On an Educational Method for Computer Graphics Introduced into Descriptive Geometry

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This short note reports on the education of *Computer Graphics*, which has been given as a part of "**Exercise of Descriptive Geometry**" to the sophomore students of the college of engineering who completed the subject of Descriptive Geometry in their freshman year.

It can be said in general, that the effective teaching of computer graphics requires the ability of instructors to let students well understand the charactors and actions of given figures in relation to the theory of Descriptive Geometry. The authors have developed some proper programs to let the students understand deeply the contents, available even if not a few of them have rather a little knowledge on the grammer of computer language, such as FORTRAN.

A discussion will be given principally on the relationship between the process of solution by using the computer algorithm and that of solution by using Descriptive Geometry. The method of education has been tried since 1973, and it shows the validity in providing easy usage and effective fruits, some of which are shown as in the **Appendix** of this paper.

1. Introduction

The value of Descriptive Geometry, which was first conceived by Gaspard Monge, has been sought[1] for a long time. In this respect, what the authors should ask of Descriptive Geometry can not be considered apart from the quintessence of Descriptive Geometry, that is; "*What is Descriptive Geometry?*" Nowadays, Descriptive Geometry is appreciated in a different manner by the men of learning and experience on the basis of each one's point of view, in large measure depending on what kind of education they have undertaken in this subject in the past.

Recently the use of computers in the field of Descriptive Geometry is gradually extending, and even the term of "Computer Descriptive Geometry" is introduced[2]. However the process of finding the solution by using a computer is essentially different from that in the traditional Descriptive Geometry, and certain regulations must definitely be taught to the students who learn the both in the class.

Thereupon, the authors would like to discuss on the relationship between Descriptive Geometry and the computer algorithm with special attention to the following two points:

(i) To see the compatibility between Descriptive Geometry dealing with *analog* media and use of the computers for processing *digital* data.

(ii) To define the region which should be covered by Descriptive Geometry.

Debates as to the propriety of using computers in the education of Descriptive Geometry

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have been frequently held in the past. However, as far as the authors know, no conclusion has been reached yet[3]. One reason seems to be a lack of investigation of the two points mentioned above; Group A agreeing to the introduction of the computer to begin with, met opposition from Group B having a doubt about the compatibility and disagreeing to it, in considering the above item (i).

The authors would like to stand by neither of the two groups but believe that the theories and studies by these two groups should be positively combined so as to yield the possibility of finding the essence of Descriptive Geometry and deterrning its region from a wider point of view. Therefore, the authors will present their opinion more fully below.

2. Compatibility of Descriptive Geometry and Use of Computer

"Compatibility", as defined in *Webster's New Collegiate Dictionary,* is equivalent to "Capability of existing together in harmony" or "Capability of cross-fertilizing freely or uniting vegetatively" in meaning. Since the algorithm of the computer method was introduced into *Monge* Descriptive Geometry, it has been asked whether the two are incongruous or harmonious. Essentially, the problem concerns how human beings and machines of computers recognize a given figure and process the information delivered from it, respectively.

Now, a simple and concrete example is given: if three spheres of various sizes is intersecting each other in the space, how many closed regions will be surrounded by them? To answer the question the following two methods may well be considered;

(a) To count the numbers of the spaces by drawing the spheres actually on a paper (projecting circles on the paper).

(b) To solve the problem by expressing it in the language of logic, and processing digital data that include relevant formulas and figures.

The former is a three-dimensional and more *human* method in the sense that it deals with the problem more analogously through a visible presentation. While, the latter is a zero-dimensional and *unhuman*, or more mechanical method, because the problem is handled digitally through the procedures of computer algorithm. In regard to the recognition of a figure, nothing is superior to human intuitive ability, but for the processing speed and precision of the involved data, the computer is the best. Thus, both human beings and machines have their advantages and disadvantages. That is why the two should ideally be made "*compatible*", and this is the point that the authors would like to emphasize.

3. The Extensive Region of Descriptive Geometry

Any entity which has a certain size and location in the space — hereafter called "*object*" in a broad sense — can be described by projection as a "*pattern*" of a plane extension usually. In recognizing the pattern, it is not necessary to explain the contents with a special language or formula, but it is only required to rely upon such a belief that the object or the information represented by the figure can be recognized at a glance. Thus, it may be said that through the pattern the intention of the drawer can be understood fully and intuitively by inspectors who have proper knowledge of it.

When a pattern is appreciated by a third person through his emotion alone, the pattern might be said to be a "***picture***" in an artistic sense. On the other hand, the object has naturally its geometrical characterics and hence, if it is possible to make an objective analysis and synthesis in the process of expressing it, the pattern can be regarded as a scientific medium that belongs to the field of "***Descriptive Geometry***". Further, taking advantage of the theoretical perfectibility attainable with Descriptive Geometry, the pattern designed in industry production can be called a "***drawing***" or "***drafting*** " usually appearing as an engineering blueprint.

The standards of value change as an *era* passes: a pattern, which seems to be far from a picture for example, is being appreciated from emotional and subjective views of human beings in the contemporary society, and likewise an engineering drawing is increasing its value in accordance with the needs of the industrial society. Then, how should be Descriptive Geometry as a science appreciated nowadays? Should its media be limited to those which are neither pictures nor drawings? Descriptive Geometry has been fully appreciated for its profound and extensive theory, which is assumed to be the opinions of Group *B*. However, the authors believe that Descriptive Geometry conceived by *G. Monge* might expand its range in the course of increasing applications with the participation of the computers, for instance, to meet new requirements.

For this reason, the authors would be convinced firmly that the following possibilities can be foreseen.

1. "Figure data" will be expressed more extensively in the digital system so that they can be fed directly to the computer. (Development of **input systems** such as the advanced digitizer, computer eyes, etc.)
2. Data processed by the computer will be presented by such analog method as figures on the screen, which is more easily understood by human beings. (Development of **output systems** such as the CRT, a refresh type display, etc.)
3. Standardization of an algorithm "language" proper to pattern processing. (Development of a **computer language**)
4. The appearance of the interface systems of communication and dialogue between human beings and machines for greater ease. (Development of **interactive method**, etc.)

When such total systems as the above are realized, won't the traditional concepts of *Monge* Descriptive Geometry be expanded consequently to meet the new requirements? And this gives the answer for the above item (ii).

Recently, figure recognition has been facilitated by two-dimensional presentations using half-tone illustrations and by a three-dimensional process making good use of holography. Accordingly, the field, which used to involve machinists and electricians principally, will be developed more by the paticipation of medical doctors, psychologists, physicists, and as well as those specialists in Descriptive Geometry. The authors presume that such research will lead to previously unknown flexibility of Descriptive Geometry which will in turn affect all aspects of science and engineering dealing with three dimensional objects.

4. Concrete Effect of Teaching Computer Programming

Effective teaching of computer programming for pattern processing requires the ability to let the students understand well about *Monge* Descriptive Geometry, by using the various numerical algorithms[4]. The students who can visualize the geometric characteristics of the objects show good understanding for the given theme in graphic design, without which it is rather difficult to illustrate the action of picture analysis clearly.

Today, the students of the engineering department are desired to have enough knowledge of using computer graphic systems as well as writing programs as the need arises, before they enter into industries or institutes.

In case of the subject "*Computer Graphics*", of which one of the authors is in charge, the lecture is given to the sophomores of the engineering courses, who completed in the freshman year the Monge Descriptive Geometry which is given over two-semesters. In the **Appendix**, several articles yielded in the class of *Computer Graphics* are shown; As the examples, they enclose an intermediate truncated polyhedrons between an original drawing ***A*** and a final drawing ***B***, and also intermediate developing and undeveloping polyhedrons from an original drawing ***A*** and a final drawing ***B***. These drawings are quite helpful and effective to let the students understand well the actions of the truncating and developing processes in three dimensional bodies performed through the method of the traditional Descriptive Geometry.

5. Conclusion

It is most advisable that the students will take up proper exercises for algorithm and programming, soon after they finish undertaking of a basic study on computer language such as FORTRAN. They take much interests in those subjects learning that accurate pictures in computer graphics help them explore the roles of the assumptions which are necessary for a mathematical proof. The authors feel strongly that incessant enthusiams of the students for programming and facilities of computer graphics displaying excellent performance are really indispensable, as well as instructors' enough ability, to execute a really effective education of computer graphics.

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Appendix

Truncation and Development of Solids

The *Platonic* solids are called as the tetrahedron, cube, octahedron, dodecahedron and icosahedron. And we symbolize them $\{p, q\}$ as $\{3, 3\}$, $\{4, 3\}$, $\{3, 4\}$, $\{5, 3\}$ and $\{3, 5\}$ respectively with regular p angles and q faces around a vertex of the solid after L.F. Tóth; *Regular Figures* 102 (1964).

When the pair of two solid has a relation of $\{p, q\}=\{q, p\}$, it is called as "**dual correspondence**". The authors notice this character of *Platonic* solids, and tried to make solids of semiregular polyhedron by computer graphics.

For example, when we chop off the four tips of the cube, we replace its eight corners with little equilateral triangles. Chopping further, that is, **truncating** the vertices of the cubic solid more and more, we reach a stage at which the triangles meet at their vertices and the triangles become hexagons. Meanwhile the relics of the original faces on the cube become smaller and smaller untill they finally disappear, leaving a regular octahedron.

Staring again with the octahedron, truncation i.e., chopping yields the same succession of solids in reverse order and ends with a cube. This is a good example of the theory of dual correspondence of $\{p, q\}=\{q, p\}$. The authors realized to simulate the process by taking the arithmetic mean of the coordinate of each vertex and face of the solids and *vice versa*, with the aid of computer graphics.

When two points belonging to each figures "***A***" and "***B***" are given, the internally dividing points are calculated as $X_k=(m_kX_i+n_kX'_j)/(m_k+n_k)$ and $Y_k=(m_kY_i+n_kY'_j)/(m_k+n_k)$ where (X_i, Y_i) belongs to the figure ***A*** and (X'_j, Y'_j) belongs to the figure ***B*** and *vice versa*. The letter m_k, and n_k (where $k=1, 2, \cdots, k$) are the integers by which the figures are divided and another intermidiate figures named "***C***" are produced. Those figures can be applied to the explanation of making semiregular solids from the

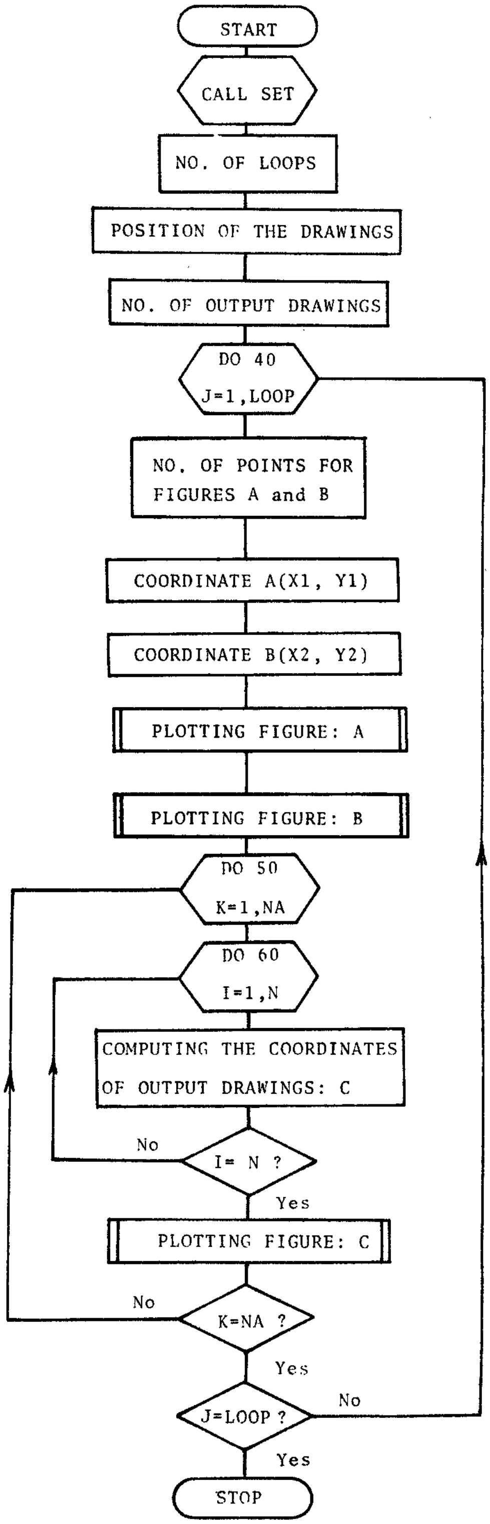


Fig. 1. Flow Chart of the Method.

```
C         **  RENZOKU ZUKEI    **
C
C
C        1978.4.5
C
      DIMENSION X1(200),Y1(200),X2(200),Y2(200)
     1   ,XX(200),YY(200),TITLE(4),A(200)
      CALL PLOTS(0.0,0.0,9)
      READ(5,11) LOOP,BUNK,(TITLE(I),I=1,4),NA
      WRITE(6,12) (TITLE(I),I=1,4), LOOP, BUNK, NA
      DO 80 JK=1,100
      A(JK)=JK
   80 CONTINUE
      DO 40 J=1,LOOP
      READ(5,41) N
      WRITE(6,52) N
      READ(5,21) (X1(I),Y1(I),I=1,N)
      READ(5,31) (X2(I),Y2(I),I=1,N)
      DO 10 IDD=1,N
      Y2(IDD)=Y2(IDD)-34.0
   10 CONTINUE
      WRITE(6,22) (X1(I),Y1(I),I=1,N)
      WRITE(6,32) (X2(I),Y2(I),I=1,N)
      IPEN=3
      DO 20 I=1,N
      CALL PLOT(X1(I),Y1(I),IPEN)
      IPEN=2
   20 CONTINUE
      IPEN=3
      DO 30 I=1,N
      CALL PLOT(X2(I),Y2(I),IPEN)
      IPEN=2
   30 CONTINUE
      DO 50 K=1,NA
      DO 60 I=1,N
      XX(I)=(X2(I)-X1(I))*A(K)/BUNK+X1(I)
      YY(I)=(Y2(I)-Y1(I))*A(K)/BUNK+Y1(I)
   60 CONTINUE
      IPEN=3
      DO 70 I=1,N
      CALL PLOT(XX(I),YY(I),IPEN)
      IPEN=2
   70 CONTINUE
   50 CONTINUE
   40 CONTINUE
      CALL PLOT(0.0,0.0,999)
      WRITE(6,42) (TITLE(I),I=1,4)
      STOP
   11 FORMAT(I10,F10.1,4A4,I4)
   12 FORMAT(1H1,10X,4A4,4X,13HLOOP COUNT = ,I10
     1   ,10X,11HBUNKATSU = ,F10.2,5X,'NA=',I4)
   21 FORMAT(6F10.1)
   22 FORMAT(1H0,15X,'X1 AND Y1 ',/(1H ,17X,6F10.2))
   31 FORMAT(6F10.1)
   32 FORMAT(1H0,15X,'X2 AND Y2 ',/(1H ,17X,6F10.2))
   41 FORMAT(I5)
   42 FORMAT(1H0,///,10X,7HEND OF   ,4A4)
   52 FORMAT(1H0,//,10X,'N=',I5)
      END
```

Fig. 2. Program for the Flow Chart.

regular solids as mentioned above. Because the semiregular solids consist of those whose faces are made of several sorts of regular polygons and whose corners are all alike. And these semiregular solids share still another property with the *Platonic* solids corresponding figures ***A*** and ***B***, which are produced by the arithmetic mean. The authors realized this method by connecting the points of (X_k, Y_k) with ***X*—*Y* plotter** after calculating the coordinates with digital computer.

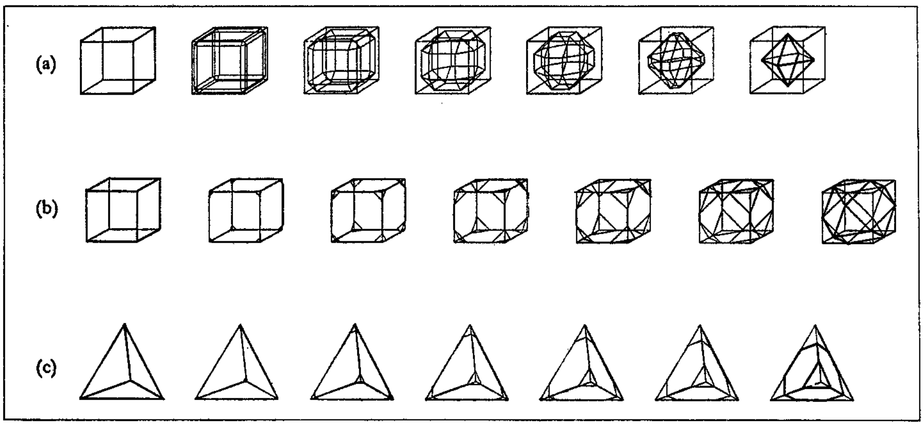


Fig. 3. Automatic Truncations where (a) is a case of *Dual Correspondence* {3, 4}↔{4, 3}, and (b) and (c) are cases of Regular-↔Semiregular Polyhedrons.

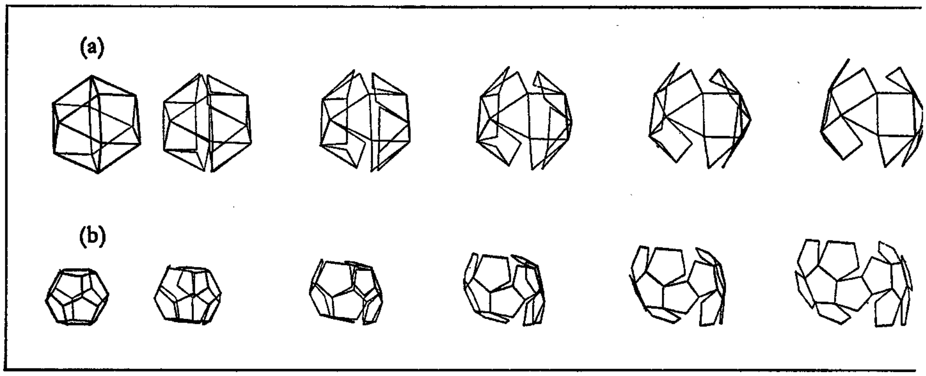


Fig. 4. Automatic *Developping* and *Assembling* for

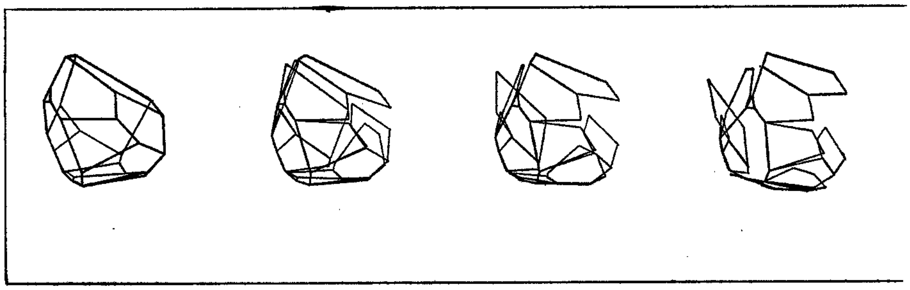
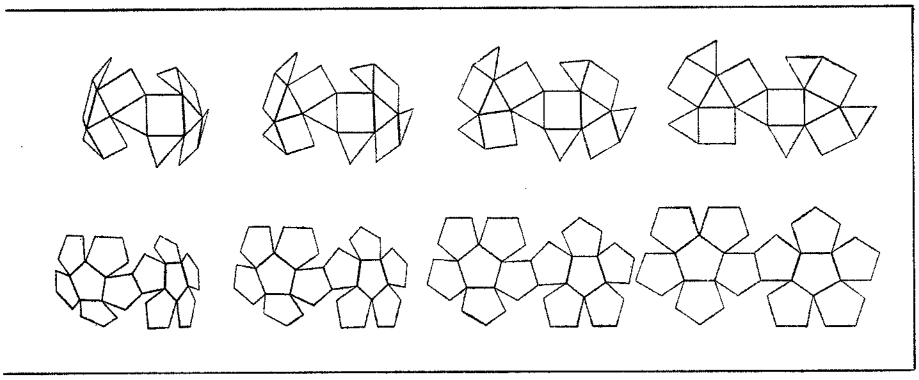


Fig. 5. A Simulation of Automatic *Developping*

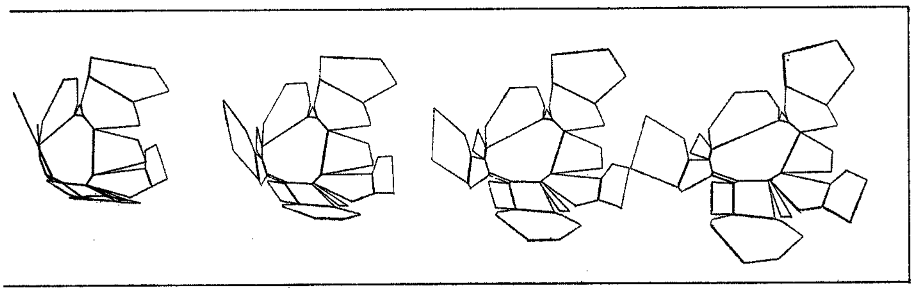
The flow chart and its program written in FORTRAN are shown in **Fig. 1** and **Fig. 2**, respectively. One of the examples is shown in **Fig. 3**, where (a) is a case of the dual correspondence between the cube {4, 3} and the octahedron {3, 4}, (b) is a case of the truncation in the cube making a new semiregular polyhedron named the cuboctahedron, and (c) is a case of also the truncation in the tetrahedron making another semiregular solid.

This algorithm can be applicable to the development of the solids. When the outer shape of the solid and that of the development are given as the input figure ***A*** and ***B*** respectively, the automatic **developing** or **assembling** processes can be simulated by the same plotting system. The processes of developping and/or assembling for the cuboctahedron and dodecahedron are shown in **Fig. 4.**

The method can be applicable to arbitrary solids. For one of the examples, a crystallization is automatically developed and/or assembled as shown in **Fig. 5.**



(a) the Cuboctahedron and (b) the Dodecahedron.



and/or *Assembling* for a Cyrstallization.