$\phi_- = \frac{a^2}{(1+jka)}\frac{V_{on}e^{jka}}{r\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}e^{j\left(\omega t-r\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}\right)}. \tag{28}$$

Since the velocity potential at the receiving point P is equal to the sum of the velocity potential ϕ_+ and ϕ_-, we have

$$\phi = \frac{a^2}{1+jka}V_{on}e^{jka}\frac{e^{j(\omega t-kr)}}{r}\left\{\frac{e^{-jkr\left(\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}-1\right)}}{\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}-\frac{e^{-jkr\left(\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}-1\right)}}{\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}\right\}. \tag{29}$$

Thus, the sound pressure p is obtained from Eqs. (30) and $p=j\omega\rho\phi$ as

$$p = j\omega\rho\frac{a^2}{1+jka}V_{on}e^{jka}\frac{e^{j(\omega t-kr)}}{r}\left\{\frac{e^{-jkr\left(\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}-1\right)}}{\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}-\frac{e^{-jkr\left(\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}-1\right)}}{\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}\right\}, \tag{30}$$

where ρ is the density of the air.

The sound pressure level is calculated by

$$\text{SPL} = 20\log_{10}\frac{p/\sqrt{2}}{p_0} \quad (\text{dB}), \tag{31}$$

where p_0 is 0.0002 μbar. Besides, by letting $A_{on}=\omega V_{on}$, Eq. (30) is rewritten as

$$p = j\rho A_{on}\frac{a^2}{1+jka}e^{jka}\frac{e^{j(\omega t-kr)}}{r}\left\{\frac{e^{-jkr\left(\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}-1\right)}}{\sqrt{1-\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}-\frac{e^{-jkr\left(\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}-1\right)}}{\sqrt{1+\frac{h}{r}\cos\theta+\frac{h^2}{4r^2}}}\right\}, \tag{32}$$

where A_{on} stands for the amplitude of acceleration at nth resonance of the vibrating beam.

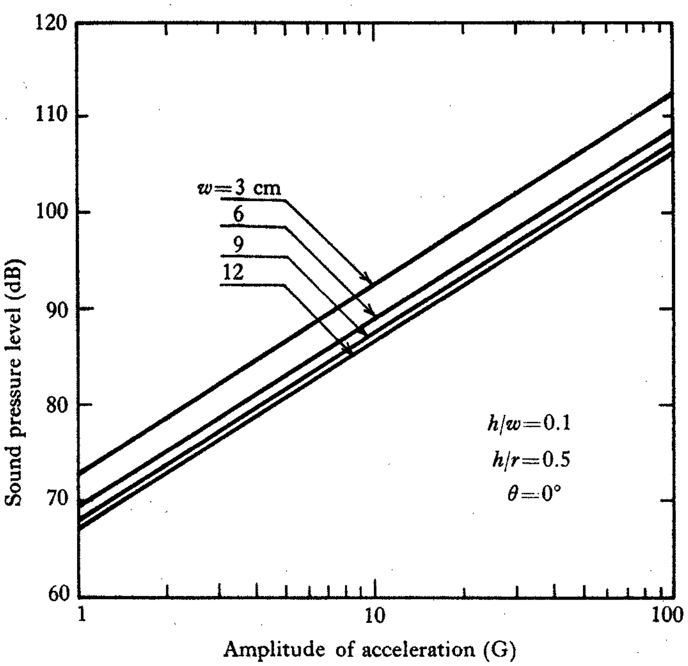
The radius of the pulsating sphere a can be determined by substituting the radiation sound pressure and the amplitude of acceleration obtained experimentally. The results of several measurements were substituted in Eq. (32). The radii of the pulsating spheres were averaged in order to minimize the effect of the errors in the measurements and the influence of background noises on the sound pressure. As the results, the radius a[7] of the equivalent sphere for the beam with the arbitrary width w and thickness h of the beam is determined as

$$a = 0.924 + 0.00774w - 0.0270h\,. \tag{33}$$

On consideration for the material damping, the amplitude of acceleration A_{on} at n th mode of vibration in the simply supported beam excited at its center is given by

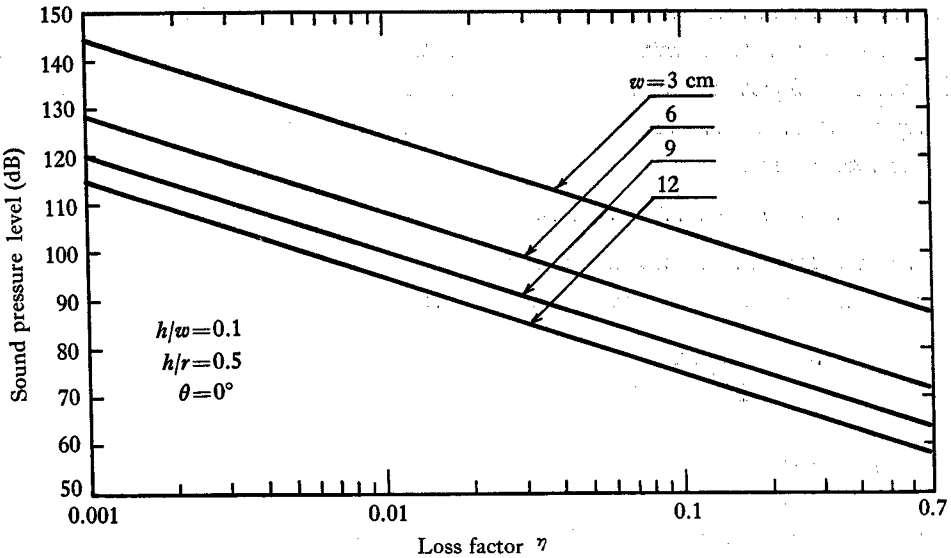
$$\frac{A_{on}}{F_0} = \frac{\omega_n^2}{\eta EI\left(\frac{n\pi}{l}\right)^4 \frac{l}{2}}\,, \tag{34}$$

where F_0 denotes the amplitude of the exciting force, ω_n the n th resonant angular frequency and I the moment of inertia of area. By taking $h/w=0.1$, $h/r=0.5$ and w as the parameter, the calculation of Eqs. (31), (32), (33) and (34) for the radiation sound pressure in relation to the amplitude of acceleration is shown in Fig. 7. Fig. 8 gives the relation between the radiation sound pressure and the material damping. It is seen from Fig. 7 and Fig. 8 that in the case of the simply supported beam, the radiation sound pressure in relation to the material damping and to the amplitude of acceleration are independent of the mode of vibration.



w: beamwidth, h: thickness of beam, r: receiving distance,
θ: sound receiving angle

Fig. 7. Relationship between sound pressure level and amplitude of acceleration



w: beam width, h: thickness of beam, r: receiving distance
θ: sound receiving angle

Fig. 8. Relationship between sound pressure level and loss factor

6. Conclusion

In order to estimate the material damping and the radiation sound pressure of machine structure, how to estimate the material damping and the radiation sound pressure of the component of machine is considered. The summary of the results is as follows:

1) The stress distribution function of the member at each vibration mode may be calculated from the normal function.
2) The stress distribution function in each vibration mode become almost similar values.
3) The material damping of the beam can be obtained from the stress distribution function of the beam at each mode of vibration and the energy absorption function.
4) The relationships between the acceleration amplitude and the radiation sound pressure, and between the material damping and the radiation sound pressure are not influenced by the mode of vibration.

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